Consumption Risk Sharing and Exchange Rates with Endogenously Segmented Asset Markets

Xuedong Wang*

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Abstract

In standard international business cycle models with complete financial markets, the correlation between the real exchange rate growth and the consumption growth ratio should be unity. However, in the data, this correlation is usually negative. International risk sharing is much poorer in reality. This consumption-real exchange rate anomaly (also called the Backus-Smith puzzle) is one of the major puzzles in international macroeconomics. Empirical results show that the nominal exchange rate movements are the main source for the Backus-Smith puzzle. In this paper, I show an endogenous segmented asset markets model may solve the consumption-real exchange rate anomaly and reveal the role of the nominal exchange rate on international risk sharing. When the nominal exchange rate is fixed, international risk sharing improves in the simulated economy.

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*Xuedong Wang, Erasmus School of Economics, Erasmus University Rotterdam and Tinbergen Institute, P.O. Box: 1738, 3000 DR, Rotterdam, The Netherlands. E-mail address: xwang@ese.eur.nl, Tel:+31-10-4088957.

1 Introduction

With a stochastic exchange economy, Backus and Smith (1993) prove that if the period utility function is isoelastic, then along any equilibrium path, there is a monotone relation between the bilateral real exchange rate, ε_{ij}, and the consumption ratio, c_i/cj: if the real exchange rate is higher in one state than in another, then so is the consumption ratio. Taking the most popular form of isoelastic utility functions, $U(c) = c^{1-\sigma}/(1-\sigma)$, the result implies

$$\sigma \Delta \log(c_i/c_j) = \Delta \log(\varepsilon_{ij}) = \Delta \log \left(e_{ij} \frac{P_j}{P_i} \right), \tag{1.1}$$

where $c_{i/j}$ indicates the consumption of countries i and j. ε_{ij} is the real exchange rate between the two countries. e_{ij} is the nominal exchange rate and $p_{i/j}$ are the price levels of the two countries. Equation (1.1) shows the time-series cross-correlation between the growth rate of relative consumptions and the growth rate of relative prices should be unity for any pair of countries. The intuition for this result is straight forward, s-ince households are more likely to consume more when their consumption basket is relatively cheap. However, this is at odds with the data. Backus and Smith (1993) test the cross-correlations between consumption and the real exchange rate with the data of eight OECD countries. Their results show the correlations between the consumption ratio and the real exchange rate are negative. The empirical evidence showing low or negative correlations between the consumption ratio and the real exchange rate is also confirmed by Lewis (1996), Chari, Kehoe, and McGrattan (2002), and Devereux and Hnatkovska (2009) *et al*. This contradiction between theory and empirical results is the famous consumption-real exchange rate anomaly (also called the Backus-Smith puzzle), one of the central puzzles in international macroeconomics (Obstfeld and Rogoff (2001)). Chari, Kehoe, and McGrattan (2002) test the correlations between bilateral real exchange rates and bilateral relative consumption for five countries. They report that the correlations vary between -0.48 and 0.24. Corsetti, Dedola, and Leduc (2008) report the annual real trade-weighted exchange rates between the United States and the other OECD countries move counter-cyclically with their consumption ratios.

More recent subtle analyses have brought some additional new insights into this anomaly. Using the data of OECD countries, Hess and Shin (2010) find that nominal exchange rate movements are the main source for the anomaly by decomposing the real exchange rate growth ($\Delta \log(\varepsilon_{ij})$) into its nominal exchange rate growth ($\Delta \log(e_{ij})$) and inflation differential ($\Delta \log(P_j/P_i)$) components, i.e. $\sigma\Delta \log(\varepsilon_{ij}) = \Delta \log(e_{ij}) + \Delta \log(P_j/P_i)$. According to their test, the average correlation between the consumption ratio growths ($\Delta \log(c_i/c_j)$) and the real exchange rate growths is -0.098. The average correlation between the consumption ratio growths and the nominal exchange rate growths is -0.174, while the average correlation between the consumption ratio growths and inflation differential is 0.163. Moreover, they use the consumption and price for the 50 U.S. states to explore the intranational evidence on risk sharing. Their evidence suggests that if the nominal exchange rate is constant, the Backus-Smith puzzle may disappear.

Hadzi-Vaskov (2008) gets similar results based on Eurozone data. He finds that there is a clear "dichotomy" between the results for the inflation differential and nominal exchange rate changes, and concludes that nominal exchange rate behavior appears crucial for understanding the consumption-real exchange rate anomaly. Hadzi-Vaskov (2008) therefore questions why is the nominal exchange rate negatively correlated to relative consumption and why does it behave so differently from the inflation differential? Devereux and Hnatkovska (2011) investigate the consumption-real exchange rate correlation with a newly constructed multi-country and multi-regional data set. The find significant evidence for within-country risk sharing. However, between countries the association is reversed. They find that the border effect is substantially, but not fully accounted for by the nominal exchange rate variability.

Table 1 shows the results of the empirical test adopting the method of Hess and Shin (2010) with the data of the U.S. and other 22 OECD counties. The results are similar to those of Hess and Shin (2010). The average consumption-real exchange rate correlation is -0.l35. The average consumption-nominal exchange rate correlation is -0.195, while the average consumption-inflation differential is 0.225. The appearance and disappearance of the Bretton Woods system provide us a natural experiment for exploring the role of the nominal exchange rate in the consumption-real exchange rate anomaly. I collect the related data of 4 Bretton Woods system members, the U.S., U.K., Australia, and South Africa, and compare the change of the correlations between the consumption ratio growths and the real/nominal exchange rate growths and inflation differential before and after the Bretton Woods system ended. Table 2 presents the results. The results show that the correlations between the consumption ratio growths and the real exchange rate growths become negative (or lower for the U.K.) after the Bretton Woods system ended. This change is clearly the result of the change in the nominal exchange rate variability after the Bretton Woods system ended.

2 Related literature

The counter-cyclical co-movements between bilateral consumption ratios and real exchange rates highlight the incompleteness of international financial risk sharing. One intuitive explanation for poor risk sharing is the existence of incomplete international financial markets. Following this idea, several models with incomplete financial markets are developed to address the consumption-real exchange rate anomaly. The successful cases include Corsetti, Dedola, and Leduc (2008) and Benigno and Thoenissen (2008), both of which allow the existence of only one non-contingent bond in the asset market. However, Benigno and Kucuk-Tuger (2008) show that quantitatively the correlation between consumption and real exchange rates is well above the one observed in the data once a second traded asset is allowed. Moreover, Levine and Zame (1998) show that the incompleteness of financial markets may not impede the risk sharing. Another branch tries to address the consumption-real exchange rate anomaly in the framework of complete financial markets. Bodenstein (2008) develops a complete asset-market model with limited enforcement for international financial contracts which can address the anomaly providing that agents are not too patient. Kollmann (2012) examines the limited asset market participation and shows this anomaly can be explained by a simple segmentation between households. That is between a subset of households trading in complete financial markets and the remaining households having hand-to-mouth existences. Cociuba and Ramanarayanan (2011) want to endogenize the limited participation with the model in Alvarez, Atkeson, and Kehoe (2002). However, they can only reduce the correlation slightly (0.62 vs. 1).

No matter whether they succeed or fail, the aforementioned research studies only consider the interactions among the real terms. None of the studies discussed above take the nominal terms into consideration. However, evidence from Hess and Shin

(2010), Hadzi-Vaskov (2008), Devereux and Hnatkovska (2011) and myself shows that the nominal exchange rate may be an important driving force behind the consumption-real exchange rate anomaly. If this is true, the reason why the nominal exchange rate moves counter-cyclically with the international consumption ratio is a question that we can not circumvent when we address the anomaly.

In this paper, I intend to reveal the role of exchange rate movements in international risk sharing. I seek to clarify how and to what extent the co-movements between the nominal exchange rate and consumption affect the incompleteness of international risk sharing and whether these should be reversed? The endogenously segmented asset markets model developed by Alvarez, Atkeson, and Kehoe (2009) is a promising framework that may help us to achieve this goal, because we can easily change the incompleteness of international risk sharing by changing the fraction of the households active in the financial markets, and we can easily change the variability of the exchange rate.

In this model, households face idiosyncratic real cost when they transfer money between the goods market and the asset market. The cost γ has a distribution $F(\gamma)$ with density $f(\gamma)$. At the beginning of each period, households receive the same real endowment y in the goods market. Inflation is distorting because it reduces the real endowment to be y/π. This effect induces some households to use the real resources to pay the transfer cost, therefore reducing the total amount of resources available for consumption. Whether households are active or inactive in the asset market depends on the costs households must pay and the level of inflation. The higher inflation is, the more households choose to be active, so the asset market is endogenously segmented. For each inflation, there is some cost level $\bar{\gamma}(\pi)$ at which the households with $\gamma \leq \bar{\gamma}(\pi)$ choose to be active and pay the cost, while all other households choose to be inactive and consume y/π. This cutoff rule is illustrated in Figure 1.

