Economic Integration and Duplication of R&D*

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June 25, 2013

Abstract

This paper studies how economic integration and duplication of research and development activities affect growth and the position, regarding innovation, which an economy takes. We find that integration leads to ambiguous effects on growth. The direction of those effects depends directly upon the human capital stock of the partner country in integration and inversely upon the degree of duplication. If such a degree is high, an innovating economy could become chronically non-innovating after integration.

JEL Codes: O41, F15, F43

Keywords: Economic Integration; Research and Development; Endogenous Growth

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*All the remaining errors are ours.

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1 Introduction

Is economic integration always good for long-run growth? If an economy is non-innovating, could it become newly innovating after integration? If innovating, could it become chronically non-innovating? Under which conditions? This paper studies this question in a model of R&D with the possibility of duplication.

The literature that combines economic integration and endogenous growth began with Rivera-Batiz and Romer [1991] and Rivera-Batiz and Xie [1993]. Both papers use Romer's [1990] model to analyze the effects of integration within similar and dissimilar countries respectively. Integration entails free trade, free capital mobility and perfect diffusion of ideas. The last assumption means that researchers of one country have access to know designs invented in the other country. The model is characterized by R&D, which is a deterministic activity that offers monopoly power and induces research spillovers and scale effects. These features together with full-integrated R&D sectors promote and strengthen positive effects on growth in both articles. As Feenstra [1996] argues, these results are built and limited by strong informational assumptions. When integration does not include perfect diffusion of ideas, it may lead to a decline in the smaller country growth rate. About these papers, two observations arise. First, perfect diffusion of ideas seems reasonable in an integrated area. Second, free flow of knowledge does not need to imply fully merger of R&D sectors, i.e. they may not become one line of research. Considering both, simultaneous duplication in R&D, i.e. two similar designs are at the same time developed in different research sectors, is a potential fact that could diminish the benefits from economic integration. Zeira (2011) argues that duplication significantly reduces the scale effect and the rate of economic growth. Peretto (2003) also points to the changes in market structure that integration could induce due to entry/exit of foreign or domestic firms.

In a model of endogenous growth via R&D, this paper studies how duplicity and integration between similar and dissimilar countries affect growth and innovation. Stability properties are also studied. Both research sectors are assumed to be able to take advantage of the knowledge generated in their neighbor's research. However, these lines of research face a high cost to integrate their activities. In this context, duplicity of simultaneous effort in R&D remains after integration takes place. This paper shows that integration between an innovating and a non-innovating country enhances growth, however, when two innovating economies integrate, the effect on growth is ambiguous. There, an innovating economy could become chronically non-innovating after integration while integration could promote innovation in an initially-non-innovating country.

The rest of the paper is organized as follows. In Section 2, the model is described. Section 3 considers integration of two innovating countries and between an innovating and a chronically-non-innovating economy. Section IV concludes. The appendix includes proofs for results.

2 The Economy

Schumpeterian models can be classified regarding the effect caused by R&D, whether it increases the number of products or its quality. Our model follows the expanding product variety approach developed by Romer [1990]. A two-country model of endogenous technological change is considered. Each economy has infinite-lived individuals and a production side that comprises three sectors: final goods', durables', and the research sector.

Agents are homogeneous to a certain extent; a proportion is human-capital-endowed H, whereas the remainder forms an unskilled-labor force L. Population and supply of labor and human capital are fixed and constant over time. Each period individuals decide consumption $c(t)$, variation on assets $\dot{d}(t)$ and, if skilled, which sector, manufacturing or research, to work for. The representative individual's optimization problem is as follows:

$$\max_{\{c(t),\dot{d}(t)\}} \int_{0}^{+\infty} \left(\frac{c(t)^{1-\sigma} - 1}{1-\sigma} \right) e^{-\rho t} dt, \tag{1}$$

subject to the budget constraint

$$c(t) + \dot{d}(t) \leq r(t)\, d(t) + w(t), \tag{2}$$

given the initial condition $d(0) = d_0$ and the no-Ponzi game condition

$$\lim_{t \to +\infty} \left\{ d(t) \, e^{-\int_0^t r(v) dv} \right\} \geq 0, \tag{3}$$

where $r(t)$ is the interest rate, $\rho > 0$ is the discount rate, and $\sigma \geq 0$ is the inverse of the elasticity of intertemporal substitution. The term $w(t)$ represents the individual's salary. If the individual is unskilled, the salary takes the value of $w_L(t)$. If skilled, the salary is $w_{H_Y}(t)$ and $w_{H_R}(t)$ depending on whether the individual works for the manufacturing or the research sector. The individual's decision about which sector to work for will be determined by comparison of the the respective watges. Soliving the individual optimization problem (1) – (3), we obtain the following interest rate:

$$r(t) = \rho + \sigma \left(\frac{\dot{c}(t)}{c(t)} \right), \tag{4}$$

which is called the preference interest rate.

