Cooperative R&D with Durable Goods*

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Abstract

The effects of the durability of the good produced by a duopolistic industry on research and development (R&D) investment in presence of R&D spillovers is analyzed. We show that relative to non-durable goods industries, the critical spillover level from which cooperation in R&D increases the level of investment is higher when firms produce durable goods in the presence of time inconsistency problems. Additionally, we find and compare the optimal R&D level when firms rent their good, sell it or both rent and sell it. The findings indicate that the durability of the product and the different commercialization practices concerning durable goods seems relevant for public policy issues regarding R&D policies.

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1 Introduction

Research and Innovation are widely accepted to be the centrepiece for long-term sustainable economic growth. Innovations stimulated by research and development are an important factor of survival, development and competitiveness, both for the company and for the national economy. However, the strong financial and economic world crisis can affect R&D investments. Besides, in the last years a growing number of firms have become involved in collaborative relationships with a variety of partners, from suppliers to customers and research institutes. European competition law regulates horizontal research and development (R&D) agreements, whereby firms coordinate their R&D operations and jointly exploit the results while still competing in the marketplace. To protect consumers, public authorities forbid firms to engage in price collusion or other agreements that restrict output[1]. The formation of research joint ventures or other cooperative R&D agreements is not forbidden, however, but encouraged by governments because of possible welfare-enhancing effects[2]. Pioneering contributions underpinning the advantages of collaboration in R&D are provided in the models of Katz (1986), d'Aspremont and Jacquemin (1988) and Kamien et al. (1992). In the d'Aspremont and Jacquemin (1988) model duopolists first decide on R&D expenditures and then compete in the product market. In their model, R&D conducted by one firm reduces its unit production cost and may have spillovers, reducing the unit cost of the other. They find that R&D investment and welfare are higher under R&D cooperation than under R&D competition if the spillover is above a certain threshold, and lower otherwise. The results are often interpreted as a rationale for governmental support of research joint ventures in industries with large knowledge spillovers. This also provides the rationale for giving cooperation in R&D special treatment.

[1] Agreements between two or more firms which restrict competition are prohibited by Article 101(1) of the Treaty, subject to some limited exceptions.

[2] The Article 101(1) prohibition does not apply to any agreement that meets the Article 101(3) conditions. Art. 101(3) exemption is normally available where the agreement or practice in question generates economic benefits in terms of lower prices, better quality products or services, faster innovation, etc. that are passed on to consumers do not impose non essential restrictions and do not eliminate competition completely.

The d'Aspremont and Jacquemin model has been generalized and extended by many economists. For instance, Henriques (1990) establish the stability conditions of the solutions. Margit (1991) explain why duopolistic firms would like to enter cooperative R&D even when they would act non-cooperatively in the product market.Kultti and Takalo (1998) endogenise the spillover parameter and find that the model of d'Aspremont and Jacquemin is outcomewise equivalent with a three stage model where in the first stage they invest in cost reducing R&D, the second stage involves the decision to exchange information and in the last stage firms decide about quantities. Goel and Haruna (2007) examine the behavior of labor-managed firms which engage in research that is subject to spillovers. They find that the equilibrium research is greatest under full cooperation, while output is greatest under full competition. Cellini et al. (2009) extend the static game examined in d'Aspremont and Jacquemin to a dynamic setup and get that private and social incentives towards R&D cooperation coincide in the sense that cartelisation dominates competition from both standpoints. Amir et al. (2008) state that a given R&D investment should always produce more cost reduction if devoted to one lab rather than two independent labs operated under natural spillovers. As the d'Aspremont and Jacquemin model does not satisfies this criterion, they modify the d'Aspremont and Jacquemin model's R&D cost function to take into account for it. They show that this change do not invalidate any of the R&D comparisons in the d'Aspremont and Jacquemin model in a qualitative sense. However, these earlier works assume that the goods produced by the firms are non-durable.

Backes-Gellner et al. (2005) investigate the determinants of inter-firm cooperation in research and development and find that neither in the durable goods industry, nor in the non durable goods industry nor in construction is the probability of firms entering R&D cooperation significantly different from the probability in business related services. Thus it appears that the likelihood of firms entering into cooperative agreements in R&D is the same whether or not they produce durable goods. The European legislation on cooperation policy makes no distinction allowing R&D agreements when firms produce durable and nondurable goods[3]. This led to the question of whether the government should take into account the fact that companies produce durable goods and the different practices used in the commercialization of durable goods in permitting cooperation agreements. In this paper the optimal R&D choice and production strategies of firms producing a durable good and operating in a duopoly market are analyzed for renting firms, selling firms and renting-selling firms. The analysis of R&D investment in durable goods markets may be important because it it precisely in durable goods markets in which much innovative activities take place and the insights gained from the nondurable case need not extend to the durable case. That's because durable goods constitute a special part of the economy because there are a number of issues that only arise or arise more frequently in those markets, such as the intertemporal inconsistency effect first noted by Coase (1972) and strategic interaction by rivals noted by Carlton and Gertner (1989).

In the setting we consider, it is shown that the dynamic reactions between oligopolists pointed out by Carlton and Gertner and the time-consistency problem identified by Coase affect the firms' levels of innovation. It is found that the spillover level for which R&D cooperation is preferable to noncooperative behavior from a social point of view is higher when firms both rent and sell their output than when firms only sell, and is higher when firms sell their output than when they rent it. The spillover level for which R&D cooperation is preferable to noncooperative behavior is the same when firms produce and rent the durable good than when firms produce non-durable goods. Thus, it seems that whether or not firms produce durable goods and the different practices used in the commercialization of durable goods should be a factor to consider when public authorities decide whether to permit R&D cooperation.

We also find and compare the optimal R&D level when firms rent their good, sell it or both rent and sell it. The level of investment is found to be higher when firms rent their good than when they sell it. Besides, the level of investment of renting-selling firms with regard to the level of investment of renting firms or

[3] See the Exemption 101(3) of the EC Treaty.

selling firms is analyzed. We find that when firms cooperate in R&D, the R&D incentives of selling firms are higher than those of renting-selling firms, but when firms do not cooperate in R&D the level of investment by renting-selling firms may be greater than that of renting or selling firms.

A standard result in the literature on the durable goods monopoly (e.g., Bulow (1982), Kahn (1986)) is that when the inverse rental demand for the good is linear and the firm may only choose the level of production, social welfare is greater if the monopoly sells its output than if it rents it. In practice, firms such as the United Shoe Company, IBM, Xerox and others began by renting their products but were later required also to sell their output. Contrary to the arguments often given as to why these firms should be required to sell, this paper shows that when firms choose not only the level of production but also their R&D level in presence of spillovers and cooperate in R&D (but not in production), because of the higher level of investment by renting firms, social welfare may be greater if firms *rent* their output than if they sell part or all of it.

The following sections of the paper first describe the framework of analysis. A modified version of the d'Aspremont and Jacquemin (1988) model presented by Amir et al. (2008) is proposed in Section 2, extended to allow for durability. In Section 3 we get the socially efficient level of R&D and production. The optimal R&D levels are obtained and compared in Section 4 in the following three cases: (i) Renting firms, where firms rent the good in question; (ii) Selling firms, where firms cannot rent but must sell their production; and (iii) Renting-Selling firms, where they may both rent and sell their production. Finally, Section 5 presents conclusions. All proofs are relegated to the Appendix.