Domestic active households share the consumption risk with foreign active households via the international financial market, while the inactive households only consume their endowment and are inert in the financial market. They only have limited international risk sharing via the international trade. Therefore, international risk sharing is incomplete at the aggregate level. In this model, the correlations between the aggregate consumption and the nominal or real exchange rate depend on which fraction of the consumption (inactive households vs. active households) is dominant. If the consumption of inactive households is dominant, then we expect to observe negative correlations between the aggregate consumption and the nominal or real exchange rate.

I adapt this model for my research by introducing the monetary policy, non-tradable goods and by allowing international trade.

3 Model

3.1 Outline

I use a model that is an extension of the model in Alvarez, Atkeson, and Kehoe (2009) with trade in goods between two countries.

This is a cash-in-advance and endowment economy with an infinite number of periods, $t = 0, 1, 2....$ There are two countries, the domestic country i (or country 1) and the foreign country j (or country 2). Each country is populated by infinitely lived households. There are two separate markets, an asset market and an goods market. In the asset market, households trade two currencies and two complete sets of one-period state-contingent bonds issued by the two countries' governments respectively. At the beginning of each period, households in each country are endowed with a certain amount of tradable goods and non-tradable goods. The tradable goods of the two counties are imperfect substitutes, so there are four goods being traded in the goods market, but only the two tradable goods can be traded internationally. Each of the non-tradable goods can only be traded in one country.

Each household has to pay a cost γ for each transfer of cash between the asset market and the goods market. In each country, this transfer cost varies across households according to a distribution $F(\gamma_i)$ with a density $f(\gamma_i)$. The difference in the transfer cost separates households into active and inactive participants in the asset market.

The sources of uncertainty in the economy include the shocks to money growth and the state of the goods endowments. In each period, the money growth and the endowment of the country determine its inflation for that period. Households in each county enter period t with cash $P_{i,t-1}Y_{i,t-1}$ that they obtained from selling their endowments $Y_{i,t-1}$ in period $t-1$ at price $P_{i,t-1}$, so households enter the goods market in period t with real value $n = P_{i,t-1}Y_{i,t-1}/P_{i,t}$. Households can choose to be an active household by paying their transfer cost γ to transfer an amount of cash $P_{i,t}x$ with real value x to or from the asset market. Hence, the cash-in-advance constraint for active households is $c = n + x$, and for inactive households is $c = n$.

3.2 Setup and Equilibrium

The economy contains four goods: two internationally tradable goods and two non-tradable goods. Let $s^t = (s_1, ..., s_t)$ denote the history of aggregate events up to period t, and $g(s^t)$ denote the density of the probability distribution over such histories. Households in country i are endowed with $Y_i^T(s_t)$ units of tradable goods and $Y_i^N(s_t)$ units of non-tradable goods. Let $Y(s_t) = (Y_i^T, Y_j^T, Y_i^N, Y_j^N)$ is the endowment vector in state s_t. Let $M_i(s^t)$ is the money stock of country i in period t, and then $\mu_i(s^t) = M_i(s^t)/M_i(s^{t-1})$ denotes the corresponding money growth rate.

The final consumption goods c_i is the aggregate of the tradable and non-tradable goods:

$$c_i(s^t) = \left[\alpha_i c_i^T(s^t)^{\frac{\phi-1}{\phi}} + (1-\alpha_i)c_i^N(s^t)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}, \tag{3.1}$$

where c_i^T is the consumption index of the aggregate of the tradable goods and c_i^N is the consumption of the non-tradable goods in country i.

The aggregate tradable goods consumption index is determined by

$$c_i^T(s^t) = \left[\alpha_{ii} c_i^{Ti}(s^t)^{\frac{\theta-1}{\theta}} + \alpha_{ij}c_i^{Tj}(s^t)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \qquad \alpha_{ii} + \alpha_{ij} = 1, \tag{3.2}$$

where c_i^{Ti} and c_i^{Tj} denote country i's consumption of the tradable goods that originate in country i and j respectively. If $\alpha_{ii} > 0.5$, then there is a home-bias in consumption.

The price index for composite consumption purchases:

$$P_i(s^t) = \left[\alpha_i^{\phi}(P_i^T(s^t))^{1-\phi} + (1-\alpha_i)^{\phi}(P_i^N(s^t))^{1-\phi}\right]^{\frac{1}{1-\phi}}, \tag{3.3}$$

where P_i^T is the price index of the aggregate of the tradable goods and P_i^N is the price of the non-tradable goods in country i.

The aggregate tradable goods price index is determined by

$$P_i^T(s^t) = \left[\alpha_{ii}^{\theta}(P_i^{Ti}(s^t))^{1-\theta} + \alpha_{ij}^{\theta}(P_i^{Tj}(s^t))^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{3.4}$$

where P_i^{Ti} and P_i^{Tj} denote the price of tradable goods that originate in country i and j, but are consumed in country i.

By the law of one price, we have

$$P_i^{Tj}(s^t) = e_{i,j}P_j^{Tj}(s^t), \text{ and } P_j^{Ti}(s^t) = \frac{P_i^{Ti}}{e_{i,j}}(s^t), \tag{3.5}$$

where $e_{i,j}$ is the nominal exchange rate between country i and j.

In period 0, all households in both countries are identical. They have the same amount of endowment ($Y_i(s_0) = Y_j(s_0)$ with $Y_i^T(s_0) = Y_j^T(s_0)$ and $Y_i^N(s_0) = Y_j^N(s_0)$), the same amount of money stock ($M_i(s_0) = M_j(s_0)$) and the same amount of government bonds ($B_i(s_0) = B_j(s_0)$), so both countries have the same price level ($P_i(s_0) = P_j(s_0)$) and the nominal exchange rate $e_{i,j}(s_0)$ equals 1.

In each period, the government of each country issues one-period bonds contingent on the aggregate state s^t. In the case of country i, the government pays off outstanding bonds $B_i(s^t)$ with currency i and issues new bonds $B_i(s^t, s_{t+1})$ at price $q_i(s^t, s_{t+1})$. Hence, the country i government budget constraint at s^t with $t \geq 1$ is

$$B_i(s^t) = M_i(s^t) - M_i(s^{t-1}) + \int_{s_{t+1}} q_i(s^t, s_{t+1})B_i(s^t, s_{t+1})ds_{t+1} \tag{3.6}$$

In period 0, we have $B_i(s_0) = \int_{s_1} q_i(s_0, s_1)B_i(s_0, s_1)ds_1$, and $M_i(s_0)$ is given. No arbitrage between the bonds of country i and j implies that

$$q_i(s^t, s_{t+1}) = q_j(s^t, s_{t+1})\frac{e_{i,j}(s^t)}{e_{i,j}(s^{t+1})} \tag{3.7}$$

Households have to pay $\gamma(s^t)$ units of tradable goods as a price, if they want to transfer cash between the asset market and the goods market. The transfer cost varies across households in each country according to a distribution $F(\gamma(s^t))$ with density $f(\gamma(s^t))$. I consider households that transfer money between the two markets as active households, otherwise they are considered inactive households. I denote B_i^i as the bonds issued by the government of country i and owned by households of country i, and B_j^i as the bonds issued by the government of country j but owned by households of country i. In the case of country i, in any period t with $t \geq 1$, households are subject to such a budget constraint in the asset market:

$$B_i^i(s^t) + e(s^t)B_j^i(s^t) = \int_{s_{t+1}} \left[q_i(s^t, s_{t+1})B_i^i(s^t, s_{t+1}) + e(s^t)q_j(s^t, s_{t+1})B_j^i(s^t, s_{t+1})\right] ds_{t+1} \tag{3.8}$$

$$+ z_i(s^t, \gamma_i(s^t)) \left[P_i(s^t)x(s^t, \gamma_i(s^t)) + P_i^T(s^t)\gamma_i(s^t)\right]$$

where $x(s^t, \gamma_i(s^t))$ is the real value that households choose to transfer between the goods market and the asset market. $P_i(s^t)x(s^t, \gamma_i(s^t))$ is the nominal value of the transfer. A positive value for x means that households have transfered money out of the asset market, otherwise, x is negative. The indicator variable $z(s^t, \gamma_i(s^t))$ is equal to zero if $x(s^t, \gamma_i(s^t))$ is equal to zero, otherwise, $z(s^t, \gamma_i(s^t))$ is equal to one.