In a competitive market, final goods are produced using human capital H_Y, labor L, and durables, which can be either domestically produced x, or imported m, through the following technology:

$$Y = H_Y^{\alpha} L^{\beta} \left[\int_{A \in A_R} x(i)^{\gamma} \, di + \int_{A^* \in A_R^*} m(i^*)^{\gamma} \, di^* \right], \tag{5}$$

where $\alpha + \beta + \gamma = 1$. The index i is a continuous variable such that for all $i > A_R$, $x(i) = 0$, i.e., only already invented designs are available.

Output can be either consumed or saved as new capital, durables. Once the patent is bought, durables are manufactured through production function (5). Purchasing patents provides perfect monopoly power to firms in durables' market, i.e. patents have infinite life. Due to free entry, the discounted stream of monopoly profits price the patent of a new design. Resulting profits are

$$\pi = p\bar{x} - r\bar{x} = \left[\frac{(1-\gamma)}{\gamma} \right] r \left(\bar{x} + \bar{m}^* \right). \tag{6}$$

The patents are therefore valued by

$$P_A = \int_0^{+\infty} \pi e^{-rt} dt = \left(\frac{\pi}{r}\right) = \left[\frac{(1-\gamma)}{\gamma}\right](\bar{x} + \bar{m}^*) . \tag{7}$$

In a model of expanding product variety, R&D consists of designs for new durables. This sector is competitive, human capital intensive and has free access to the stock of knowledge, A_R. The research technology is described by

$$\dot{A} = \delta H_R A_R, \tag{8}$$

where δ is a productivity parameter, H_R is human capital in research. This technology allows for deterministic research, i.e., without uncertainty a new design is achieved. In steady state, consumption, knowledge, capital and final output growth rates coincide. Then, equation (4) can be written as

$$r = \rho + \sigma \delta H_R \left(\frac{A_R}{A}\right) . \tag{9}$$

All factors are paid their respective marginal product in the corresponding sector. The labor market condition, $H_R = H - H_Y$, and human capital wages give the proportion of human capital employed in each sector. The ratio between human capital wages in both sectors determines whether a country is either innovating or not. If the ratio equals one, both sectors are active. When the manufacturing sector gives greater wages, the research sector remains inactive; i.e. the economy is non-innovating. This ratio can be expressed in terms of technology interest rate as follows:

$$r \geq \left(\frac{\delta}{\Lambda}\right)\left[\frac{A_R (x + m^*)}{x^{1-\gamma}(A x^{\gamma} + A^* m^{\gamma})}\right](H - H_R) , \tag{10}$$

with equality in an innovating economy and inequality for a non-innovating.

An equilibrium for this model are paths for prices and quantities such that: (i) individuals solve their planning problem; (ii) firm maximize profits taking prices and interest rates as given; (iii) markets clear.

3 Prior to Economic Integration

This section analyses integration of innovating economies and between an innovating and a chronically non-innovating country. Economies are assumed to employ the same technology and share the same parameter values; they may differ in the knowledge stock, the population size and its distribution. For the time being, the domestic country is assumed to innovate, whereas the foreign one does not research. Foreign economy's variables are marked with an asterisk. Initially these economies are assumed to be completely isolated. Even though this conjecture is unrealistic, it highlights the effects of integration. By means of a subsequent comparison, let us first study the economy in isolation.