2 Theoretical framework

An industry with $n = 2$ identical firms that produce a homogeneous durable good is considered. Entry into the industry is assumed to be unprofitable. All agents have perfect and complete information and potential buyers of the durable good have perfect foresight. There are two discrete periods of time $t = 1, 2$. The good is assumed to be perfectly durable, so that consumers view units previously produced and units just produced as identical. Without loss of generality it is assumed that the discount factor is 1 and there is a perfect second hand market for the durable good. This gives a two-stage game with the following timing. In the first stage the firms decide on the levels of R&D expenditures. In the second stage the firms set the output that they will sell or rent in two periods of time, to maximize the present value of their total profits.

The notation used is similar to the d´Aspremont and Jacquemin (1988) model. The inverse rental demand function for the services of the durable good in each period is $p_t(Q_t) = a - bQ_t$, with a, $b > 0$, and Q_t represents the quantity used by consumers in that period. The cost of production of each firm i is $C_i = [A - x_i - \beta x_j] q_i$, $i, j = 1, 2$, $i \neq j$, $a > A > 0$, $0 < \beta < 1$, where x_i is the amount of research that the firm i undertakes, x_j is the amount of research that its rival undertakes and β is the cost reduction experienced by a firm due to a unit of a rival‘s R&D expenditure[4]. The cost of R&D, as suggested in Amir et al.(2008), is assumed to be quadratic, of the form $r(x_i) = \delta x_i + \frac{\gamma x_i^2}{2}$[5]. Parameter γ is assumed to be positive.

The following notation is used

q_{1i}^s: quantity sold by firm i in the first period

q_{1i}^r: quantity rented by firm i in the first period

q_{2i}: quantity sold (or rented) by firm i in the second period.

We assume throughout the paper that parameters and functions are such that we obtain interior solutions in each optimization problem and, therefore, non-negative quantities and prices of the durable good in each period.

[4] According to Katz (1986) spillovers refer to research done by one firm which can be used by another firm even though the latter does not receive permission (i.e. purchase a license) to use the inventive output.

[5] Amir et al. (2008) show that positive marginal cost at zero or a minimum level of fixed costs suffices for the R&D cost function to be consistent with the criterion that it should always be more efficient to achieve a given cost reduction by investing in one R&D lab only instead of investing in several distinct labs operating under natural spillovers. However, as they indicated, initial positive marginal cost alone is not compatible with full spillovers values (the maximum spillover parameter is such that $\beta_{\max} = \frac{\delta}{\delta + \gamma A}$).

3 Socially efficient level of R&D

Social welfare is defined as the sum of the consumer surplus and the producer‘s profits. The goal is to find the optimal quantities and the optimal investment level that maximize social welfare. We denote by Q_t the output used in the market in each period t, where $t = 1, 2$[6]. Consumer surplus and the firms' profits are respectively:

$$CS(Q) = \frac{b}{2} \sum_{t=1,2} (Q_t)^2$$

$$\pi(Q) = (a - bQ_1) Q_1 + (a - bQ_2) Q_2 - (A - (1+\beta) x) Q_2 - 2\delta x - \gamma x^2.$$

So, assuming symmetric interior solution the efficient level of R&D and the socially efficient amount of productions that are obtained from the maximization of the social welfare can be written respectively as[7]

$$x^{**} = \frac{(1+\beta)(2a - A) - 4b\delta}{4b\gamma - (1+\beta)^2}$$

$$Q_1^{**} = Q_2^{**} = \frac{2\left[(2a - A)\gamma - (1+\beta)\delta\right]}{4b\gamma - (1+\beta)^2}.$$

4 Market decisions when goods are durable

In this section we study the R&D and output decisions of firms that produce durable goods. The first case analyzed is that in which firms behave noncooperatively in both output and research. The second case analyzed is that in which firms cooperate in R&D investment, and they behave noncooperatively in the product market. These two cases are analyzed when firms rent their output, when they sell it and when they both rent and sell it.

[6] The quantity used in the second period is the sum of the quantities sold in each period. The quantity used in the first period is the sum of the quantity sold and the quantity rented in that period.

[7] We have that both when firms cooperate in R&D as when there is R&D competition, the level of investment that maximize social welfare is higher than the level of investment that is chosen by firms when the good is rented, sold or rented and sold simultaneously (this levels of investment are found in the following section).

4.1 Renting Firms

4.1.1 R&D competition

First we focus on the case where firms do not cooperate at the R&D stage, that is firms choose their R&D expenditure independently.

In the second stage each active firm chooses the quantity to be produced in periods 1 and 2 in order to maximize the discounted value of its total profits, given the production costs that they have inherited from the first stage. As the rental prices in each period $t = 1, 2$ are given by $p_1^r = a - bq_{1i}^r - bq_{1j}^r$ and $p_2 = a - bq_{2i} - bq_{2j}$ respectively, each firm i's present value of their total profits would be:

$$\pi_i^{*r} = p_1^r q_{1i}^r - (A - x_i - \beta x_j) q_{1i}^r + p_2 q_{2i} - (A - x_i - \beta x_j)(q_{2i} - q_{1i}^r) - \delta x_i - \frac{\gamma x_i^2}{2}, \tag{1}$$

subject to $q_{2i} \geq q_{1i}^r$.

Differentiating equation (1) with respect to first and second period outputs and solving gives[8]:

$$q_{2i} = q_{1i}^r = \frac{1}{6b}(2a - A + 2x_i - x_j - \beta x_i + 2\beta x_j). \tag{2}$$

In the first stage, each firm chooses the level of R&D to maximize its own profits, taking into account the second-stage equilibrium outputs. Thus, the firms maximize Eq. (1) subject to Eq. (2) to obtain the optimal investment level.

Assuming interior symmetric solution, the optimal investment level of each firms i and the numbers of units rented each period are:

$$\begin{aligned} x_i^{*r} &= \frac{(2-\beta)(2a-A) - 9b\delta}{9b\gamma - (2-\beta)(1+\beta)} \qquad (3)\\ q_{1i}^{*r} &= q_{2i}^* = \frac{3((2a-A)\gamma - \delta(1+\beta))}{2(9b\gamma - (2-\beta)(1+\beta))}. \end{aligned}$$

By differentiating the Eq. (1) with respect to x_i, we can derive the first-order

[8] We assume throughout the paper that the second-order conditions are satisfied.

condition for profit maximization as follows[9]:

$$\frac{d\pi_i^{*r}}{dx_i} = 0 \Leftrightarrow \underbrace{\frac{\partial \pi_i^r}{\partial C_i^t}\frac{\partial C_i^t}{\partial x_i}}_{\text{Direct Effect}} + \underbrace{\frac{\partial \pi_i^r}{\partial q_{1j}}\frac{\partial q_{1j}}{\partial x_i}}_{\substack{\text{Strategic Effect 1} \\ \text{t=1}}} + \underbrace{\frac{\partial \pi_i^r}{\partial q_{2j}}\frac{\partial q_{2j}}{\partial x_i}}_{\substack{\text{Strategic Effect 1} \\ \text{t=2}}} - \underbrace{r'(x_i)}_{\text{Cost Effect}} = 0. \quad (4)$$

As shown in (4), there are four terms that jointly determine the renting firm's optimal R&D. The first term is called the Direct effect, which is positive. It represent a further cost reduction effect on its profits. The second and third terms are the Strategic effects in periods $t = 1$ and $t = 2$ respectively. These strategic effects are negative (positive) if $\beta > \frac{1}{2}$ ($\beta < \frac{1}{2}$). When firm i decides its R&D investment level, it takes into account the effect that it will have on its competitor's production. If spillovers are high ($\beta > \frac{1}{2}$), an increase in firm i's R&D investment reduces firm j's marginal cost. Firm j will increase its production and this will reduce firm i's profits. The last term is negative and represents the R&D cost effect.