In period 0, the asset market budget constraint for households is given by

$$B_i^i(s_0) + e(s_0)B_j^i(s_0) = \int_{s_1} \left[q_i(s_0, s_1)B_i^i(s_0, s_1) + e(s^t)q_j(s_0, s_1)B_j^i(s_0, s_1)\right] ds_1 \tag{3.9}$$

Household consumption is subject to the cash-in-advance constraint. The nominal budget constraint for the goods market is

$$P_i(s^t)c_i(s^t, \gamma_i(s^t)) \leq P_i^{Ti}(s^{t-1})Y_{i,t-1}^T + P_i^N(s^{t-1})Y_{i,t-1}^N + z_i(s^t, \gamma_i(s^t))P_i(s^t)x(s^t, \gamma_i(s^t)), \tag{3.10}$$

The nominal resource constraint is given by

$$\int \left[P_i(s^t)c_i(s^t, \gamma_i(s^t)) + z_i(s^t, \gamma_i(s^t))P_i^T(s^t)\gamma_i(s^t)\right] f_i(\gamma_i(s^t))d\gamma_i(s^t) = P_i^{Ti}(s^t)Y_{i,t}^T + P_i^N(s^t)Y_{i,t}^N \tag{3.11}$$

Since the asset market is complete, the competitive equilibrium allocation and asset prices can be determined from the solution to the following planning problem:

$$\max \sum_{t=1}^{\infty} \beta^t \int_{s^t} \int_{\gamma_i(s^t)} \left[U\left(c_i(s^t, \gamma_i(s^t))\right)\right] f(\gamma_i(s^t))g(s^t)d\gamma_i(s^t)ds^t \tag{3.12}$$

subject to the resource constraint (3.11) and the following additional constraint

$$c_i(s^t, \gamma_i(s^t)) = z(s^t, \gamma_i(s^t))c_{iA}(s^t, \gamma_i(s^t)) + \left[1 - z(s^t, \gamma_i(s^t))\right] c_{i\bar{A}}(s^t) \tag{3.13}$$

where c_{iA} denotes the consumption of active households and $c_{i\bar{A}}$ denotes the consumption of inactive households. The budget constraint (3.10) indicates that the consumption of inactive households is independent of $\gamma_i(s^t)$ and is equal to

$$c_{i\bar{A}}(s^t) = \frac{P_i^{Ti}(s^{t-1})Y_{i,t-1}^T + P_i^N(s^{t-1})Y_{i,t-1}^N}{P_i(s^t)} \tag{3.14}$$

The first-order condition for the active household consumption c_{iA} is reduced to

$$\beta^t U' \left[c_{iA}(s^t, \gamma_i(s^t))\right] g(s^t) = \lambda_i(s^t)P_i(s^t) \tag{3.15}$$

$\lambda_i(s^t)$ is the multiplier on the resource constraint of country i. This first-order condition clearly implies that all households that pay the fixed cost choose the same consumption level, which means that $c_{iA}(s^t, \gamma_i(s^t))$ is independent of $\gamma_i(s^t)$. So the active household consumption can be denoted as $c_{iA}(s^t)$.

Combining this first order condition for both the home and the foreign counties, we get the international risk sharing condition:

$$\frac{\lambda_i(s^t)P_i(s^t)}{\lambda_j(s^t)P_j(s^t)} = \frac{U_i'(c_{iA}(s^t))}{U_j'(c_{jA}(s^t))} \tag{3.16}$$

Now, we know there are two consumption styles in each country, the active household consumption $c_A(s^t)$ and the inactive household consumption $c_{\bar{A}}(s^t)$. Both of them are independent of the transfer cost $\gamma(s^t)$. Given the state, the only factor that determines households being active or inactive is the level of their transfer cost. So it is clear that for any state, there is a threshold transfer cost $\bar{\gamma}(s^t)$ at which the households with $\gamma(s^t) \leq \bar{\gamma}(s^t)$ pay the cost and consume $c_A(s^t)$. All the households with $\gamma(s^t) > \bar{\gamma}(s^t)$ choose to be inactive in the asset market and consume $c_{\bar{A}}(s^t)$.

The planner's problem reduces to

$$\max_{c_{iA}(s^t), \bar{\gamma}_i(s^t)} U(c_{iA}(s^t))F(\bar{\gamma}_i(s^t)) + U(c_{i\bar{A}}(s^t))[1 - F(\bar{\gamma}_i(s^t))] \tag{3.17}$$

subject to

$$P_i(s^t)c_{iA}(s^t)F(\bar{\gamma}_i(s^t)) + \int_0^{\bar{\gamma}_i(s^t)} P_i^{Ti}(s^t)\gamma_i(s^t)f(\gamma_i(s^t))d\gamma_i(s^t) + P_i(s^t)c_{i\bar{A}}(s^t)[1 - F(\bar{\gamma}_i(s^t))] \tag{3.18}$$

$$=P_i^{Ti}(s^t)Y_{i,t}^T + P_i^N(s^t)Y_{i,t}^N$$

The first order condition is

$$U(c_{iA}(s^t)) - U\left(c_{i\bar{A}}(s^t)\right) - \frac{U'(c_{iA}(s^t))}{P_i(s^t)}\left[P_i(s^t)c_{iA}(s^t) + P_i^{Ti}(s^t)\bar{\gamma}_i(s^t) - P_i(s^t)c_{i\bar{A}}(s^t)\right] = 0 \tag{3.19}$$

As in Alvarez, Atkeson, and Kehoe (2009) claim: "In the decentralized economy corresponding to the planning problem, asset prices are given by the multipliers on the resource constraints for the planning problem." According to (3.15), these multipliers are equal to the marginal utility of active households.

Hence, the pricing kernels for the domestic and the foreign country are

$$m_i(s^t, s_{t+1}) = \beta\frac{U'\left(c_{iA}(s^{t+1})\right)}{U'(c_{iA}(s^t))}\frac{1}{\pi_{i,t+1}} \tag{3.20}$$

$$m_j(s^t, s_{t+1}) = \beta\frac{U'\left(c_{jA}(s^{t+1})\right)}{U'(c_{jA}(s^t))}\frac{1}{\pi_{j,t+1}} \tag{3.21}$$

In complete asset markets, the pricing kernels for domestic and foreign assets are related:

$$m_{j,t+1} = m_{i,t+1}\frac{e_{t+1}}{e_t} \tag{3.22}$$

I assume that two counties are identical in period 0, so $\lambda_i(s_0)$ equals $\lambda_j(s_0)$ and $e(s_0)$ equals 1. Given such an assumption, combining equations (3.16), (3.20), (3.21), and (3.22), we can solve the international risk sharing condition recursively:

$$\frac{U_i'(c_{iA}(s^t))}{U_j'(c_{jA}(s^t))} = \frac{P_i(s^t)}{e(s^t)P_j(s^t)} \tag{3.23}$$

Goods market-clearing conditions are

$$c_{1A}^{T1}F(\bar{\gamma}_1) + \int_0^{\bar{\gamma}_1} \gamma_1 f(\gamma_1)d\gamma + c_{1\bar{A}}^{T1}[1 - F(\bar{\gamma}_1)] + c_{2A}^{T1}F(\bar{\gamma}_2) + c_{2\bar{A}}^{T1}[1 - F(\bar{\gamma}_2)] = Y_1^T \tag{3.24}$$

$$c_{2A}^{T2}F(\bar{\gamma}_2) + \int_0^{\bar{\gamma}_2} \gamma_2 f(\gamma_2)d\gamma + c_{2\bar{A}}^{T2}[1 - F(\bar{\gamma}_2)] + c_{1A}^{T2}F(\bar{\gamma}_1) + c_{1\bar{A}}^{T2}[1 - F(\bar{\gamma}_1)] = Y_2^T \quad (3.25)$$

$$c_{1AN}(s^t)F(\bar{\gamma}_1(s^t)) + c_{1\bar{A}N}(s^t)[1 - F(\bar{\gamma}_1(s^t))] = Y_{1,t}^N \quad (3.26)$$

$$c_{2AN}(s^t)F(\bar{\gamma}_2(s^t)) + c_{2\bar{A}N}(s^t)[1 - F(\bar{\gamma}_{2,t}(s^t))] = Y_{2,t}^N \quad (3.27)$$

Money market-clearing conditions are

$$P_1^{T1}(s^t)Y_{1,t}^T + P_1^N(s^t)Y_{1,t}^N = M_1(s^t) \quad (3.28)$$

$$P_2^{T2}(s^t)Y_{2,t}^T + P_2^N(s^t)Y_{2,t}^N = M_2(s^t) \quad (3.29)$$

For each period, I solve the three market-clearing conditions and the risk-sharing condition (3.23) and the first order condition (3.19) for the active and inactive consumption, the threshold transfer cost and the prices for tradable and non-tradable goods.