3.1 An Innovating and a Non-Innovating Economy in Isolation

In isolation, trade and diffusion of ideas between economies are not allowed, i.e. $m = m^* = 0$, A_R equals A in the innovating country and the initial stock, A_0^* in the non-innovating. Thus, technological and preference interest rates are

$$r = \left(\frac{\delta}{\Lambda}\right)(H - H_R) \text{ and } r = \rho + \sigma\delta H_R, \tag{11}$$

and

$$r^* \geq \left(\frac{\delta}{\Lambda}\right)(H^* - H_R^*) \textit{ and } r^* = \rho, \tag{12}$$

in an innovating and a non-innovating economy respectively.

An innovating economy employs a positive amount of human capital in research and reaches the following steady-state rate of growth:

$$g_{isolation} = \left(\frac{\dot{A}}{A}\right) = \delta H_R = \left[\frac{\delta H - \Lambda\sigma}{1 + \Lambda\sigma}\right]. \tag{13}$$

Note that this implies scale effects, i.e., the greater human capital endowment an economy has, the faster it will grow. If a country does not research, then both human capital in research and the growth rate are zero. The condition for being an initially-non-innovating economy is

$$H \leq \left(\frac{\Lambda\rho}{\delta}\right). \tag{14}$$

Hence the smaller human capital and the less productive the research sector is, the more likely the economy could be non-innovating.

Duplicity in R&D and isolation merit closer examination. As these economies have developed apart, it is reasonable that equivalent lines of research have grown in both economies. Separately in each country, similar goods could have been invented to cover needs that are alike. Thus, some degree of initial, pre-integration, duplicity in R&D can be expected. Nevertheless, given that these countries are isolated, this redundancy does not affect their economies.

4 Economic Integration

Integration comprehends free trade, free capital mobility and perfect diffusion of ideas. The first two conditions imply identical durables' prices, $p = p^*$, demands for durables, $m = x$ and $m^* = x^*$, and interest rates, $r = r^*$, in both countries. Perfect diffusion of ideas implies a common stock of knowledge.

About duplicity in R&D, some degree of initial, pre-integration, redundancy can be expected, Φ_0. When an innovating and a chronically non-innovating country integrate, no post-integration duplicity appears. However, when two innovating countries merge, two lines of research coexist, which are assumed to be competitors, equally efficient and face a high cost to integrate. There, some possible post-integration redundancy, Φ, should also be included.[1]

Let us characterize these redundancy parameters, Φ_0 and Φ. First, they are less than one if there is redundant effort in R&D and one otherwise. Second, if $A_i \geq A_i^*$, for all i, time intervals, we can expect that

$$\Phi_0 \in \left(\left[\frac{A_T}{A_T + A_T^*}\right], 1\right) \text{ and } \Phi \in \left(\left[\frac{A_Z}{A_Z + A_Z^*}\right], 1\right), \tag{15}$$

[1] Note that duplicity is inter-country, but not inside them. This argument could be justified by the high proportion of unsuccessful research joint ventures and the high costs associated to co-ordinating activities in a greater country.

where T denotes the date when integration took place and Z denotes the time interval between T until present. Besides, it can be clearly seen that $\Phi_0 \leq \Phi$.

By means of simplification, any degree of redundancy, if exists, is assumed to be uniformly spread within the line of research of each country. Then, the stock of knowledge available for both economies is described by

$$A_R = \Phi_0 \left(A_T + A_T^*\right) + \Phi \left(A_Z + A_Z^*\right), \tag{16}$$

that can be rewritten as

$$A_R = \Phi \left(A + A^*\right) - \left(\Phi - \Phi_0\right) \left(A_T + A_T^*\right), \tag{17}$$

when two innovating economies integrate and

$$A_R = \Phi_0 \left(A_T + A_0^*\right) + A_Z \text{ or } A_R = A - \left[\left(1 - \Phi_0\right) A_T - \Phi_0 A_0^*\right], \tag{18}$$

when an innovating and a non-innovating integrate.

This new aspect modifies our setting. Regarding patents, all projects for new designs have a probability of being part of this duplicity, $1 - \Phi'$. Once a design is duplicated, it can be either better or worse than the similar design invented in the other line, better with probability ξ. The firm with better-quality design sets the maximum price that gives the other enterprise non-positive profits, satisfying all the demand at that price and leaving none to the other firm.[2] In this case, profits are no longer monopoly gains, but either zero or $\hat{\pi} = \lambda\pi$, where $0 \leq \lambda \leq 1$, corresponding to lower and higher quality design respectively. This introduces a kind of accidental "creative destruction" in the sense that new designs destroy future rents of other products. By means of simplification, we assume that the quality jump, λ, of all better products relative to worse ones, will be identical. This makes equation (7) become

$$P_A = \Psi \left[\frac{(1-\gamma)}{\gamma}\right] \left(x + x^*\right), \tag{19}$$

[2]This is known as limit pricing.

where $\Psi = [\Phi' + (1-\Phi')\,\xi\lambda]$ and $\Phi' \leq \Psi \leq 1$. Without post-integration redundancy, Ψ is one. This parameter represents the reduction in the durable producers' profits due to redundancy in R&D.