We next solve the case where firms choose their R&D level cooperatively.

4.1.2 R&D cooperation

Firms coordinate their R&D decisions in the first stage, so as to maximize the present value of joint profits, but remain rivals in the marketplace. In this case the second stage is identical to the one obtained when firms do not cooperate in R&D, so first stage can now be solved taking into account the restrictions imposed in the second stage.

At the first stage the firms maximize their joint profits, as a function of x_i and x_j:

$$\max_{\{x_i, x_j\}} \hat{\pi}^r = \pi_1^{*r} + \pi_2^{*r} = \sum_{i=1}^{2} \left(p_1^r q_{1i}^r + p_2 q_{2i} - (A - x_i - \beta x_j)\, q_{2i} - \delta x_i - \frac{\gamma x_i^2}{2} \right),\ i \neq j, \quad (5)$$

subject to Eq. (2). Solving the maximization problem, we find that the interior

[9] C_i^t represent the total (first period plus second period) production costs of the firm i.

symmetric solution is:

$$\begin{aligned}
\hat{x}_i^r &= \frac{(\beta+1)(2a-A)-9b\delta}{9b\gamma-(\beta+1)^2} \quad i=1,2 \\
\hat{q}_{1i}^r &= \hat{q}_{2i} = \frac{3\left((2a-A)\gamma-\delta(\beta+1)\right)}{2\left(9b\gamma-(\beta+1)^2\right)}.
\end{aligned} \tag{6}$$

By differentiating the equation (5) with respect to x_i, we can derive the first-order condition for profit maximization as follows:

$$\frac{d\hat{\pi}^r}{dx_i} = \frac{d\pi_i^{*r}}{dx_i} + \underbrace{\frac{\partial \pi_j^r}{\partial q_{1i}}\frac{\partial q_{1i}}{\partial x_i}}_{\substack{\text{Strategic Effect 2} \\ t=1}} + \underbrace{\frac{\partial \pi_j^r}{\partial q_{2i}}\frac{\partial q_{2i}}{\partial x_i}}_{\substack{\text{Strategic Effect 2} \\ t=2}} + \underbrace{\frac{\partial \pi_j^r}{\partial C_j^t}\frac{\partial C_j^t}{\partial x_i}}_{\text{Spillover Effect}} = 0. \tag{7}$$

Under R&D cooperation, when the firms choose how much to spend on R&D so as to maximize joint profits, besides the effects mentioned for R&D competition, they also take into account the effects of their R&D expenditures on the profits of the other firm. So, there are two additional opposing forces when a firm decides its R&D level. First, when cooperative research lowers the production costs of the firm i, it will increase the units rented in the first and second periods, reducing its competitor's profits (strategic effects 2). However, there is a spillover effect working in the opposite direction. Because of spillovers, an increase in x_i reduce firms j's marginal cost and, thereby, increase its profits. So, the spillover effect is positive, and increases with the spillover parameter.

Comparing the level of investment and the quantity used in the market each period of firms that cooperate in R&D with the levels when there is no cooperation, we may establish the following proposition:

Proposition 1. *If firms producing a durable good can commit to renting their production, then the level of R&D and the quantity used in the market each period is higher in the case of cooperative research than in the non-cooperative case if and only if* $\beta > \frac{1}{2}$.

Proof: See Appendix. ■

In line with previous studies of non-durable goods[10], the comparison of R&D competitive and R&D cooperative investment by durable good produc-

[10] See d'Aspremount and Jacquemin (1988) and Kamien et al. (1992).

ers is driven by critical spillover levels. In this case, the value of β from which cooperation in R&D increases the level of investment is the same if the firms produce durable goods and rent them as when firms produce nondurable goods. The level of investment that would be obtained in a two period repeated Cournot game that arises with non-durable goods when firms do not cooperate in R&D is $x_i^{*nd} = \frac{4(a-A)(2-\beta)-9b\delta}{9b\gamma-4(1+\beta)(2-\beta)}$ and when firms cooperate in R&D is $\hat{x}_i^{nd} = \frac{4(a-A)(1+\beta)-9b\delta}{9b\gamma-4(1+\beta)^2}$. Hence, as $\hat{x}_i^{nd} - x_i^{*nd} = \frac{36(\delta+A\gamma-a\gamma+\beta\delta)(1-2\beta)b}{\left(9b\gamma-4(\beta+1)^2\right)(9b\gamma-4(2-\beta)(1+\beta))}$, $\hat{x}_i^{nd} > x_i^{*nd}$ if and only if $\beta > \frac{1}{2}$.

When firms produce a durable good and rent it, the quantity produced in the second period is zero. Production only takes place in the first period, and the output is rented in this period and the next. But when firms produce non-durable goods, they produce and sell the good in both periods, so they have to cope with positive production costs in each period. The investment of firms reduces production costs in both periods, so the level of R&D investment by non-durable goods firms is higher than that of durable goods firms that rent their output.

Now we consider the case where the duopoly sells the good to consumers.

4.2 Selling Firms

4.2.1 R&D competition

Coase (1972) studied the consequences of rational expectations for market power. He argued that buyers of durable goods will correctly recognize that the firm will have an incentive to reduce its prices (increase production) in future periods. This tends to reduce the value of the existing stock of durables. Since the existing stock of units is held by buyers, in the absence of explicit contracts of guarantees, the firm has no incentive to take this capital loss into consideration in its future pricing behavior. This causes buyers to substitute current consumption by future consumption and decreases current demand. Thus, in order to calculate the intertemporal consistent schedule of production that maximizes the discounted value of profits for firm i, the maximization problem has to be resolved recursively by backward induction: first the optimal production for period $t = 2$ must be determined given any production in period $t = 1$, and then the optimal production corresponding to period 1 must be calculated. The sale prices in each period $t = 1, 2$ are given by $p_1^s = a - bq_{1i}^s - bq_{1j}^s + p_2$ and $p_2 = a - b\left(q_{1i}^s + q_{1j}^s + q_{2i} + q_{2j}\right)$ respectively.

At $t = 2$ each firm sells the quantity that maximizes its profits corresponding to the second period, given the quantity sold in the first period:

$$\max_{q_{2i}} p_2 q_{2i} - (A - x_i - \beta x_j)\, q_{2i} - \delta x_i - \frac{\gamma x_i^2}{2}. \tag{8}$$

Maximizing we can get the quantity produced in the second period in equilibrium by each firm i:

$$q_{2i} = \frac{1}{3b}\left(a - A + (2-\beta)\, x_i + (2\beta - 1)\, x_j - bq_{1i}^s - bq_{1j}^s\right). \tag{9}$$

In $t = 1$, each firm i choose the level of first period production that maximizes the present value of its total profits, that can be written as:

$$\pi_i^{*s} = p_1^s q_{1i}^s - (A - x_i - \beta x_j)\, q_{1i}^s + p_2 q_{2i} - (A - x_i - \beta x_j)\, q_{2i} - \delta x_i - \frac{\gamma x_i^2}{2}. \tag{10}$$

Thus, maximizing Eq. (10) subject to (9) we can get the first period sales, and therefore the second period output:

$$\begin{aligned} q_{1i}^s &= \frac{20a - 2A + (9 - 7\beta)\, x_i + (9\beta - 7)\, x_j}{64b}, \\ q_{2i} &= \frac{4a - 10A + (21 - 11\beta)\, x_i + (21\beta - 11)\, x_j}{32b}. \end{aligned} \tag{11}$$

