Optimal strategies for long-term sustainability in pay-as-you-go pension systems using control theory

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February 28, 2014

Abstract

The aim of this paper is to design an automatic balancing mechanism, based on minimizing changes in the main variables that play a part in a Pay-As-You-Go (PAYGO) pension system (contribution rate, normal retirement age and indexation of pensions) using nonlinear programming. This mechanism identifies and applies an optimal path of these variables to a PAYGO system and absorbs fluctuations in longevity, fertility rates, life expectancy, salary growth or any other random events in a pension plan.

JEL Classification: E62, H55, J11, J26

Keywords: Defined Benefit, Financial equilibrium, Pay-as-you-go, Public pensions, Risk, Transparency.

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‡María del Carmen Boado-Penas is grateful for the financial assistance received from the Spanish Ministry of Education and Science (project ECO2009-13616) and from the Spanish Ministry of Economy and Competitiveness (project ECO2012-36685).

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1 Introduction

Pension provision is a topic of on-going discussion within the policy decisions of countries. The public pensions are usually based on a Pay-As-You-Go financing (PAYGO) basis where pensions for retirees are paid by active people. It is well understood that PAYGO systems require a balance between the benefits paid to the pensioners and the contributions made by the active workers. For this reason, Milevsky (2010) [8] considers a PAYGO system an extreme funding method where the sponsors provide benefits to retirees when they are due and payable and an actual fund is never accumulated. In the same line, De la Croix et al. (2013) [2] argue that the decline in fertility, increase in longevity, and the aging of the baby-boom generation will all contribute to substantial increases of the old-age dependency ratio and this has raised concern about the sustainability of the pension system.

According to The European Commission (White Paper (2012) [4]), the population of Europe by 2060 is expected to increase its life expectancy at birth for males by 7.9 years and by 6.5 years for females, when compared to 2010. Furthermore, an annual increase of around two million people aged 60 and more is reflected in the European population estimated to become almost twice as high in 2060 as in the late 1990s and early 2000s. Moreover, in 2012, pension expenditure represented a very large and rising share of public expenditure: more than 10% of GDP on average and expecting to rise to 12.5% in 2060 in the EU as a whole.

Valdés-Prieto (2006) [15] also points out that defined benefit PAYGO systems tend to require periodic adjustments because of demographic and economic uncertainty. In fact, the common trend of the crisis response is a wave of parametric pension adjustments in Europe including countries like France, Greece, Hungary, Romania and Spain, in the last four years. These parametric reforms include changes in the contribution ceilings, revaluation of past wages, increases in the retirement age or indexation on the amount of the pension.

Following this process of reforming the pension system, Vidal-Meliá et al. (2009) and (2010) [17] [18] define an Automatic Balance Mechanism [1] (ABM) as a set of pre-determined measures established by law to be applied immediately as required according to the solvency or sustainability [2] indicator. Its purpose, through successive application, is to re-establish the financial equilibrium of PAYGO pension systems with the aim of making those systems viable without the repeated intervention of the legislators. They advocate that the purposes of introducing an ABM method are three: to adapt the system to changing socio-economic and demographic conditions; to create a credible institutional framework to increase the likelihood that promises of pension payments will be respected; and to minimize the use of the pension system as an electoral weapon. As examined by Tuner (2009) [14] at least 12 countries have life expectancy indexing of benefits or automatic adjustment tied to an indicator of social security insolvency. However, Sweden, Germany, Japan, Canada and Finland are the most relevant, with Sweden the leader and pioneering of the ABM. (See for instance Tuner (2009) [14], Vidal-Meliá et. al. (2009) [17], OECD (2012) [9]).

It is important to highlight that two papers (Haberman and Zimbidis (2002) [5] and Pantelous and Zimbidis (2008) [11]) propose parametric reforms introducing the concept of a liquidity or contingency fund, in order to absorb events that might affect liquidity in a PAYGO pension plan. As emphasized by Pantelous and Zimbidis (2008) [11] this non-zero reserve fund is acting as a buffer, fluctuating deliberately (in the short run) and absorbing partially or completely

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[1] See Appendix C.