About capital, two aspects are noted. First, under free trade, prices of imported and domestic durables coincide; so do their demands, $x = m$. Second, duplicity reduces the capital stock available. Capital will not comprise all designs invented, $x(A + A^*)$, but a smaller set, xA_R, which does not include low-quality redundant durables. Thus, the technology in the goods' sector becomes

$$Y = H_Y^{\alpha} L^{\beta} \left[\int\limits_{A \in A_R} x(a)^{\gamma}\, da + \int\limits_{A^* \in A_R} m(a^*)^{\gamma}\, da^* \right] = H_Y^{\alpha} L^{\beta} \int\limits_{A_R} x(a)^{\gamma}\, da \tag{20}$$

Then, technological and preference interest rates can be written as follows:

$$r = \Psi \left(\frac{\delta}{\Lambda}\right) \left[\frac{x + x^*}{x}\right] (H - H_R), \tag{21}$$

and

$$r = \rho + \sigma\delta H_R \Phi \left[\frac{(A + A^*)}{A}\right] - \sigma\delta H_R (\Phi - \Phi_0) \left[\frac{(A_T + A_T^*)}{A}\right], \tag{22}$$

for the domestic country and either equivalent for the foreign one, if innovating or the following when is not innovating:

$$r \geq \left(\frac{\delta}{\Lambda}\right) \left[\frac{(x + x^*)}{x^*}\right] H^*. \tag{23}$$

When two economies integrate, A_T and A_T^* are constant and either A and A^* increase over time if innovating, or just A if the partner in integration is non innovating, then the negative term in (22) becomes negligible in the long run. Then, the preference interest rate becomes

$$r = \rho + \sigma\delta H_R \Phi \left[\frac{(A + A^*)}{A}\right], \tag{24}$$

when two innovating economies integrate, and

$$r = \rho + \sigma\delta H_R, \tag{25}$$

when an innovating and a chronically non-innovating economy integrate.

4.1 Economic Integration within two Innovating Economies

When two innovating economies integrate, perfect capital mobility yields the following world interest rates:

$$r = \Psi \left(\frac{\delta}{\Lambda} \right) \left(H_W - H_{W_R} \right), \tag{26}$$

and

$$r = \rho + \sigma \delta \Phi H_{W_R}, \tag{27}$$

where the sub-index W denotes world variables, i.e., the new integrated area. Equating both world interest rates, the human capital employed in the new integrated country is found, which generates a common rate of growth

$$g_{integration} = \left(\frac{\dot{A}_R}{A_R} \right) = \Phi \delta H_{W_R} = \left[\frac{\Psi \Phi \delta H_W - \Lambda \rho \Phi}{\Psi + \Lambda \sigma \Phi} \right]. \tag{28}$$

Hence, under integration two innovating economies have a common steady-state growth rate described by equivalent relations with respect to preference and technological parameters than under isolation. Scale effects are also predicted. In addition, post-integration redundancy and the decrease in patent value affect negatively the growth rate. Thus, the higher degree of duplicity and the stronger effect of this duplicity in the patent value, i.e. smaller Φ and Ψ, the lower rate of growth we attained. This can be summarized in

Proposition 1 $g_{isolation} \underset{(\leq)}{\geq} g_{integration}$ *if and only if* $(1-\varphi) H + \Theta \underset{(\leq)}{\geq} \varphi H^*$*, where the constant* $\varphi = \left[\frac{\Phi\Psi + \Psi\Lambda\sigma\Phi}{\Psi + \Lambda\sigma\Phi} \right]$ *is positive and less than one, and* $\Theta = \left[\frac{\Phi - \Psi}{\Psi + \Lambda\sigma\Phi} \right] \left(\frac{\Lambda\rho}{\delta} \right)$ *is also positive.*