In the first stage, the firms choose R&D levels taking into account the consequences on the equilibrium output levels. Thus, the firms maximize Eq. (10) subject to Eq. (11) to obtain the optimal investment level. Assuming interior symmetric solution, the optimal investment level of each firms i and the numbers of units rented each period are:

$$x_i^{*s} = \frac{(110 - 66\beta)\, a - (119 - 65\beta)\, A - 256b\delta}{256b\gamma - (\beta + 1)(119 - 65\beta)},$$

$$\begin{aligned} q_{1i}^{*s} &= \frac{32\gamma b\,(10a - A) - a\,(\beta + 1)(135 - 73\beta) - 32\delta b\,(\beta + 1)}{4b\,(256b\gamma - (\beta + 1)(119 - 65\beta))}, \\ q_{2i}^{*} &= \frac{32\gamma b\,(2a - 5A) + a\,(\beta + 1)(39 - 25\beta) - 160b\delta\,(\beta + 1)}{2b\,(256b\gamma - (\beta + 1)(119 - 65\beta))}. \end{aligned} \tag{12}$$

By differentiating the equation (10) with respect to x_i and taking into account equation (12), we can derive the first-order condition for profit maximization as follows:

$$\frac{d\pi_i^{*s}}{dx_i} = 0 \Leftrightarrow \underbrace{\frac{\partial \pi_i^s}{\partial C_i^t}\frac{\partial C_i^t}{\partial x_i}}_{\text{Direct Effect}} + \underbrace{\frac{\partial \pi_i^s}{\partial q_{1j}}\frac{\partial q_{1j}}{\partial x_i}}_{\substack{\text{Strategic Effect 1} \\ \text{t=1}}} + \underbrace{\frac{\partial \pi_i^s}{\partial q_{2j}}\left(\frac{\partial q_{2j}}{\partial x_i} - \frac{\partial q_{2j}}{\partial q_{1i}}\frac{\partial q_{1i}}{\partial x_i}\right)}_{\substack{\text{Strategic Effect 1} \\ \text{t=2}}} +$$

$$+ \underbrace{\frac{\partial \pi_{1i}^s}{\partial q_{2i}}\left(\frac{\partial q_{2i}}{\partial x_i} - \frac{\partial q_{2i}}{\partial q_{1i}}\frac{\partial q_{1i}}{\partial x_i}\right)}_{\textbf{Intertemporal inconsistency effect}} - \underbrace{r'(x_i)}_{\text{Cost Effect}} = 0. \quad (13)$$

If we compare equations (4) and (13), we can see that in the case of selling firms there is an extra term named the Intertemporal inconsistency effect that is not present when firms rent their output. This extra term is always negative, and is due to the commitment problem faced by a sales firms. That is, since the selling firm does not own the units previously sold (unlike renting firms where control of the stock of durables is retained), it has no incentive to account for this in future periods. This leads the firm to market in the second period a quantity that is higher than the quantity marketed by renting firms, reducing the selling price of the second period, and therefore also the price of first period and its profits. Thus, the selling firm would like to commit to low future production, and this can be achieved with high future production costs. So the intertemporal inconsistency effect makes selling firms choose to spend less on R&D, so as to keep their marginal costs high.

We next consider the case where there is R&D cooperation.

4.2.2 R&D cooperation

In this case the second stage is identical to the one obtained when firms do not cooperate in R&D, so the first stage is now solved taking into account the restrictions imposed in the second stage.

At the first stage the firms maximize the joint profits, as a function of x_i

and x_j:

$$\max_{\{x_i,x_j\}} \hat{\pi}^s = \pi_1^{*s}+\pi_2^{*s} = \sum_{i=1}^{2}\left(p_1^s q_{1i}^s + p_2 q_{2i} - (A - x_i - \beta x_j)\left(q_{1i}^s + q_{2i}\right) - \delta x_i - \frac{\gamma x_i^2}{2}\right), \tag{14}$$

subject to (11).

Solving the firms maximization problem, the interior solution is found to be:

$$\hat{x}_i^s = \frac{(\beta+1)(22a-27A) - 128b\delta}{128b\gamma - 27(\beta+1)^2}$$

$$\begin{aligned}
\hat{q}_{1i}^s &= \frac{16\gamma b(10a-A) - 31a(\beta+1)^2 - 16b\delta(\beta+1)}{4b\left(128b\gamma - 27(\beta+1)^2\right)}, \\
\hat{q}_{2i} &= \frac{16\gamma b(2a-5A) + 7a(\beta+1)^2 - 80\delta b(\beta+1)}{2b\left(128b\gamma - 27(\beta+1)^2\right)}.
\end{aligned} \tag{15}$$

The comparison of the R&D investment level of selling firms in the case of competitive research and in the case of cooperative research yield the following result:

Proposition 2. *If firms sell their output, then the level of R&D and the quantity used in the market in each period is higher in the case of cooperative research than in the non-cooperative case if and only if* β *is higher than a critical value* β^* *that satisfies* $0.6 < \beta^* < 0.65$.

Proof: See Appendix. ■

If we compare propositions 1 and 2, we have that the critical spillover level from which cooperation in R&D increases the level of investment is higher when firms produce and sell the durable good than when they only rent it.

Previous work on non-durable goods has focused on the impact of market structure and market power on the incentives of firms to engage in innovative activities. The conclusion of these studies is not clear: some find a positive relationship between innovation and monopoly power and conclude that increased concentration improves the conditions for achieving technological progress[11],

[11] See Schumpeter (1942) and Grossman and Helpman (1991) and Romer (1990).

but others argue that firms invest less in a monopolistic environment than in competitive markets[12]. The empirical evidence on the relationship between innovation and competition is also ambiguous[13]. From the literature on durable goods it is known that the market power held by a firm that produce a durable good and sells it can be substantial but is notably less than the power held by a firm that produces a durable good which is rented rather than sold. The reason lies in the inability of the firm to credibly commit itself not to expand output. Thus, it is desirable to analyze how market power influences the level of R&D in this context. If the level of investment of renting firms is compared to the level of investment of selling firms, the following result is obtained.

Proposition 3. *Both when firms cooperate in R&D and when firms act non-cooperatively in R&D, the amount of R&D in the case of renting firms is higher than the amount of R&D in the case of selling firms.*

Proof: See Appendix. ■

It is found that renting firms choose higher R&D levels in the first stage than selling firms. As mentioned previously, when firms sell their good, they take into account an additional effect of changes in R&D on their profits that is not present when firms rent their good, that is, they take into account the intertemporal inconsistency effect (see equations (4) and (13)). Therefore, due to the time-inconsistency problem, the selling firms choose to spend less on R&D than renting firms, so as to keep their marginal costs high. Bulow (1982), considering a durable goods monopolist firm, shows that the selling firm chooses to make lower fixed investments that the renter[14]. Bulow's model is extended here in two ways: first, considering a duopoly instead of a monopoly, and second

[12] See Arrow (1962).

[13] Nickell (1996) and Blundell et al. (1999) report a positive relationship between innovation and competition intensity. Mansfield (1968) and Tang (2006) show that this relationship may be positive or negative depending on how competition is perceived in the particular innovation activity.

[14] Bond and Samuelson (1986) analyze the incentives to innovate in the case where a monopolist either rents or sells a durable good. They found that the monopoly seller may spend more on innovation than is socially optimal when innovation occurs in two periods. The additional possibility of period 2 innovation allows the monopoly to exploit the period 2 residual demand more effectively, and hence induces the monopoly to spend more on innovation than is socially optimal.

by introducing a first stage in which firms choose their R&D level.