[2] Identifying the difference between the concepts of solvency and sustainability is not immediate. According to Knell et al. (2006), the term sustainability has many definitions, though it almost always refers to the fiscal policies of a government, the public sector or the pension system. One of the most widely accepted definitions in the area of pensions is that of a position where there is no need to increase the contribution rate in the future. On the other hand, the concept of solvency refers to the ability of a pension scheme's assets to meet the scheme's liabilities (See Green Paper (2010) [21])

the uncertainty in mortality, fertility rates or other random events. Afterwards, the contingency fund returns to zero when the fluctuations disappear, leaving the system at a new equilibrium point.

The aim of this paper, extending Haberman and Zimbidis (2002) [5] and Pantelous and Zimbidis (2008) [11], is to design an automatic balancing mechanism, based on nonlinear programming and minimizing of a logarithmic function, that identifies and applies an optimal path for the contribution rate, retirement age and indexation of pensions into a PAYGO system and absorbs into a pension plan fluctuations in longevity, fertility rates, life expectancy, salary growth or any other random events.

Following this introduction, the paper is organized as follows. The next section describes the model together with the numerical optimization and discusses its main properties. Section 3 shows a practical application to a simulated population and suggests how an ABM may be designed. Section 4 provides concluding comments. Finally, three appendices show a more detailed explanation about control theory, present the dynamics for total contributions and benefits, and describe the main automatic balancing mechanisms of some countries.

2 Model formulation for a PAYGO financing

Bertsekas (1999) [1] explained that mathematical models of optimization can be generally represented by a constraint set X and a cost function f that maps elements X into real numbers. The set X consists of the available decisions x and the cost $f(x)$ is scalar measure of undesirability of choosing decisions x. In general, what we want is to find an optimal $x^* \in X$ such that $f(x^*) \leq f(x), \forall x \in X$. Furthermore, non-linear programming is required for the solution where either the cost function f is non-linear or the constraint set X is specified by non-linear equations and inequalities. On the other hand, (Sontag (1998) [12]) states that control theory is the area of application-oriented mathematics that deals with the basic principles underlying the analysis and design of control systems. To control an object means to influence its behaviour so as to achieve a desired goal. [3]

The contribution rate, age of normal retirement and the indexation of pensions are the main variables that the authority can control over time. Using an objective function, described in eq.(2), we determine the dynamic system where the fund, $F(t)$, is the state variable and the contribution rate, $c(t)$, the age of retirement, $x(t)$, and the indexation of pensions, $\lambda(t)$, are the control variables. The variable $F(t)$ fluctuates deliberately to absorb changes in fertility, mortality and any other events that might affect the liquidity indicator in a pension plan. The aim of the control variables is to re-establish the liquidity of the PAYGO pension system over the time with pre-established rules without the intervention of the state.

Haberman and Zimbidis (2002) [5] propose two different models[4] to identify an optimal path of the contribution rate and retirement age. The functional objective that determines the smoothness of the path includes weight changes in the two variables. These weights reflect the expectations of the participants in the pension system as well as the underlying demographic trends. The functional objective has a quadratic form that minimizes the distance between the control variables and their pre-defined value. Using standard linearization procedures the authors obtained a closed solution to the optimal path for the control variables. Both models try to keep the control variables at the same level over time.

Following Haberman and Zimbidis (2002) [5], Pantelous and Zimbidis (2008) [11] construct a discrete-time stochastic model including several control vari-

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[3] See Appendix A.

[4] The first deterministic model is based on a continuous framework while the second one is a stochastic model in discrete time as, in practice, it is impossible to change the control variables in continuous time.

ables such as different investment strategies, contribution rates, retirement ages and levels of pension benefits. Finally, the authors allow the fund take positive or negative values. Again, the functional objective has a quadratic form and the optimal path of the control variables is trying to be constant over the time. Haberman and Zimbidis (2002) [5] and Pantelous and Zimbidis (2008) [11] does not try to find an analytical solution to the optimization problem.

Approximating a nonlinear dynamic system by employing a linear model permits the application of simple and systematic linear control techniques and the solutions can be found explicitly. On the other hand, a linearized model is not always valid. When the dynamic system does not fluctuate in a sufficiently small neighborhood around the point of equilibrium the linear model does not give an accurate solution. Furthermore, we can derive the solution of a nonlinear system straight away using analytical methods.