Proof. See the Appendix. ■

Some results can be inferred. First, integration does not have an always-positive effect on growth. Second, integration lowers growth when redundancy in research is high and implies a strong effect on the patent value. Those negative effects could be offset when the foreign human capital stock is enough high relative to ours. Under redundancy, there is a threshold level of human capital stock above which integration is growth-enhancing. Another question of interest is what would happen when both countries are identical. Under this setting, the answer to that ques tion is

Corollary 1 *In the case of two identical economies,* $g_{isolation} \underset{(\leq)}{\geq} g_{integration}$ *if and only if* $\Theta \underset{(\leq)}{\geq} (2\varphi - 1)\, H$.

Proof. Introduce $H = H^*$ into Proposition 1. ■

Therefore, a high degree of post-integration redundancy can lead to a common lower growth rate than under isolation for similar and dissimilar countries. Rivera-Batiz and Romer [1991] studied the same degree of integration for identical economies, disregarding redundancy, and it always enhanced growth. Our analysis complements theirs by introducing a figure that bounds their results.

4.2 Integration between an Innovating and a Non-Innovating Economy.

The initially-non-innovating country is assumed to remain so. Note that otherwise, i.e. if it became newly innovating, the study of integration would be equivalent to the carried in previous section.

Integration is clearly growth-enhancing for non-innovating economies. For innovating countries, interest rates need to be compared. In isolation, we had

$$r_{isolation} = \left(\frac{\delta}{\Lambda}\right)(H - H_R)\,. \tag{29}$$

Under integration, we have

$$r_{integration} = \left(\frac{\delta}{\Lambda}\right)(H - H_R) + \left(\frac{\delta}{\Lambda}\right)\left[\frac{x^*}{x}\right](H - H_R)\,. \tag{30}$$

[Insert Figure 1 about here.]

Human capital in research is determined by the intersection between technology and preference interest rates. Since the first term in the left-hand side of equation (30) is the interest rate in isolation and the second term is positive, integration yields higher technology interest rate. A common preference interest rate implies more human capital in research and therefore faster growth in both countries. This is shown in Figure 1, where taking values from Benhabib, Perli and Xie [1994), the preference (upward-sloped line) and the technology interest rates are simulated under isolation (the thicker from downward-sloped lines) and in integration (the remainder). This allows us to establish the following result:

Proposition 2 *Economic integration between an innovating and a chronically-non-innovating economy has an always-positive effect on growth for both countries.*

Using perfect capital mobility and definitions for durables' demands, a condition for being chronically-non-innovating can be found

$$\left[\frac{L^*}{L}\right] H_Y \geq H^*. \tag{31}$$

In this equation, human capital in the manufacturing sector is the unique endogenous variable, or using the clearing market condition, the human capital employed in research. Then the position regarding innovation, which a country has, could be altered by whichever policy or situation that would affect the amount of human capital employed in the research sector of the partner in integration. Since here integration promotes research in the domestic country, and hence employs more human capital in research, integration facilitates that a non-innovating country could become newly innovating. Hence, we can assert the next statement:

Corollary 2 *Economic integration within an innovating and a non-innovating economy promotes that the last country could become newly-innovating.*

This proposition brings up another interesting postulate related with the previous section. If integration within innovating economies lowers growth, it reduces the research activity and human capital in research falls. Therefore, the condition for being chronically-non-innovating is easier fulfilled. Consequently, we can state the following:

Corollary 3 *When economic integration within innovating countries reduces growth, an innovating economy could become chronically non-innovating.*

5 Conclusions

Under a model of expanding product variety, this paper explores how integration affects growth and the position, regarding innovation, which an economy takes. Integration between an innovating and a non-innovating country enhances growth, however when two innovating economies integrate, the effect on growth is ambiguous. There, an innovating economy could become chronically non-innovating after integration and integration could promote innovation in an initially-non-innovating country.

This paper is related to several lines of recent research: economic growth theory, industrial organization, international trade, etc. Then, the extensions could take very different approaches. Two straightforward extensions would be welfare analysis and stochastic R&D. The former would allow a normative approach to the issue. The later would make our analysis more realistic. We could see how shocks, and its persistence, would affect short and long run. On the other hand, accidental quality improvements emerged in our model. Nevertheless, in reality they occur as an intended activity. The understanding of R&D would be better approached if these aspects were considered.