3.3 Model Implications

With the assumption of a CRRA utility, and taking log for both sides of the equation (3.23), we get:

$$\begin{aligned} \sigma \log(c_{Ai}/c_{Aj}) &= \log(\varepsilon_{i,j}) \\ &= \log(e_{i,j}) + \log(P_j/P_i) \end{aligned} \quad (3.30)$$

Equation (3.30) indicates that the log of the consumption ratio between the domestic and foreign active households correlates perfectly with the log of the real exchange rate, i.e. $\rho[\log(c_{Ai}/c_{Aj}), \log(\varepsilon_{i,j})] = 1$. Therefore, the active households have no contribution to the consumption-real exchange rate anomaly. Proposition 3 of Alvarez, Atkeson, and Kehoe (2009) shows that the log of the consumption of active households is strictly increasing in the log of inflation, when inflation is around 1. According to the proposition, the log of the consumption ratio of active households should negatively correlate with the log of the price ratio, i.e. $\rho[\log(c_{Ai}/c_{Aj}), \log(P_j/P_i)] < 0$. If this is true, we can immediately deduce that $\rho[\log(c_{Ai}/c_{Aj}), \log(e_{i,j})] > 0$ and $\rho[\log(e_{i,j}), \log(P_j/P_i)] < 0$, based on $\rho[\log(c_{Ai}/c_{Aj}), \log(\varepsilon_{i,j})] = 1$. It is not easy to obtain the data for the consumption of active households in each country, so it is difficult to check the value of $\rho[\log(c_{Ai}/c_{Aj}), \log(P_j/P_i)]$ and $\rho[\log(c_{Ai}/c_{Aj}), \log(e_{i,j})]$ empirically. However, we can use the data of the nominal exchange rate and the price ratio to check whether the correlation between the log of the nominal exchange rate and the log of the price ratio in the real world is negative or not. With the U.S. as country j, the first column of Table 3 shows the correlation between the log of the nominal exchange rate and the log of the price ratio for some of the OECD countries. We can see the correlations for all of the countries are negative. Equations (3.4) and (3.5) show us why the nominal exchange rate and the price ratio should be negatively correlated. When substituting equation (3.5) into equation (3.4), we can see that, given P_i^{Ti} and P_j^{Tj}, the higher the nominal exchange rate, then the lower is the tradable goods price ratio (P_j^T/P_i^T), so that given the non-tradable goods price (P_i^N and P_j^N), the higher the nominal exchange rate, then the lower is the price ratio (P_j/P_i). Hence, the nominal exchange rate negatively correlates with the price ratio.

The correlation between the consumption ratio of the inactive households ($\log(c_{\bar{A}i}/c_{\bar{A}j})$) and the real exchange rate is revealed by equation (3.14), (3.4) and (3.5). According to equation (3.14), the consumption of inactive households is proportional to their nominal endowment ($P_i^{Ti}(s^{t-1})Y_{i,t-1}^T + P_i^N(s^{t-1})Y_{i,t-1}^N$), but inversely proportional to the price level ($P_i(s^t)$), so we get $\rho[\log(c_{\bar{A}i}/c_{\bar{A}j}), \log(P_j/P_i)] > 0$. Since the nominal exchange rate negatively correlates with the price ratio, we immediately get $\rho[\log(c_{\bar{A}i}/c_{\bar{A}j}), \log(e_{i,j})] < 0$.

The model indicates clearly that the consumption of the inactive households decrease with inflation, but increase with the appreciation of the nominal exchange rate since it improve the terms of trade. When inflation is moderate, the consumption of the active households increase with inflation, but decrease with the appreciation of the nominal exchange rate since it can reduce inflation. The correlations between the aggregate consumption ratio and the real exchange rate and the price ratio and the nominal exchange rate are the aggregate results of these two types of consumption. If the consumption of the inactive households dominate the consumption of the active households, then the correlation between the aggregate consumption ratio and the price ratio is positive, but the correlation between the aggregate consumption ratio and the nominal exchange rate is negative, and vice versa. Now the causality between international risk sharing and the nominal exchange rate has been revealed. The existence of inactive households reduces international risk sharing for aggregate consumption and causes the negative correlation between the aggregate consumption and the nominal exchange rate. However, the variability of the nominal exchange rate can in turn influence international risk sharing by changing inflation. Therefore, when variability of the nominal exchange rate is removed, international risk sharing may improve.

We will see more details with the simulated economy in the next section.

4 Calibration and results

4.1 Calibration

Table (4) shows the main parameter values. Preference is represented by the constant relative risk aversion preference of the form $U(c) = c^{1-\sigma}/(1-\sigma)$. The coefficient of relative risk aversion σ is set to 2. The discount factor β is set to 0.95. The elasticity of the substitution between tradable and non-tradable goods, ϕ, is set to 0.44, and the elasticity of the substitution between home and foreign-originated tradable goods, θ, is set to 2, following the suggestion of Stockman and Tesar (1995). In line with the calibration of Corsetti, Dedola, and Leduc (2008) and Benigno and Thoenissen (2008), the value of tradable goods share in the final consumption goods, α, is set to 0.55, while the weight of the domestic tradable goods, α_{ii}, is set to 0.72.

The state for the household endowments of the two countries follows a Markov chain. The endowment vector and the transition matrix are taken from Bodenstein (2008), who calibrates the parameters based on the GDP of G7 countries.

The calibration of the transfer cost γ is not easy, because it cannot be directly observed. I have chosen the level of the transfer cost based on the fraction of the active and inactive households we observe in the real world. Based on the U.S. data, empirical results of Mankiw and Zeldes (1991), Haliassos and Bertau (1995), Vissing-Jrgensen (2002), and Christelis and Georgarakos (2011) show that most U.S. households do not hold stocks. The fraction of households that possess stocks in United States is around 25%. However, Bucks, Kennickell, Mach, and Moore (2009) in a later study show that about 50% of U.S. households own stock directly or indirectly. Moreover, Bonaparte and Cooper (2009) find that only 8.6% of households owning stocks actually adjust their portfolio of common stocks on a monthly basis and the fraction is no more than

71.0% on an annual basis. Based on the above empirical results, I have chosen the level of the transfer cost so that the active households are no more than 50% of the total number of households. I assume γ has a uniform distribution with 0 as the lower bound, so the fraction of the active households depends on the largest value of the transfer cost. The greater the value is of the transfer cost, then the lower the fraction is of active households.

I have chosen for inflation the annual mean, $\bar{\pi}$, as 3%. The inflation target has a volatility of 0.0115. This is based on the empirical results of de Vries and Wang (2013). I assume that the governments of the two countries make their decisions regarding each period's money stock using the following rule: at the beginning of each period, the governments observe the realization of the endowment $Y_{i,t}$ and $Y_{j,t}$, and then they decide the money stock of this period based on the endowment and the target inflation. So the period t money stock can be summarized in the following equation:

$$M_i(s^t) = \left[p_i^{Ti}(s^{t-1})Y_{i,t}^T + p_i^N(s^{t-1})Y_{i,t}^N\right](\bar{\pi} + \varepsilon_t) \tag{4.1}$$

$$M_j(s^t) = \left[p_j^{Tj}(s^{t-1})Y_{j,t}^T + p_j^N(s^{t-1})Y_{j,t}^N\right](\bar{\pi} + \varepsilon_t) \tag{4.2}$$

4.2 Numerical results

The economy is simulated 50 times over 300 periods for each set of transfer cost. Table 5 shows the main results of the economy. The reported numbers are the average over the 50 simulations. The results are highly consistent with my analysis and prediction in Section 3.3. The fraction of active households continues to shrink with the increase of the transfer cost. The model succeeds in generating the negative correlations between the consumption ratio and the real exchange rate. More importantly, it generates negative correlations between the consumption ratio and the nominal exchange rate and it generates positive correlations between the consumption ratio and the price ratio. This is fully consistent with the empirical results of Hess and Shin (2010) and Hadzi-Vaskov (2008) and myself (Table 1). Therefore, the extension of the segmented asset markets model from Alvarez, Atkeson, and Kehoe (2009) helps to solve the Backus-Smith puzzle at both the real and the nominal level. As the model predicts, the negative correlation between the consumption ratio and the real and nominal exchange rates are caused by the inactive households. So these correlations become more and more negative as the fraction of the inactive households increases. It is reasonable to think that the difference in the fraction of the active and inactive households in different countries is one of the reason for their difference in the consumption-exchange rate correlation. The model also predicts the negative correlation between the price ratio and the active-household consumption, and the negative correlation between the nominal exchange rate and the price ratio. The later has been confirmed by my empirical test with the data of OECD countries, which is shown in the first column of Table 3.