Lastly, we consider the case in which firms may both rent and sell their output but they do not coordinate to rent them.

4.3 Renting-Selling Firms

4.3.1 R&D competition

In an oligopoly setting, strategic reasons lead an oligopolist to choose to both rent and sell, even if it can choose only to rent. The intuition is that by selling an extra unit today, a firm deprives its rivals of current and future sales (See Carlton and Gertner (1989)). The prices each period t, where $t = 1, 2$ of the good produced by the industry for renting-selling firms are $p_1^r = a - bq_{1i}^s - bq_{1j}^s - bq_{1i}^r - bq_{1j}^r$, $p_1^s = p_1^r + p_2$, and $p_2 = a - bq_{1i}^s - bq_{1j}^s - bq_{2i} - bq_{2j}$.

In $t = 2$, each firm i, $i, j = 1, 2$, $i \neq j$ will solve:

$$\max_{q_{2i}} p_2 q_{2i} - (A - x_i - \beta x_j)\left(q_{2i} - q_{1i}^r\right) - \delta x_i - \frac{\gamma x_i^2}{2}, \tag{16}$$

subject to $q_{2i} \geq q_{1i}^r$.

Solving the maximization problem, the second period equilibrium levels of output are found to be:

$$q_{2i} = \frac{\left(a - A + 2x_i - x_j - bq_{1i}^s - bq_{1j}^s - \beta x_i + 2\beta x_j\right)}{3b}. \tag{17}$$

In period $t = 1$, each firm chooses the levels of sales and rentals, q_{1i}^s and q_{1i}^r, that maximize the present value of its total profits, given by:

$$\pi_i^{*r-s} = p_1^s q_{1i}^s + p_1^r q_{1i}^r + p_2 q_{2i} - (A - x_i - \beta x_j)\left(q_{1i}^s + q_{2i}\right) - \delta x_i - \frac{\gamma x_i^2}{2}, \tag{18}$$

subject to (17).

Maximizing Eq. (18) with respect to q_{1i}^s and q_{1i}^r subject to (17) gives the equilibrium levels of output:

$$\begin{aligned} q_{1i}^s &= q_{2i} = \frac{1}{5b}\left(a - A + 3x_i - 2x_j - 2\beta x_i + 3\beta x_j\right) \\ q_{1i}^r &= \frac{1}{15b}\left(2a + 3A - 9x_i + 6x_j + 6\beta x_i - 9\beta x_j\right). \end{aligned} \tag{19}$$

In the first stage, each firm chooses its level of R&D to maximize its own profits. Firm i maximize Eq. (18) subject to Eq. (19) to obtain the optimal investment level. Assuming interior symmetric solution, we have:

$$\begin{aligned} x_i^{*r-s} &= \left(\frac{4(a-A)(3-2\beta)-25b\delta}{25b\gamma-4(\beta+1)(3-2\beta)}\right) \\ q_{1i}^{*s} &= q_{2i}^{*} = \left(\frac{5(a-A)\gamma-5\delta(\beta+1)}{25b\gamma-4(\beta+1)(3-2\beta)}\right) \\ q_{1i}^{*r} &= \left(\frac{5\gamma b(3A+2a)-4a(\beta+1)(3-2\beta)+15\delta b(\beta+1)}{3b(25b\gamma-4(\beta+1)(3-2\beta))}\right). \end{aligned} \tag{20}$$

By differentiating the equation (18) with respect to x_i, we can derive the first-order condition for profit maximization as follows:

$$\begin{aligned} \tfrac{d\pi_i^{*r-s}}{dx_i} = 0 \Leftrightarrow & \underbrace{\frac{\partial\pi_i^{r-s}}{\partial C_i^t}\frac{\partial C_i^t}{\partial x_i}}_{\text{Direct Effect}} + \underbrace{\frac{\partial\pi_i^{r-s}}{\partial q_{1j}^s}\frac{\partial q_{1j}^s}{\partial x_i}}_{\substack{\text{Strategic selling Effect}\\ \text{t=1}}} + \underbrace{\frac{\partial\pi_i^{r-s}}{\partial q_{1j}^r}\frac{\partial q_{1j}^r}{\partial x_i}}_{\substack{\text{Strategic renting Effect}\\ \text{t=1}}} + \\ & \underbrace{+\frac{\partial\pi_i^{r-s}}{\partial q_{2j}}\left(\frac{\partial q_{2j}}{\partial x_i}-\frac{\partial q_{2j}}{\partial q_{1i}^s}\frac{\partial q_{1i}^s}{\partial x_i}\right)}_{\substack{\text{Strategic Effect}\\ \text{t=2}}} + \underbrace{\frac{\partial\pi_{1i}^{r-s}}{\partial q_{2i}}\left(\frac{\partial q_{2i}}{\partial x_i}-\frac{\partial q_{2i}}{\partial q_{1i}^s}\frac{\partial q_{1i}^s}{\partial x_i}\right)}_{\text{Intertemporal inconsistency effect}} - \underbrace{r'(x_i)}_{\text{Cost Effect}} = 0. \end{aligned} \tag{21}$$

If we compare equations (13) and (21), we have that in the case of renting-selling firms the first period strategic effects has two terms, named strategic selling effect and strategic renting effect. If $\beta > \frac{2}{3}$ ($\beta < \frac{2}{3}$), the strategic selling effect is negative (positive) and the strategic renting effect is positive (negative). So, if firm i increases its R&D efforts, for high spillovers ($\beta > \frac{2}{3}$,), its competitors will sell more units but will rent fewer. The reduction in units rented will increase firm's R&D but the increase in units sold will reduce the firm's R&D level.

The case where renting-selling firms choose their R&D level cooperatively is now solved.

4.3.2 R&D cooperation

As the second stage is identical to the one obtained when firms do not cooperate in R&D, first stage is now solved.

At the first stage, the firms maximize their joint profits as a function of x_i and x_j:

$$\max_{\{x_i,x_j\}} \hat{\pi}^{r-s} = \sum_{i=1}^{2} \left(p_1^s q_{1i}^s + p_1^r q_{1i}^r + p_2 q_{2i} - (A - x_i - \beta x_j)(q_{1i}^s + q_{2i}) - \delta x_i - \frac{\gamma x_i^2}{2} \right) \tag{22}$$

subject to the restrictions imposed by the second stage, given by the equation (19).

Solving the maximization problem, the optimal investment level and the quantity sold and rented in the first period, and the quantity sold in the second period proves to be:

$$\begin{aligned}
\hat{x}_i^{r-s} &= \left(\frac{4(a-A)(\beta+1) - 25b\delta}{25b\gamma - 4(\beta+1)^2} \right) \quad i = 1,2 \\
\hat{q}_{1i}^s &= \hat{q}_{2i} = \frac{5(a-A)\gamma - 5\delta(\beta+1)}{25b\gamma - 4(\beta+1)^2} \quad i = 1,2 \\
\hat{q}_{1i}^r &= \frac{5\gamma b(3A+2a) - 4a(\beta+1)^2 + 15\delta b(\beta+1)}{3b\left(25b\gamma - 4(\beta+1)^2\right)} \quad i = 1,2.
\end{aligned} \tag{23}$$

The comparison of the quantity used in the market each period and the comparison of the R&D investment level of renting-selling firms in the case of competitive research and in the case of cooperative research yield the following result:

Proposition 4. *If in equilibrium firms producing a durable-good rent and sell output, then the level of R&D and the quantity used in the market in the second period is higher in the case of cooperative research than in the non-cooperative case if and only if $\beta > \frac{2}{3}$. The quantity used in the first period when firms cooperate in R&D is equal to the quantity used when there is no R&D cooperation.*

Proof: See Appendix. ■

From Propositions 1, 2 and 4 it can be concluded that the critical spillover level from which cooperation in R&D increases the level of investment is the highest when firms both rent and sell their product, and is the lowest either when firms produce durable goods and rent it or when firms produce non-durable goods. Therefore, the durability of the good produced by the industry and the different practices used in the commercialization of the goods seems to be a factor to take account in enabling cooperation agreements.