The European economic integration serves to illustrate that free trade, capital mobility, etc. do not come alone, but simultaneously with governmental policy agreements. This model could be improved and the questions that could be answered multiplied by introducing policy design in our setting.

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6 APPENDIX A: STEADY STATE PROOFS.

Patent Value under Post-Integration Redundancy

Let denote Φ' the probability of being a non-redundant design. If a design is duplicated, let ξ name the probability of being better than the similar design. Profits are either zero or $\hat{\pi} = \lambda\pi$, where $0 \leq \lambda \leq 1$, for the lower and higher-quality-design producer respectively. Then, the patent value is

$$P_A = \Phi' \int_0^{+\infty} \pi e^{-rt} dt + (1-\Phi') \left[\xi \int_0^{+\infty} \hat{\pi} e^{-rt} dt + (1-\xi) \int_0^{+\infty} 0 e^{-rt} dt \right] =$$

$$\Phi' \left(\frac{\pi}{r} \right) + (1-\Phi') \xi \left(\frac{\hat{\pi}}{r} \right) = \left[\Phi' + (1-\Phi') \xi \lambda \right] \left(\frac{\pi}{r} \right) = \Psi \left(\frac{\pi}{r} \right) = \Psi \left[\frac{(1-\gamma)}{\gamma} \right] (x + x^*), \tag{32}$$

where Ψ is such that $\Psi = \left[\Phi' + (1-\Phi') \xi \lambda \right]$.

Proof of Proposition ??

The economy would grow more (less) under isolation if and only if

$$\left[\frac{\delta H - \Lambda\sigma}{1 + \Lambda\sigma} \right] \underset{(\leq)}{\geq} \left[\frac{\Psi\Phi\delta H_W - \Lambda\rho\Phi}{\Psi + \Lambda\sigma\Phi} \right], \tag{33}$$

which is equivalent to

$$\left[1 - \Phi\Psi \left(\frac{1+\Lambda\sigma}{\Psi + \Lambda\sigma\Phi} \right) \right] H + \left[\left(\frac{\Phi - \Psi}{\Psi + \Lambda\sigma\Phi} \right) \left(\frac{\Lambda\rho}{\delta} \right) \right] \underset{(\leq)}{\geq} \Phi\Psi \left(\frac{1+\Lambda\sigma}{\Psi + \Lambda\sigma\Phi} \right) H^*, \tag{34}$$

which in turns implies that

$$(1-\varphi) H + \Theta \underset{(\leq)}{\geq} \varphi H^*,$$

where the following is satisfied:

1.- The constant $\varphi = \Phi\Psi \left(\frac{1+\Lambda\sigma}{\Psi+\Lambda\sigma\Phi} \right) = \left[\frac{\Phi\Psi + \Psi\Lambda\sigma\Phi}{\Psi+\Lambda\sigma\Phi} \right]$ is positive and less than one. All terms are positive. Besides, duplicity implies $\Psi \geq \Phi\Psi$ and $\Psi\Lambda\sigma\Phi \geq \Lambda\sigma\Phi$. Hence, $0 \leq \varphi \leq 1$.

2.- The constant $\Theta = \left[\frac{\Phi-\Psi}{\Psi+\Lambda\sigma\Phi} \right] \left(\frac{\Lambda\rho}{\delta} \right)$. See that $\Psi = \left[\Phi' + (1-\Phi') \xi\lambda \right]$ and $\Phi = \left[\Phi' + \left(\frac{1}{2} \right) (1-\Phi') \right]$, since Φ covers designs in the knowledge stock: non-redundant and better-quality redundant durables. Since both lines are equally efficient, ξ should be one half. Given that $\lambda \leq 1$, then $\lambda\xi \leq \left(\frac{1}{2}\right)$ is satisfied. Then, Φ is greater than Ψ and hence Θ is positive. ■