In the standard business cycle models with a representative agent and complete markets, the cross-country correlation for consumption growth rate is usually much higher than the data. Backus, Kehoe, and Kydlan (1992) report that the correlations between Japan and the major European countries lie between 0.22 and 0.48. The correlation between the U.S. and the European aggregate is 0.46. Brandt, Cochrane, and Santa-Clara (2006) test the annual consumption growth correlations between the U.S. and three other countries, the U.K., Germany and Japan. They report correlations ranging from 0.24 to 0.42. I test the annual correlations for consumption growth between the U.S. and some OECD counties. The results are shown in the second column of Table 3. The correlations range from -0.376 (Sweden vs. U.S.) to 0.605 (Canada vs. U.S.) with an average of 0.289. Standard business cycle models usually predict a very high correlation. In my model, the cross country correlation for consumption growth is 0.820 when there is no limited participation ($\gamma = 0$).[1] However, the cross-country consumption correlation is reduced to the level of the data with the presence of inactive households. The results clearly show that the correlations continue to decrease and can even be negative as the fraction of inactive households increases.

The model clearly demonstrates the mechanisms that influence how the nominal exchange rate affects international risk sharing. The negative correlation between the aggregate consumption ratio and the nominal exchange rate is due to the negative correlation between the nominal exchange rate and inactive households consumption, while the reason for their negative correlation is that the nominal exchange rate may

[1]When there is no transfer cost between the goods market and the financial market, the model degenerates to the standard international business model with a representative agent who is active in the financial market.

dampen the effect of inflation on the consumption of the inactive households by changing the terms of trade. The appreciation of the nominal exchange rate may increase the consumption of the inactive households. Therefore, the effect of the nominal exchange rate on international risk sharing depends on the existence of the inactive households in the economy. The first column of Table 5 shows that the correlation between the consumption ratio and the real exchange rate is unity and the correlation between the consumption ratio and the nominal exchange rate is no longer negative when there are no inactive households in the economy.

In order to explore more details regarding the causality between the nominal exchange rate and international risk sharing, I will now assume that the government of one country decides to fix the exchange rate of its currency to that of another country in order for us to see how the results might change when the variability of the nominal exchange rate is removed.

According to the classic open-economy trilemma, a country cannot simultaneously achieve exchange rate stability, capital market openness, and monetary policy autonomy. Therefore, any country that wants to peg their currency to the currency of another country has to make a trade-off between the freedom of financial flows and monetary sovereignty. For example, within the Euro area, all the Member States have adopted the Euro as their common currency, i.e. a fixed 1:1 exchange rate, and all have open capital markets, so they do not have monetary sovereignty. They have to follow the edicts of the European Central Bank. In contrast to the Euro area, the Chinese government has chosen to maintain their monetary sovereignty although they peg the Renminbi to the U.S. dollar.[2] As a result, the Chinese financial market is closed to foreign capital.

In my simulated economies under fixed exchange rate (e=1), both conditions are considered, i.e. whether the international financial market is either open or closed. Each economy is simulated with or without transfer costs. When there is a transfer cost, I assume the transfer cost has a uniform distribution with the largest value of 2% of Y^T. Table 6 shows the main results of the fixed exchange rate economies. The results under the flexible exchange rate are labeled as "Flexible Ex.", the results under the

[2]From 1995 to 2005, the Chinese government pegged the Renminbi to the U.S. dollar with a target exchange rate of 8.28 RMB for each US$ and only allowed variations in the exchange rate within a very narrow margin. After 2005, the Chinese government announced the reform of the exchange rate. They have allowed the RMB to appreciate gradually and have loosened the constraints on the exchange rate variation.

fixed exchange rate with an open international financial market are labeled as "Fixed Ex. & Open", and the results under the fixed exchange rate with a closed international financial market are labeled as "Fixed Ex. & Closed". The results show that when the exchange rate is fixed, the fractions of the active households is larger than that of the flexible exchange rate under the same inflation and transfer cost. This is because when the exchange rate is fixed, the dilution effect of inflation on the consumption of the inactive households is more significant, so more households are willing to pay the transfer cost and become active households. When the exchange rate are fixed and there are open international financial markets, the correlation between the aggregate consumption and the real exchange rate is positive (0.488), while this correlation is negative when the exchange rate is flexible (-0.534). Moreover, the aggregate consumption growth correlation between the two counties increase to 0.758 from 0.356 under the flexible exchange rate. It is clear that the change in the exchange rate regime mainly affects the inactive households. The correlation between the inactive household consumption and the real exchange rate increase to 0.929 under the flexed exchange rate and open international financial markets from -0.586 under the flexible exchange rate. The consumption growth correlation for active households also increase a little, from 0.931 to 0.986. When there are only active households in the economy, this correlation increase from 0.804 to 0.828. Therefore, we see the elimination of the variability in the exchange rate may increase the consumption growth correlation for both the active and inactive households. However, the effect is more significant on inactive households than on active households.

The results also indicate that the freedom of international financial flows are also necessary for international risk sharing. If the international financial markets are closed, then the consumption-real exchange rate correlation is -0.550 when all the households are active and -0.442 when some of the households are inactive, and the consumption correlations are both -0.075. Specifically, the consumption-real exchange rate correlation for inactive households increases to 0.367 from -0.586 and their consumption growth correlation increases to 0.007 from -0.152, when compare with the results under the flexible exchange rate. However, the block of the international financial flows is a disaster for international risk sharing of active households. Comparing with the results under the flexible exchange rate, the consumption-real exchange rate correlation for active households decreases to -0.333 from 1.00 and their consumption growth correlation decrease to 0.015 from 0.931. As a result, international risk sharing is poor when we look at the aggregate household consumption once international financial flows are blocked.

China is the only main economy in the world that does not open its financial market to the foreign capital. Lets use the example of China to see what is the effect of blocking in international financial flows on international risk sharing in reality. Table 7 shows the correlations between China and several main economies in the world, Australian, Japan, the U.K. and U.S. We can see not only the real and the nominal exchange rates have negative correlations with the consumption, but the price ratios are also negatively correlated with consumption. China's consumption growth is negatively correlated with the consumption growths of all the countries in the table except Japan. The consumption growth correlation between China and Japan is 0.062, almost 0. The empirical results for China are consistent with the model results for a closed economy.

Besides looking at the correlations, I also use the international risk sharing index developed by Brandt, Cochrane, and Santa-Clara (2006) to measure the level of international risk sharing. This index is labeled as the BCSW index in Table 6.[3] The value of this index lies between 1 and -1, with 1 indicating perfect international risk sharing and 0 indicating the two countries' marginal utility growths that are uncorrelated. If the index is -1, then the marginal utility growth of one country is just the opposite of the other. In other words, the higher the index value is, then the higher is the level of international risk sharing.

[3]The original definition for the index of international risk sharing in Brandt, Cochrane, and Santa-Clara (2006) is:

$$1 - \frac{var(\log m_{j,t+1} - \log m_{i,t+1})}{var(\log m_{j,t+1}) - var(\log m_{i,t+1})} = 1 - \frac{var\left(\log \frac{e_{t+1}}{e_t}\right)}{var(\log m_{j,t+1}) - var(\log m_{i,t+1})}.$$

I have some disagreement with this definition. In an open economy with variability in the exchange rate, full international risk sharing is equivalent to the condition of $m_{j,t+1} = m_{i,t+1}\frac{e_{t+1}}{e_t}$ not $m_{j,t+1} = m_{i,t+1}$. However, when we calculate the index for international risk sharing following the definition of Brandt, Cochrane, and Santa-Clara (2006), we can not get the highest value of 1 even if the full international risk sharing condition is satisfied as long as there is variability in the exchange rate. Therefore, I make the following amendment to the definition for the index of international risk sharing:

$$1 - \frac{var(\log m_{j,t+1} - \log m_{i,t+1} - \log \frac{e_{t+1}}{e_t})}{var(\log m_{j,t+1}) - var(\log m_{i,t+1})}.$$

I refer to the new index as the BCSW index. This amendment is particularly important for studies which are designed to investigate the effect of the exchange rate on international risk sharing, because the value of the new index is controlled for variability in the exchange rate.

From the results in Table 6, we see that when there is an open international financial market, then the index value for active households is 1 despite whether the exchange rate is fixed or flexible. The difference in the index values between the fixed and flexible exchange economies shows us that the adverse effects of the exchange rate movement on international risk sharing mainly affect the inactive households. The BCSW index value is -0.202 for inactive households when the exchange rate is flexible, while the index value is 0.848 which causes the index values for all households to increase from 0.155 to 0.802. We see again the importance of openness in international financial markets. When there are no financial flows between the two countries, international risk sharing is almost 0 for the active households of the two countries (BCSW index: -0.018). And while, removing the exchange rate variability may slightly improve international risk sharing for inactive households (-0.007 vs. -0.202), this effect is too small however to prevent aggregate international risk sharing from slipping into the negative (-0.153 vs. 0.155).