Due to the problems of time inconsistency and strategic behavior of firms, the level of investment may vary depending on whether the companies rent, sell or both rent and sell the good. So, it would be desirable to compare the optimal R&D decision in the case of renting-selling firms with that of selling and renting firms.

Proposition 5. *The amount of R&D in the case of renting-selling firms is such that:*

(i). When firms cooperate in R&D, the amount of R&D in the case of selling firms is higher than the amount of R&D in the case of renting-selling firms.

(ii). When firms act non-cooperatively in R&D, the amount of R&D in the case of renting-selling firms may be higher than the amount of R&D in the case of renting firms.

Proof: See Appendix. ■

From propositions 3 and 5 and taking into account the efficient level of R&D, we have that $x^{**} > \hat{x}_i^r > \hat{x}_i^s > \hat{x}_i^{r-s}$. Thus, when firms cooperate in R&D, the amount of research which is the closest to the social optimum is the one achieved by renting firms. Hence, if cooperation agreements are allowed by antitrust authorities, the highest level of investment is obtained when firms only rent their good. When firms act non-cooperatively in both output and R&D however, the level of R&D expenditure by renting-selling firms may be greater than the level of investment by renting firms.

In the analysis it is assumed that firms can cooperate in R&D, sharing basic information and efforts at the R&D stage, but remain rivals in the marketplace. But partners who have produced inventions together may wish to cooperate in the production process as well, in order jointly to recoup their R&D investment. I would like to briefly discuss the implications of this. If firms are allowed to cooperate in the product market as well, the results for renting-selling firms are the same as those for renting firms. In a duopolistic environment when firms can rent and sell their production, the time consistency problem and dynamic interaction between competitors make them sell part of their production. But in the full cooperation case, when firms cooperate in R&D and in the production process, it is as if there were a monopoly and a monopoly never chooses to sell part of its product: it prefers to rent all units of the good. Hence, the level of investment of renting firms and renting-selling firms is the same, and higher than the level of investment of firms that may only sell their output (selling firms)[15].

The last result concerns which of the three different scenarios - renting firms, selling firms and renting-selling firms- leads to a higher level of social welfare. A comparison between these scenarios is interesting in the light of cases in which certain companies have tried to exclusively (or almost exclusively) to rent their products rather than sell them. Antitrust litigation forced these companies to make their products available for sale. Two notable cases are the United Shoe Corporation, which in the 1930s rented its equipment to shoe manufacturers, and the IBM Corporation, which in the 1960s rented mainframe computers to firms using office equipment. Contrary to the arguments often made as to why these firms should be required to sell, this paper shows that when firms cooperate in R&D, social welfare may be greater when firms rent their products than when they sell them totally or partially. To show this, consider the following example: $A = 18$, $a = 130$, $\beta = 0.4$, $\delta = 25$, $\gamma = 2$, and $b = 1$. With these parameter specifications, if we calculate and compare the social welfare and consumer surplus[16] obtained in each case, we have that $\hat{W}^r > \hat{W}^{r-s} > \hat{W}^s$ and $\hat{C}S^s > \hat{C}S^{r-s} > \hat{C}S^r$. Renting firms invest more in R&D than selling-firms or renting-selling firms, which means that their benefits are greater, which in turn

[15] In the total cooperation case, we get that the level of investment with renting firms and with renting-selling firms is $\tilde{x}^r = \tilde{x}^{r-s} = \frac{(\beta+1)(2a-A)-8b\delta}{8b\gamma-(\beta+1)^2}$, whereas the level of investment with selling firms is $\tilde{x}^s = \frac{(\beta+1)(a-A)-4b\delta}{4b\gamma-(\beta+1)^2}$. If we compare these levels of investment, we get that $\tilde{x}^r - \tilde{x}^s = \frac{(\beta+1)\left(4\delta b(\beta+1)-a(\beta+1)^2+4Ab\gamma\right)}{\left(4b\gamma-(\beta+1)^2\right)\left(8b\gamma-(\beta+1)^2\right)}$, which is always positive if we take into account the restriction $A \geq \frac{(1+\beta)(a(1+\beta)-4b\delta)}{4b\gamma}$ that assure us positive marginal production cost with renting and renting-selling firms.

[16] Let $W^j(Q)$ be social welfare and $CS^j(Q)$ be consumer surplus, where $j = r, s, r-s$, denote the cases of renting firms, selling firms, and renting-selling firms respectively.

increases social welfare. So, if firms are allowed to cooperate in R&D, social welfare may decrease when authorities require companies to sell part of their products[17].

5 Conclusions

This paper describes the optimal R&D choice of firms operating in imperfect competitive markets and producing durable goods, and shows that the time-consistency problem and dynamic reactions between oligopolists affect firms' levels of innovation.

We get that the spillover level for which R&D cooperation is preferable to noncooperative behavior is higher in a durable good firm setting, when firms sell their output totally or partially, than in a non-durable goods setting. This indicates that the durability of the product may be an important element in the debate about the desirability or implementability of policy instruments allowing R&D cooperation between firms. Thus, the topic has important policy implications as a blanket policy covering durable and nondurable goods would not work very well.

It is shown that the optimal R&D choice by renting firms is higher than the optimal R&D choice by selling firms. When firms cooperate in R&D, the optimal R&D decision by selling firms is higher than the optimal R&D by renting-selling firms. Therefore, when firms are allowed to cooperate in R&D, the highest level of R&D effort is obtained when firms only rent their good.

A result in the literature on durable goods monopolist is that social welfare is higher when firms are not permitted to rent. We find that when firms cooperate in R&D, because of the higher level of investment by renting firms, social welfare may be higher if firms only rent their good.

These results may be relevant in an economic context in which governments want to increase the level of R&D of the firms. R&D and innovation is needed

[17] When firms cooperate in R&D, if we compare social welfare for renting, selling and renting-selling firms, we can find numerical examples where any of the results are posible, that is, $\hat{W}^r \gtreqless \hat{W}^s$, $\hat{W}^s \gtreqless \hat{W}^{r-s}$ and $\hat{W}^r \gtreqless \hat{W}^{r-s}$.

to create smart, sustainable growth and get Europe out of the current economic crisis.

Some extensions are left for further research. In the present paper it is allowed innovation to occur only in period 1, that is, firms decide in the first stage what their level of investment will be, and then they decide their production levels. It would be interesting to learn whether the results change if the additional possibility of period 2 innovation is introduced.