6.1 Figures and Tables

7 PROOFS

The individual's optimization problem

The representative individual's optimization problem is as follows:

$$\max_{\left\{c(t),\dot{d}(t)\right\}} \int_{0}^{+\infty} \left(\frac{c\left(t\right)^{1-\sigma}-1}{1-\sigma} \right) e^{-\rho t} dt,$$

subject to the budget constraint

$$c\left(t\right)+\dot{d}\left(t\right) \leq r\left(t\right) d\left(t\right)+w\left(t\right),$$

given the initial condition $d\left(0\right)=d_{0}$ and the no-Ponzi game condition

$$\lim_{t\to+\infty} \left\{ d\left(t\right) e^{-\int_{0}^{t} r(v)dv} \right\} \geq 0.$$

We solve this problem by solving the following Hamiltonian:

$$\max_{\left\{c(t),\dot{d}(t)\right\}} \int_{0}^{+\infty} J\left(d\left(t\right),\dot{d}\left(t\right),t\right) dt,$$

where

$$J\left(d\left(t\right),\dot{d}\left(t\right),t\right)=\left[\frac{\left(r\left(t\right)d\left(t\right)+w\left(t\right)-\dot{d}\left(t\right)\right)^{1-\sigma}-1}{1-\sigma} \right] e^{-\rho t}.$$

We compute

$$\begin{aligned} J_{\dot{d}(t)}\left(d(t), \dot{d}(t), t\right) &= -c(t)^{-\sigma} e^{-\rho t}, \\ \frac{\partial J_{\dot{d}(t)}\left(d(t), \dot{d}(t), t\right)}{\partial t} &= \left[\rho + \sigma\left(\frac{\dot{c}(t)}{c(t)}\right)\right] c(t)^{-\sigma} e^{-\rho t}, \\ J_{d(t)}\left(d(t), \dot{d}(t), t\right) &= r(t)\, c(t)^{-\sigma} e^{-\rho t}. \end{aligned}$$

Taking into accoun that the solution of the Hamiltonian is

$$J_{d(t)}\left(d(t), \dot{d}(t), t\right) = \frac{\partial J_{\dot{d}(t)}\left(d(t), \dot{d}(t), t\right)}{\partial t},$$

we obtain

$$r(t) = \rho + \sigma\left(\frac{\dot{c}(t)}{c(t)}\right). \tag{35}$$

The optimization problem of the firms in the manufacturing sector

Final output in the home country is given by equation (5). This constant returns to scale production function leads to the usual indeterminacy in the number of price-taking firms and the scale of production for each firm. Since aggregate output is determined, we can focus on the sector's total output and derive the aggregate demand for each good x by solving a maximization problem that is conditional on given levels of H_Y and L for the industry as a whole. The maximization problem can be written as:

$$\max_{H_Y, L, x(i), m(i)} Y\left(H_Y, L, x(i), m(i)\right) - w_{H_Y} H_Y - w_L L - \int\limits_{A \in A_R} p(i)\, x(i)\, di - \int\limits_{A^* \in A_R^*} p^*(i^*)\, m(i^*)\, di^* =$$

$$H_Y^{\alpha} L^{\beta}\left[\int\limits_{A \in A_R} x(i)^{\gamma}\, di + \int\limits_{A^* \in A_R^*} m(i^*)^{\gamma}\, di^*\right] - w_{H_Y} H_Y - w_L L - \int\limits_{A \in A_R} p(i)\, x(i)\, di - \int\limits_{A^* \in A_R^*} p^*(i^*)\, m(i^*)\, di^*.$$

This problem gives the following first order conditions:

$$\begin{aligned}
w_{H_Y} &= \alpha H_Y^{\alpha-1} L^{\beta} \left[\int\limits_{A \in A_R} x(i)^{\gamma}\, di + \int\limits_{A^* \in A_R^*} m(i^*)^{\gamma}\, di^* \right], \\
w_L &= \beta H_Y^{\alpha} L^{\beta-1} \left[\int\limits_{A \in A_R} x(i)^{\gamma}\, di + \int\limits_{A^* \in A_R^*} m(i^*)^{\gamma}\, di^* \right], \\
p(i) &= \gamma H_Y^{\alpha} L^{\beta} x(i)^{\gamma-1}, \\
p^*(i^*) &= \gamma H_Y^{\alpha} L^{\beta} m(i^*)^{\gamma-1}.
\end{aligned}$$