5 Conclusions

Hess and Shin (2010) show that the nominal exchange rate variability is the main source of the consumption-real exchange rate anomaly. This conclusion is confirmed by my empirical tests. Moreover, the change of the correlations for the consumption and real exchange rate anomaly before and after the ending of the Bretton Woods system (the fixed exchange rate regime) provides new evidence that it is the nominal exchange rate fluctuation, not the price ratio, that causes the consumption-real exchange rate anomaly.

In this paper, I try to explore the mechanisms which underpin the co-movement between consumption, the nominal exchange rate and inflation within the framework of limited participation in the asset markets. By introducing non-tradable goods, international trade and home bias into the endogenously segmented asset markets model developed by Alvarez, Atkeson, and Kehoe (2009), I show that this framework can solve the consumption-real exchange rate anomaly. Just with standard calibration, it generates correlations between the relative consumption and real exchange rate, nominal exchange rate and the price ratio close to the data under either the flexible or fixed exchange rate regime. The model shows that the existence of inactive households in the financial markets is a cause of the poor international risk sharing we have observed. Furthermore, it reveals how the nominal exchange rate influences international risk sharing by affecting the consumption of inactive households.

The model implies the negative correlations between the nominal exchange rate and the price ratio, which is confirmed by the data of OECD counties. The presence of inactive households also reduces the cross-country correlations of consumption growth to the level of the data. The role of inactive households in reducing the cross-country consumption correlations are indirectly confirmed by Zhang (2013), who uses micro-level household consumption data in the U.S. and U.K. and shows that the stockholders' consumption correlation is considerably higher than that of the aggregate consumption growth.

APPENDIX

A variables and equations

A.1 Exogenous variables

M_1: Money supply in domestic country;

M_2: Money supply in foreign country;

Y_1^T: Tradable good endowment in domestic country;

Y_1^N: Non-tradable good endowment in domestic country;

Y_2^T: Tradable good endowment in foreign country;

Y_2^N: Non-tradable good endowment in foreign country;

$e(s_0) = 1$: The nominal exchange rate at period 0;

$P_1^T(s_0) = 1$: The tradable goods price in domestic country at period 0;

$P_2^T(s_0) = 1$: The tradable goods price in foreign country at period 0;

A.2 Endogenous variables need to be solved

1. c_{1A}^T: Tradable good consumption of active households in domestic country;
2. P_1^T: Tradable good price of domestic country;
3. P_1: Price level of domestic country;
4. c_{1A}: Aggregate consumption of active households in domestic country;
5. c_{1A}^N: Non-tradable good consumption of active households in domestic country;
6. P_1^N: Non-tradable good price of domestic country;
7. $c_{1\bar{A}}^T$: Tradable good consumption of inactive households in domestic country;
8. $c_{1\bar{A}}$: Aggregate consumption of inactive households in domestic country;
9. $c_{1\bar{A}}^N$: Non-tradable good consumption of inactive households in domestic country;
10. c_{2A}^T: Tradable good consumption of active households in foreign country;
11. P_2^T: Tradable good price of foreign country;
12. P_2: Price level of foreign country;
13. c_{2A}: Aggregate consumption of active households in foreign country;
14. c_{2A}^N: Non-tradable good consumption of active households in foreign country;
15. P_2^N: Non-tradable good price of foreign country;
16. $c_{2\bar{A}}^T$: Tradable good consumption of inactive households in foreign country;

17. $c_{2\bar{A}}$: Aggregate consumption of inactive households in foreign country;

18. $c^N_{2\bar{A}}$: Non-tradable good consumption of inactive households in foreign country;

19. e: Exchange rate between domestic and foreign currencies;

20. $\bar{\gamma}_1$: Cutoff level of transfer cost in domestic country;

21. $\bar{\gamma}_2$: Cutoff level of transfer cost in foreign country;

22. $c^{T1}_{1\bar{A}}$: Tradable good originated in domestic country and consumed by inactive households in domestic country;

23. $c^{T2}_{1\bar{A}}$: Tradable good originated in foreign country but consumed by inactive households in domestic country;

24. $c^{T2}_{2\bar{A}}$: Tradable good originated in foreign country and consumed by inactive households in foreign country;

25. $c^{T1}_{2\bar{A}}$: Tradable good originated in domestic country but consumed by inactive households in foreign country;

26. P^{T1}_1: The price of tradable good originated in domestic country and consumed in domestic country;

27. P^{T2}_1: The price of tradable good originated in foreign country but consumed in domestic country;

28. P^{T2}_2: The price of tradable good originated in foreign country and consumed in foreign country;

29. P^{T1}_2: The price of tradable good originated in domestic country but consumed in foreign country;

30. c^{T1}_{1A}: Tradable good originated in domestic country and consumed by active households in domestic country;

31. c^{T2}_{1A}: Tradable good originated in foreign country but consumed by active households in domestic country;

32. c^{T2}_{2A}: Tradable good originated in foreign country and consumed by active households in foreign country;

33. c^{T1}_{2A}: Tradable good originated in domestic country but consumed by active households in foreign country;

34. c_1: Total consumption in domestic country;

35. c_2: Total consumption in foreign country;

A.3 Model equations

$$c_{1A}^{T} = \left(\frac{\alpha}{P_1^T} P_1\right)^{\phi} c_{1A} \tag{A.1}$$

$$c_{1A}^{N} = \left(\frac{1-\alpha}{P_1^N} P_1\right)^{\phi} c_{1A} \tag{A.2}$$

$$c_{1\bar{A}}^{T} = \left(\frac{\alpha}{P_1^T} P_1\right)^{\phi} c_{1\bar{A}} \tag{A.3}$$

$$c_{1\bar{A}}^{N} = \left(\frac{1-\alpha}{P_1^N} P_1\right)^{\phi} c_{1\bar{A}} \tag{A.4}$$

$$c_{2A}^{T} = \left(\frac{\alpha}{P_2^T} P_2\right)^{\phi} c_{2A} \tag{A.5}$$

$$c_{2A}^{N} = \left(\frac{1-\alpha}{P_2^N} P_2\right)^{\phi} c_{2A} \tag{A.6}$$

$$c_{2\bar{A}}^{T} = \left(\frac{\alpha}{P_2^T} P_2\right)^{\phi} c_{2\bar{A}} \tag{A.7}$$

$$c_{2\bar{A}}^{N} = \left(\frac{1-\alpha}{P_2^N} P_2\right)^{\phi} c_{2\bar{A}} \tag{A.8}$$

$$c_{1A} = \left[\alpha (c_{1A}^{T})^{\frac{\phi-1}{\phi}} + (1-\alpha)(c_{1A}^{N})^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{A.9}$$

$$c_{2A} = \left[\alpha (c_{2A}^{T})^{\frac{\phi-1}{\phi}} + (1-\alpha)(c_{2A}^{N})^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{A.10}$$

$$c_{1A}^{T} = \left[\alpha_{11} (c_{1A}^{T1})^{\frac{\theta-1}{\theta}} + (1-\alpha 11)(c_{1A}^{T2})^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{A.11}$$

$$c_{2A}^{T} = \left[\alpha_{22} (c_{2A}^{T2})^{\frac{\theta-1}{\theta}} + (1-\alpha_{22})(c_{2A}^{T1})^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{A.12}$$

$$c_{1A}^{T1} = \left(\frac{\alpha_{11}}{P_1^{T1}} P_1^T\right)^{\theta} c_{1A}^{T} \tag{A.13}$$

$$c_{1A}^{T2} = \left(\frac{1-\alpha_{11}}{P_1^{T2}} P_1^T\right)^{\theta} c_{1A}^{T} \tag{A.14}$$

$$c_{2A}^{T2} = \left(\frac{\alpha_{22}}{P_2^{T2}} P_2^T\right)^{\theta} c_{2A}^{T} \tag{A.15}$$

$$c_{2A}^{T1} = \left(\frac{1-\alpha_{22}}{P_2^{T1}} P_2^T\right)^\theta c_{2A}^T \tag{A.16}$$

$$c_{1\bar{A}}^{T1} = \left(\frac{\alpha_{11}}{P_1^{T1}} P_1^T\right)^\theta c_{1\bar{A}}^T \tag{A.17}$$