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Appendix

This appendix gives the proofs of the propositions in the text. The following restrictions are used in the proofs:

- Renting-firms, R&D cooperation, restriction for $x_i^* + \beta x_j^* \le A$:

$$A > \frac{(1+\beta)\,(2a\,(1+\beta) - 9b\delta)}{9b\gamma}. \tag{A.1}$$

- Selling-firms, R&D cooperation, restrictions that ensure positive production figures:

$$(\delta + A\gamma)\, b < \left(\frac{32b\gamma a - 7a\,(\beta+1)^2 - 80b\delta\beta}{80} \right). \tag{A.2}$$

- Renting firms, restriction for $x_i^{*r} > 0$:

$$(2a - A)\,\gamma > \delta\,(1+\beta)\,. \tag{A.3}$$

- Renting-firms, R&D competition, restriction for $x_i^* + \beta x_j^* \le A$:

$$A > \frac{(1+\beta)\,(2a\,(2-\beta) - 9b\delta)}{9b\gamma}. \tag{A.4}$$

- Renting-selling firms, R&D competition, restriction for $x_i^* + \beta x_j^* \le A$:

$$b\,(\delta + A\gamma) \ge \frac{4a\,(1+\beta)\,(3-2\beta) - 25b\delta\beta}{25}. \tag{A.5}$$

- Renting-selling firms, R&D competition, restriction for $q_{2i} > q_{1i}^r$:

$$Ab\gamma < \frac{4a\,(1+\beta)\,(3-2\beta) + 5ab\gamma - 30b\delta\,(1+\beta)}{30}. \tag{A.6}$$

- Renting-selling firms, R&D cooperation, restriction for $q_{2i} > q_{1i}^r$:

$$Ab\gamma \le \frac{5\gamma ba + 4a\,(\beta+1)^2 - 30b\delta\,(\beta+1)}{30}. \tag{A.7}$$

- Renting-selling firms, R&D cooperation, second order condition:

$$b\gamma > \frac{4\,(1+\beta)^2}{25}. \tag{A.8}$$

- Renting firms, R&D cooperation, second order condition:

$$b\gamma > \frac{(1+\beta)^2}{9}. \tag{A.9}$$

- Selling firms, restriction for $x_i^* + \beta x_j^* \le A$:

$$Ab\gamma > \frac{(1+\beta)\,(22a\,(1+\beta) - 128b\delta)}{128}. \tag{A.10}$$

Proof of Proposition 1.

$$
\begin{aligned}
x_i^{*r} - \hat{x}_i^r &= \frac{(2-\beta)(2a-A) - 9b\delta}{9b\gamma - (2-\beta)(1+\beta)} - \frac{(\beta+1)(2a-A) - 9b\delta}{9b\gamma - (\beta+1)^2} = \\
&= \frac{9\left((2a-A)\gamma - (1+\beta)\delta\right)(1-2\beta)\, b}{\left(9b\gamma - (2-\beta)(1+\beta)\right)\left(9b\gamma - (\beta+1)^2\right)}.
\end{aligned}
$$

Since the denominator of x_i^{*r} is positive for the second order condition, the numerator of x_i^{*r} also must be positive, so it results that $(2-\beta)(2a-A) > 9b\delta$ and $9b\gamma > (2-\beta)(1+\beta)$. Multiplying these two inequalities, it results that $(2a-A)\gamma > (1+\beta)\delta$. Indeed, as the denominator of $(x_i^{*r} - \hat{x}_i^r)$ is positive for the second order conditions, we have that $\hat{x}_i^r - x_i^{*r} > 0$ if $(2\beta - 1) > 0$,.

The second part of the proof is that $Q_1^{*r} > \hat{Q}_1^r$ and $Q_2^{*r} > \hat{Q}_2^r$ if $\beta > \frac{1}{2}$.

We have that $Q_1^{*r} - \hat{Q}_1^r = Q_2^{*r} - \hat{Q}_2^r = \frac{3((2a-A)\gamma-(1+\beta)\delta)(1-2\beta)(\beta+1)}{(9b\gamma-(2-\beta)(1+\beta))\left(9b\gamma-(\beta+1)^2\right)}$. Then, it is straighforward to prove that $Q_1^{*r} > \hat{Q}_1^r$ and $Q_2^{*r} > \hat{Q}_2^r$ if $\beta > \frac{1}{2}$ ■

Proof of Proposition 2. It needs to be shown that $\hat{x}_i^s > x_i^{*s}$ for β higher than a critical value β^* that satisfies $0.6 < \beta^* < 0.65$.

We have that

$(\hat{x}_i^s - x_i^{*s}) = \frac{32\left(11a(1+\beta)+260b(\delta+A\gamma)-264ab\gamma-216b\beta\delta-476Ab\beta\gamma+440ab\beta\gamma-11a\beta^2(1+\beta)-476b\beta^2\delta\right)}{(256b\gamma-(\beta+1)(119-65\beta))\left(128b\gamma-27(\beta+1)^2\right)}$

Let us denote

$H = (11a + 11a\beta + 260b\delta + 260Ab\gamma - 264ab\gamma - 216b\beta\delta - 476Ab\beta\gamma + 440ab\beta\gamma -$
$-11a\beta^2 - 11a\beta^3 - 476b\beta^2\delta)$ and

$J = (256b\gamma - (\beta+1)(119-65\beta))\left(128b\gamma - 27(\beta+1)^2\right),$

The second order conditions assure us that the denominator J is always positive.

First, it is demonstrated that $\frac{dH}{d\beta} > 0$.

$\frac{dH}{d\beta} = 11a - 22a\beta - 216b(\delta + A\gamma) - 260Ab\gamma + 440ab\gamma - 952b\beta\delta - 33a\beta^2.$

Taking into account the restriction (A.2), we have that $11a - 22a\beta - 216b(\delta + A\gamma) -$

$260Ab\gamma + 440ab\gamma - 952b\beta\delta - 33a\beta^2 >$

$\frac{1}{10}\left(943a + 1446a\beta + 7360b\delta + 4760Ab\gamma + 592ab\gamma + 503a\beta^2\right) > 0$

Second, it is demonstrated that $x_i^{*s} > \hat{x}_i^s$ if $\beta = 0.6$.

Let us denote $G = -H$. Taking into account restriction $(A.10)$, we have that $G > \frac{11}{16}a\,(3 - 5\beta)\left(128b\gamma - 27\,(\beta+1)^2\right)$

So, if $\beta = 0.6$, then $G > 0$.

Finally, it remains to be proved that $\hat{x}_i^s > x_i^{*s}$ if $\beta = 0.65$.

Taking into account the restriction $(A.2)$, it results that

$H > \frac{a}{20}\left(143\beta - 3200b\gamma + 4992b\beta\gamma + 991\beta^2 + 613\beta^3 - 235\right).$

So, if $\beta = 0.65$, then $H > \frac{1}{20}a\,(44.8b\gamma + 444.99)$ and this expression is always positive.

Taking into account that $x_i^{*s} > \hat{x}_i^s$ if $\beta = 0.6$ and that $\hat{x}_i^s > x_i^{*s}$ if $\beta = 0.65$, that J is positive, and that $\frac{dH}{d\beta} > 0$, it results that there must exist a critical value β^* that satisfies $0.6 < \beta^* < \frac{2}{3}$ such that $\hat{x}_i^s > x_i^{*s}$ for β higher than the critical value. ■

Proof of Proposition 3. We will first prove that $\hat{x}_i^r > \hat{x}_i^s$ and then that $x_i^{*r} > x_i^{*s}$:

1. We want to show that $\hat{x}_i^r > \hat{x}_i^s$

$$\hat{x}_i^r - \hat{x}_i^s = \frac{(\beta+1)\left((-32)a(\beta+1)^2 + \gamma b(115A+58a) + 115\delta b(\beta+1)\right)}{\left(9b\gamma - (\beta+1)^2\right)\left(128b\gamma - 27(\beta+1)^2\right)}$$

As the denominator is positive because of the second order conditions, $\hat{x}_i^r > \hat{x}_i^s$ if the numerator is positive

Taking into account the restriction $(A.1)$, substituting in the numerator and simplifying, it results that $\left(-32a\,(\beta+1)^2 + \gamma b\,(115A + 58a) + 115\delta b\,(\beta+1)\right) >$

$29b\,((2a - A)\,\gamma - (1+\beta)\,\delta)$

It can be ensured that this expression is always positive by considering restriction $(A.3)$.