The price of the singlefinal output good, which is freely traded, must be the same in the two countries. It is taken as a numeraire. The first order conditions for the durables yield an implied derived demand

$$\begin{aligned}
x(i) &= \left[\gamma H_Y^{\alpha} L^{\beta} p(i)\right]^{\frac{1}{1-\gamma}}, \\
m(i^*) &= \left[\gamma H_Y^{\alpha} L^{\beta} p^*(i^*)\right]^{\frac{1}{1-\gamma}},
\end{aligned}$$

and analogously in the foreign country. In equilibrium the production for a rperesentative domestic input must be the sum of domestic usage $x(i)$ and exports $m^*(i)$. Since all domestic producers face the same demand and have identical cost functions we can ignore the index i and write them as functions of the quantities. The revenue from selling a durable is $p(x)x + p(m^*)m^*$. Because it costs one unit of forgone output to produce one unit of capital, the flow opportunity cost of these units is $r(x+m^*)$. Therefore the instantaneous rate of profit earned by the holder of the patent is

$$\pi = \max_{x,m^*} \left\{ p(x)x + p(m^*)m^* - r(x+m^*) \right\}.$$

Equating each market's marginal revenue to marginal costs and substituting $p(i)$ from the FOC of the domestic and foreign firms, yields a pair of pricing equations for domestic and foreign sales:

$$p(x) = p(m^*) = \frac{r}{\gamma}.$$

These pricing equations represent identical markups over marginal cost, thus we can drop the distinction between prices for selling in the domestic country or abroad. Similar considerations for the foreign firms give a similar result. Furthermore, free flows of the final consumption good and free borrowing and lending imply that the interest rate must be the same, $r = r^*$, and therefore that $p = p^* = \frac{r}{\gamma}$.This fixed price is asssociated with constant values for $x = x^*$ and $m = m^*$ that are denoted with overbars: $\bar{x}$, $\bar{m}$.

Note that given the value of p,

$$r = \gamma p = \gamma^2 H_Y^\alpha L^\beta x^{\gamma-1}.$$

Free entry into the durables sector ensures that the discounted value of revenue minus variable costs equals design costs

$$P_A(t) = \int_0^{+\infty} \pi(s') \, e^{-\int_t^{s'} r(s)s} ds'.$$

Differenctiating with respect to time yields an arbitrage equation relating interest rate to current profits per dollar invested plus the percentage change in the value of designs overtime,

$$r = \frac{\pi(t)}{P_A} + \frac{\dot{P}_A}{P_A}.$$

We seek for a solution characterized by a constant value for P_A, in which case the arbitrage equation reduces to

$$P_A = \frac{\pi(t)}{r} = \frac{[p(\bar{x} + \bar{m}^*) - r(\bar{x} + \bar{m}^*)]}{r} = \left[\frac{(1-\gamma)}{\gamma}\right](\bar{x} + \bar{m}^*).$$

The firm in the research sector must decide how much human capital to employ. Provided the following research technology

$$\dot{A} = \delta H_R A_R,$$

the marginal product of research is

$$\delta P_A A_R = w_{H_R}.$$

It is always satisfied that

$$
\begin{aligned}
\frac{w_{H_Y}}{w_{H_R}} &\geq 1, \text{ that is,} \\
\frac{w_{H_Y}}{w_{H_R}} &= \frac{\alpha H_Y^{\alpha-1} L^{\beta} \left[\tilde{A} x^{\gamma} + \tilde{A}^{*} m^{\gamma} \right]}{\delta \left[\frac{(1-\gamma)}{\gamma} \right] (\bar{x} + \bar{m}^{*}) A_R} \geq 1, \\
r &\geq \frac{\delta}{\Lambda} \left[\frac{A_R (\bar{x} + \bar{m}^{*})}{x^{1-\gamma} \left(\tilde{A} x^{\gamma} + \tilde{A}^{*} m^{\gamma} \right)} \right] (H - H_R),
\end{aligned}
$$

where

$$
\Lambda = \left[\frac{\alpha}{(1-\gamma)\gamma} \right],
$$

And we know that

$$
r = \gamma^2 H_Y^{\alpha} L^{\beta} x^{\gamma-1}.
$$

The optimization of a firm in the research sector

$$
\max_{H_R} P_A \dot{A} - w_{H_R} H_R,
$$

where

$$
\dot{A} = \delta H_R A_R.
$$

The first order condition says

$$
w_{H_R} = \delta P_A A_R.
$$