$$c_{1\bar{A}}^{T2} = \left(\frac{1-\alpha_{11}}{P_1^{T2}} P_1^T\right)^\theta c_{1\bar{A}}^T \tag{A.18}$$

$$c_{2\bar{A}}^{T2} = \left(\frac{\alpha_{22}}{P_2^{T2}} P_2^T\right)^\theta c_{2\bar{A}}^T \tag{A.19}$$

$$c_{2\bar{A}}^{T1} = \left(\frac{1-\alpha_{22}}{P_2^{T1}} P_2^T\right)^\theta c_{2\bar{A}}^T \tag{A.20}$$

$$P_1^{T2} = eP_2^{T2} \tag{A.21}$$

$$P_2^{T1} = \frac{1}{e} \cdot P_1^{T1} \tag{A.22}$$

$$c_{1\bar{A}} = \frac{P_{1,-1}^{T1} Y_{1,-1}^T + P_{1,-1}^N Y_{1,-1}^N}{P_1} \tag{A.23}$$

$$c_{2\bar{A}} = \frac{P_{2,-1}^{T2} Y_{2,-1}^T + P_{2,-1}^N Y_{2,-1}^N}{P_2} \tag{A.24}$$

$$\frac{c_{1A}^{1-\sigma}}{1-\sigma} - \frac{c_{1\bar{A}}^{1-\sigma}}{1-\sigma} - \frac{c_{1A}^{-\sigma}}{P_1}(P_1 c_{1A} + P_1^{T1}\bar{\gamma}_1 - P_1 c_{1\bar{A}}) = 0 \tag{A.25}$$

$$\frac{c_{2A}^{1-\sigma}}{1-\sigma} - \frac{c_{2\bar{A}}^{1-\sigma}}{1-\sigma} - \frac{c_{2A}^{-\sigma}}{P_2}(P_2 c_{2A} + P_2^{T2}\bar{\gamma}_2 - P_2 c_{2\bar{A}}) = 0 \tag{A.26}$$

$$\frac{P_1}{eP_2} = \frac{c_{1A}^{-\sigma}}{c_{2A}^{-\sigma}} \tag{A.27}$$

$$c_{1A}^{T1} F(\bar{\gamma}_1) + \int_0^{\bar{\gamma}_1} \gamma_1 f(\gamma_1) d\gamma + c_{1\bar{A}}^{T1}[1 - F(\bar{\gamma}_1)] + c_{2A}^{T1} F(\bar{\gamma}_2) + c_{2\bar{A}}^{T1}[1 - F(\bar{\gamma}_2)] = Y_1^T \tag{A.28}$$

$$c_{2A}^{T2} F(\bar{\gamma}_2) + \int_0^{\bar{\gamma}_2} \gamma_2 f(\gamma_2) d\gamma + c_{2\bar{A}}^{T2}[1 - F(\bar{\gamma}_2)] + c_{1A}^{T2} F(\bar{\gamma}_1) + c_{1\bar{A}}^{T2}[1 - F(\bar{\gamma}_1)] = Y_2^T \tag{A.29}$$

$$c_{1A}^N F(\bar{\gamma}_1) + c_{1\bar{A}}^N [1 - F(\bar{\gamma}_1)] = Y_1^N \tag{A.30}$$

$$c_{2A}^N F(\bar{\gamma}_2) + c_{2\bar{A}}^N [1 - F(\bar{\gamma}_2)] = Y_2^N \tag{A.31}$$

$$P_{1,-1}^{T1} Y_1^T + P_{1,-1}^N Y_1^N = M_1 \tag{A.32}$$

$$P_{2,-1}^{T2} Y_2^T + P_{2,-1}^N Y_2^N = M_2 \tag{A.33}$$

$$c_1 = F_1(\bar{\gamma}_1) c_{1A} + (1 - F_1(\bar{\gamma}_1)) c_{1\bar{A}} \tag{A.34}$$

$$c_2 = F_2(\bar{\gamma}_2)c_{2A} + (1 - F_2(\bar{\gamma}_2))c_{2\bar{A}} \tag{A.35}$$

For each period, we have 35 equations to solve 35 variables. Given the state of the first period (period 0), we can solve these variables at any period numerically.

A.4 Model equations at period 0

$$c_1^T = Y_1^T \tag{A.36}$$

$$c_1^N = Y_1^N \tag{A.37}$$

$$c_2^T = Y_2^T \tag{A.38}$$

$$c_2^N = Y_2^N \tag{A.39}$$

$$c_1 = \left[\alpha_1(c_1^T)^{\frac{\phi-1}{\phi}} + (1-\alpha_1)(c_1^N)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}, \tag{A.40}$$

$$c_2 = \left[\alpha_2(c_2^T)^{\frac{\phi-1}{\phi}} + (1-\alpha_2)(c_2^N)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}, \tag{A.41}$$

$$c_1^T = (\alpha P_1)^{\phi} c_1 \tag{A.42}$$

$$c_1^N = \left(\frac{1-\alpha}{P_1^N} P_1\right)^{\phi} c_1 \tag{A.43}$$

$$c_2^T = (\alpha P_2)^{\phi} c_2 \tag{A.44}$$

$$c_2^N = \left(\frac{1-\alpha}{P_2^N} P_2\right)^{\phi} c_2 \tag{A.45}$$

$$P_1 = \left[\alpha^{\phi} + (1-\alpha)^{\phi}(P_1^N)^{1-\phi}\right]^{\frac{1}{1-\phi}} \tag{A.46}$$

$$P_2 = \left[\alpha^{\phi} + (1-\alpha)^{\phi}(P_2^N)^{1-\phi}\right]^{\frac{1}{1-\phi}} \tag{A.47}$$

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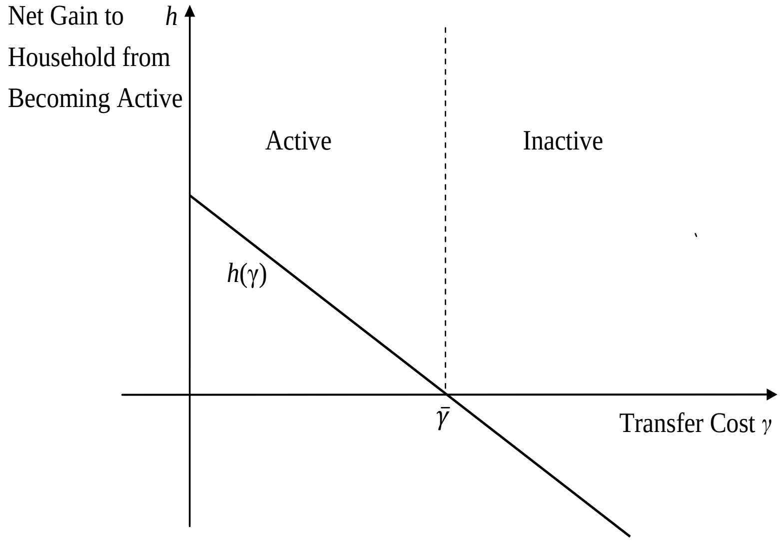
Table 1: Consumption-exchange rate correlation between USA and the OECD countries

Country	$\rho\left[\sim, \Delta\log(\varepsilon_{ij})\right]$	$\rho\left[\sim, \Delta\log(e_{ij})\right]$	$\rho\left[\sim, \Delta\log\left(\frac{p_j}{p_i}\right)\right]$
AUS	0.033	0.007	0.128
AUT	-0.093	-0.150	0.306
BEL	-0.104	-0.147	0.233
CAN	-0.207	-0.324	0.395
DNK	-0.230	-0.246	0.108
FIN	-0.268	-0.308	0.164
FRA	-0.107	-0.117	0.083
GRC	-0.231	-0.294	0.182
HUN	-0.040	-0.006	-0.031
ISL	-0.623	-0.450	0.120
IRL	0.006	-0.147	0.418
ITA	-0.135	-0.080	-0.135
JPN	-0.003	-0.014	0.037
KOR	0.249	0.213	0.134
LUX	0.074	-0.012	0.432
MEX	0.105	-0.309	0.451
NLD	-0.155	-0.185	0.141
NEL	-0.293	-0.412	0.370
NOR	-0.092	-0.170	0.370
PRT	-0.231	-0.319	0.254
ESP	-0.367	-0.459	0.333
SWE	-0.267	-0.355	0.450
AVE	-0.135	-0.195	0.225

Notes: The annual data are from the World Bank, 1971-2012.where the sign "$\sim$" represents the consumption growth differential between the U.S.A and the other countries, i.e. $\Delta\log\left(\frac{c_i}{c_j}\right)$.