2. Lastly, it needs to be shown that $x_i^{*r} > x_i^{*s}$.

$x_i^{*r} > x_i^{*s}$ if $\frac{(2-\beta)(2a-A)-9b\delta}{9b\gamma-(2-\beta)(1+\beta)} > \frac{(110-66\beta)a-(119-65\beta)A-256b\delta}{256b\gamma-(\beta+1)(119-65\beta)}$.

$$x_i^{*r}-x_i^{*s} = \frac{\left(559b\delta-256a+559Ab\gamma+34ab\gamma+230b\beta\delta-329Ab\beta\gamma+82ab\beta\gamma+192a\beta^2-64a\beta^3-329b\beta^2\delta\right)}{[9b\gamma-(2-\beta)(1+\beta)](256b\gamma-(\beta+1)(119-65\beta))}$$

As the second-order conditions ensure that the denominator is positive, consider the numerator

$$(559b\delta - 256a + 559Ab\gamma + 34ab\gamma + 230b\beta\delta - 329Ab\beta\gamma + 82ab\beta\gamma + 192a\beta^2 -$$
$$-64a\beta^3 - 329b\beta^2\delta) =$$

$$= (230b(\delta + A\gamma + \beta\delta) - 256a + 329b\delta + 329Ab\gamma + 34ab\gamma -$$
$$-329Ab\beta\gamma + 82ab\beta\gamma + 192a\beta^2 - 64a\beta^3 - 329b\beta^2\delta)$$

Taking into account the restriction $(A.4)$,substituting in the above expression and simplifying, it results that:

$$(230b(\delta + A\gamma + \beta\delta) - 256a + 329b\delta + 329Ab\gamma + 34ab\gamma -$$
$$-329Ab\beta\gamma + 82ab\beta\gamma + 192a\beta^2 - 64a\beta^3 - 329b\beta^2\delta) >$$
$$\left(\tfrac{1}{9}\right)(460a\beta - 1384a + 2961b(\delta + A\gamma) + 306ab\gamma - 2961Ab\beta\gamma + 738ab\beta\gamma +$$
$$+1268a\beta^2 - 576a\beta^3 - 2961b\beta^2\delta)$$

Taking into account the restrictions $(A.5)$ and $(A.6)$ and substituting in the above expression it results that

$$\tfrac{1}{9}(460a\beta - 1384a + 2961b(\delta + A\gamma) + 306ab\gamma - 2961Ab\beta\gamma + 738ab\beta\gamma +$$
$$+1268a\beta^2 - 576a\beta^3 - 2961b\beta^2\delta)$$
$$> \tfrac{1}{450}a\left(15\,300b\gamma - 12\,532\beta + 12\,225b\beta\gamma - 3716\beta^2 + 10\,680\beta^3 + 1864\right)$$

Taking into account restriction $(A.9)$ it results that

$$\tfrac{1}{450}a\left(15\,300b\gamma - 12\,532\beta + 12\,225b\beta\gamma - 3716\beta^2 + 10\,680\beta^3 + 1864\right) >$$

$\frac{1}{1350}\left(36\,115\beta^2-34\,013\beta+10\,692\right)(\beta+1)\,a$

And this expression will be positive if $0<\beta<1$. ■

Proof of Proposition 4.

$$\begin{aligned}\hat{x}_i^{r-s}-x_i^{*r-s} &= \left(\frac{4\,(a-A)\,(\beta+1)-25b\delta}{25b\gamma-4\,(\beta+1)^2}\right)-\left(\frac{4\,(a-A)\,(3-2\beta)-25b\delta}{25b\gamma-4\,(\beta+1)\,(3-2\beta)}\right)= \\ &= \frac{100b\,(3\beta-2)\,((a-A)\,\gamma-\delta\,(\beta+1))}{\left(25b\gamma-4\,(\beta+1)^2\right)(25b\gamma+4\,(\beta+1)\,(2\beta-3))}\end{aligned}$$

Since the numerator and denominator of x_i^{*r-s} must be positive, $4\,(\beta+1)\,(a-A)>25b\delta$ and $25b\gamma>4\,(1+\beta)^2$.

Multiplying these two inequalities, it results that $(a-A)\,\gamma>(1+\beta)\,\delta$. So, since the denominator of $\left(\hat{x}_i^{r-s}-x_i^{*r-s}\right)$ is positive for the second order conditions, it is found that $\hat{x}_i^{r-s}>x_i^{*r-s}$ if $(3\beta-2)>0$ or $\beta>\frac{2}{3}$.

Now, it must be proved that $Q_2^{*r-s}>\hat{Q}_2^{r-s}$ if $\beta>\frac{2}{3}$.
It is known that $Q_2^{*r-s}-\hat{Q}_2^{r-s}=\frac{80((a-A)\gamma-(\beta+1)\delta)(2-3\beta)(\beta+1)}{(25b\gamma+4(\beta+1)(2\beta-3))\left(25b\gamma-4(\beta+1)^2\right)}$.
Then, $Q_2^{*r-s}>\hat{Q}_2^{r-s}$ if $\beta>\frac{2}{3}$. ■

Proof of Proposition 5.

1.It is to be proved that $\hat{x}_i^s>\hat{x}_i^{r-s}$

$$\hat{x}_i^s-\hat{x}_i^{r-s}=\frac{(\beta+1)\left(20a\,(\beta+1)^2-163b\delta-163Ab\gamma+38ab\gamma-163b\beta\delta\right)}{\left(128b\gamma-27\,(\beta+1)^2\right)\left(25b\gamma-4\,(\beta+1)^2\right)}$$

As the second-order conditions ensure that the denominator is positive, consider the numerator

$\hat{x}_i^s>\hat{x}_i^{r-s}$ if $\left(20a\,(\beta+1)^2-163b\delta-163Ab\gamma+38ab\gamma-163b\beta\delta\right)>0$

Taking into account restriction $(A.7)$, substituting in the expression above, it results that

$$\left(20a\left(\beta+1\right)^2-163b\delta-163Ab\gamma+38ab\gamma-163b\beta\delta\right)>\frac{13}{30}a\left(25b\gamma-4\left(\beta+1\right)^2\right)$$

And this expression will be positive as the restriction (A.8) holds.

2. It needs to be shown that $x_i^{*r} \gtreqless x_i^{*r-s}$.

Consider the following example: $a = 130$, $A = 18$, $\beta = 0.4$, $\delta = 25$, $\gamma = 2$, and $b = 1$. In this case, $x_i^{*r} = 10.29$ and $x_i^{*r-s} = 9.57$. Therefore, $x_i^{*r} > x_i^{*r-s}$.

However, if $a = 200$, $A = 10$, $\beta = 0.3$, $\delta = 35$, $\gamma = 8$, and $b = 1$, then $x_i^{*r} = 4.986$ and $x_i^{*r-s} = 5.060$. Therefore, $x_i^{*r} < x_i^{*r-s}$. ■