Figure 1: Cutoff rule defining zones of activity and inactivity



Given the state, the cutoff rule function h is a function of γ, which measures the net gain to a household from switching from being an inactive household with consumption y/π to an active household with consumption c_A. The expression of h is just the right side of equation (3.19). The first two terms on the right side of (3.19) measure the utility gain from the household's switch from inactivity to activity, and the third term measures the utility cost of such a switch. When the transfer cost of the household is at $\bar{\gamma}$, the utility gain is equal to the utility cost, and the household is indifference to being active or inactive. For more detail of the cutoff rule function, please refer to Alvarez, Atkeson, and Kehoe (2002).

Table 2: Correlations before and after the end of the Bretton Woods system

Country pair	$\rho\left[\sim, \Delta\log(\varepsilon_{ij})\right]$		$\rho\left[\sim, \Delta\log(e_{ij})\right]$		$\rho\left[\sim, \Delta\log\left(\frac{p_j}{p_i}\right)\right]$	
	before	after	before	after	before	after
US-AUS	0.070	-0.034	0	-0.073	0.070	0.196
US-SA	0.061	-0.105	-0.010	-0.143	0.208	0.251
US-UK	-0.005	-0.068	0.113	-0.136	-0.009	0.266

Notes: In order to obtain enough observations, the quarterly data are used for this analysis. All the data are from Datastream. The starting period is the first quarter of 1959, and the end period is the second period of 2013. The word "before" represents the periods before the end of Breton Woods system from Q1 1959-Q3 1971. The word "after" represents the periods after the end of Breton Woods system from Q4 1971-Q2 2013. Again, the sign "$\sim$" represents the consumption growth differential between the U.S.A and the other countries, $\Delta\log\left(\frac{c_i}{c_j}\right)$.

Table 3: Exchange rate-price correlation and consumption growth correlation between U.S. and other OECD countries

Country	$\rho[\log(e_{i,j}), \log(P_j/P_i)]$	$\rho[\Delta\log(c_i), \Delta\log(c_j)]$
AUS	-0.742	0.090
AUT	-0.782	0.227
BEL	-0.432	0.366
CAN	-0.501	0.605
DNK	-0.347	0.518
FIN	-0.391	0.401
FRA	-0.628	0.453
GRC	-0.983	0.290
ISL	-0.996	0.321
IRL	-0.775	0.589
ISR	-0.995	0.425
ITA	-0.922	0.254
JPN	-0.884	0.441
KOR	-0.917	0.157
LUX	-0.484	0.268
MEX	-0.998	-0.219
NLD	-0.735	0.454
NEL	-0.810	0.401
NOR	-0.460	0.319
PRT	-0.956	0.063
ESP	-0.856	0.449
SWE	-0.692	0.304
SWZ	-0.896	-0.376
TUR	-0.998	0.131
AVE	-0.758	0.289

Notes: All the annual data are from the World Bank, from 1971-2012.

Table 4: Parameter values

risk aversion (σ)	2
discounter factor (β)	0.95
elasticity of substitution:	
domestic and foreign tradables (θ)	2
tradables and non-tradables (ϕ)	0.44
share of tradables (α)	0.55
share of domestic tradables	α_{11}=α_{22}=0.72
endowment vector:	
$y_i^T(h)$	1.0257;
$y_i^T(l)$	0.9743;
$y_i^N(h)$	2.4684;
$y_i^N(l)$	2.4316;

Table 5: Correlations under limited participation

$\gamma_{max.}$ $\left[\%Y^T(s_t)\right]$	0	1.00	2.00	5.00	10.00	20.00
Active households (%)	100	42.2	33.0	22.9	16.9	12.2
a) Correlations for all households:						
$\rho\left[\Delta\log\left(\frac{c_i}{c_j}\right), \Delta\log(\varepsilon_{i,j})\right]$	1.00	-0.457	-0.502	-0.591	-0.669	-0.731
$\rho\left[\Delta\log\left(\frac{c_i}{c_j}\right), \Delta\log(e_{i,j})\right]$	0.863	-0.422	-0.470	-0.565	-0.646	-0.709
$\rho\left[\Delta\log\left(\frac{c_i}{c_j}\right), \Delta\log\left(\frac{P_j}{P_i}\right)\right]$	-0.402	0.293	0.353	0.463	0.553	0.620
$\rho[\Delta\log(c_i), \Delta\log(c_j)]$	0.804	0.346	0.356	0.304	0.160	-0.012
b) Correlations for inactive households:						
$\rho\left[\Delta\log\left(\frac{c_{i\bar{A}}}{c_{j\bar{A}}}\right), \Delta\log(\varepsilon_{i,j})\right]$	–	-0.521	-0.586	-0.671	-0.729	-0.764
$\rho\left[\Delta\log\left(\frac{c_{i\bar{A}}}{c_{j\bar{A}}}\right), \Delta\log(e_{i,j})\right]$	–	-0.521	-0.581	-0.660	-0.714	-0.747
$\rho\left[\Delta\log\left(\frac{c_{i\bar{A}}}{c_{j\bar{A}}}\right), \Delta\log\left(\frac{P_j}{P_i}\right)\right]$	–	0.512	0.555	0.611	0.652	0.676
$\rho[\Delta\log(c_{i\bar{A}}), \Delta\log(c_{j\bar{A}})]$	–	-0.141	-0.151	-0.176	-0.190	-0.183
c) Correlations for active households:						
$\rho\left[\Delta\log\left(\frac{c_{iA}}{c_{jA}}\right), \Delta\log(\varepsilon_{i,j})\right]$	–	1.00	1.00	1.00	1.00	1.00
$\rho\left[\Delta\log\left(\frac{c_{iA}}{c_{jA}}\right), \Delta\log(e_{i,j})\right]$	–	0.999	0.999	0.999	0.999	0.999
$\rho\left[\Delta\log\left(\frac{c_{iA}}{c_{jA}}\right), \Delta\log\left(\frac{P_j}{P_i}\right)\right]$	–	-0.979	-0.981	-0.984	-0.985	-0.984
$\rho[\Delta\log(c_{iA}), \Delta\log(c_{jA})]$	–	0.747	0.816	0.899	0.922	0.946
$\rho\left[\log(e_{i,j}), \log\left(\frac{P_j}{P_i}\right)\right]$	-0.949	-0.983	-0.980	-0.978	-0.976	-0.972

Table 6: Model results under fixed exchange rate

	Flexible Ex.		Fixed Ex. & Open		Fixed Ex. & Closed	
$\gamma_{max.}$ $[\%Y^T(s_t)]$	0	2.00	0	2.00	0	2.00
Active households (%)	100	33.0	100	35.9	100	47.9
a) Correlations for all households:						
$\rho\left[\Delta\log\left(\frac{c_i}{c_j}\right), \Delta\log(\varepsilon_{i,j})\right]$	1.00	-0.534	1.00	0.488	-0.550	-0.442
$\rho[\Delta\log(c_i), \Delta\log(c_j)]$	0.804	0.356	0.828	0.758	-0.075	-0.075
b) Correlations for inactive households:						
$\rho\left[\Delta\log\left(\frac{c_{i\bar{A}}}{c_{j\bar{A}}}\right), \Delta\log(\varepsilon_{i,j})\right]$	–	-0.586	–	0.929	–	0.367
$\rho[\Delta\log(c_{i\bar{A}}), \Delta\log(c_{j\bar{A}})]$	–	-0.151	–	0.833	–	0.007
c) Correlations for active households:						
$\rho\left[\Delta\log\left(\frac{c_{iA}}{c_{jA}}\right), \Delta\log(\varepsilon_{i,j})\right]$	–	1.00	–	1.00	–	-0.333
$\rho[\Delta\log(c_{iA}), \Delta\log(c_{jA})]$	–	0.931	–	0.986	–	0.015
d) BCSW international risk-sharing index:						
For all households	1.00	0.155	1.00	0.802	-0.148	-0.153
For active households	-	1.00	-	1.00	-	-0.018
For inactive households	-	-0.202	-	0.848	-	-0.007

Table 7: Correlations between China and other main economies

Country	$\rho\left[\sim, \Delta\log(\varepsilon_{i,j})\right]$	$\rho\left[\sim, \Delta\log(e_{i,j})\right]$	$\rho\left[\sim, \Delta\log\left(\frac{P_j}{P_i}\right)\right]$	$\rho[\Delta\log(c_i), \Delta\log(c_j)]$
AUS	-0.342	-0.395	-0.213	-0.106
JPN	-0.560	-0.651	-0.360	0.062
UK	-0.443	-0.666	-0.160	-0.518
US	-0.673	-0.676	-0.492	-0.329

Notes: The annual data are from World bank, 1994-2012, where the sign "$\sim$" represents the consumption growth differential between China and the other countries, $\Delta\log\left(\frac{c_i}{c_j}\right)$.