In this paper, we propose a different approach to control the contribution rate and the age of normal retirement. In particular, we discuss a nonlinear framework and develop an optimal path for these two control variables in a pension scheme and, additionally, we introduce a third control, the indexation of pensions. The main difference with previous papers is that the functional objective to determine an optimal and smooth path for the control variables is defined to minimize the percentage of change over time using the logarithm function. So we are introducing the idea of minimizing the growth rate of the control variables (contribution rate, normal retirement age and indexation of the pensions), instead of trying to keep them at a certain level.

The control model is derived from the basic equation in a PAYGO system and, in the same way as Haberman and Zimbidis (2002) [5], we introduce a contingency fund into the basic equation. The dynamics of the fund can be expressed as:

$$F(t+1) = J(t)F(t) + c(t)W(t,g,x(t)) - B(t,g,x(t),\lambda(t)), \tag{1}$$

where $F(t)$ is the contingency fund at the beginning of year t; $J(t)$ is the growth rate of the fund during year t; $c(t)$ is the contribution rate (control variable) during year t; $W(t)$ is the total contribution base paid at t; $x(t)$ is the age of normal retirement (control variable) fixed at t; $B(t)$ is the total expenditure on pensions at t, $\lambda(t)$ is the indexation of pensions during year t and g the growth of salaries.

The model is studied in both symmetric and asymmetric cases. Under the symmetric scenario, the model determines if the contribution rate and the age of normal retirement will be reduced if the buffer fund has a surplus or is increased in periods of deficit. For the indexation of the pensions we also impose a lower bound, so we are allowing the model to calculate the indexation within a fixed interval. Under an asymmetric design, changes in the control variables only occur when a deficit in the fund arises.

Finally, we are going to maintain the funding level above a certain minimum. The minimum level will be imposed at more than zero thus avoiding a deficit in the system.[5].

Analytically, the optimal deterministic non-linear control problem can be expressed as:

$$min_{c_t,x_t,\lambda_t}\left[\sum_{t=1}^{n}\left[\theta_1 log\left(\frac{c_{t+1}}{c_t}\right)+\epsilon_1\theta_2 log\left(\frac{x_{t+1}}{x_t}\right)+\epsilon_2\theta_3 log\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\right]\right]$$

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[5] No maximum level has been imposed.

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s.t. = \begin{cases}
F(t+1) = J(t)F(t) + c(t)W(t,g,x(t)) - B(t,g,x(t),\lambda(t)); \\
c_{min} \leq c(t) \leq c_{max}; \\
x_{min} \leq x(t) \leq x_{max}; \\
\lambda_{min} \leq \lambda(t) \leq \lambda_{max}; \\
\dfrac{c(t+1)}{c(t)} \leq \Delta c; \\
\dfrac{x(t+1)}{x(t)} \leq \Delta x; \\
\Delta\lambda_2 \leq \dfrac{\lambda(t+1)}{\lambda(t)} \leq \Delta\lambda_2; \\
\theta_1 + \theta_2 + \theta_3 = 1; \\
F(n) = 0; \\
\epsilon_1 = \dfrac{log(\Delta c)}{log(\Delta x)}; \\
\epsilon_2 = \dfrac{log(\Delta c)}{log(\Delta\lambda_1)} 1_{\{\frac{\lambda(t+1)}{\lambda(t)} \geq 0\}} + \dfrac{log(\Delta c)}{log(\Delta\lambda_2)} 1_{\{\frac{\lambda(t+1)}{\lambda(t)} \leq 0\}}
\end{cases} \tag{2}
$$

The coefficients ϵ_1 and ϵ_2 have been introduced in the growth rate of the age of normal retirement and in the indexation of the pensions to deal with the metric problems that are present due to the nature of the logarithmic function, as $c(t)$, $x(t)$ and $\lambda(t)$ have different units and constraints. Additionally, using the idea in Haberman and Zimbidis (2002) [5] we define θ_1, θ_2, and θ_3 as the weights that measure the impact that occurs when the control variables $c(t)$, $x(t)$ and $\lambda(t)$ change over time.

The dynamics behind the total contribution base, $W(t)$, and the total expenditure on pensions, $B(t)$, are complex as these variables depend on the normal retirement age, $x(t)$, and the indexation of pensions, $\lambda(t)$, respectively. [6]

To enable numerical computation methods (LaValle (2006) [7]), a family of trajectories need to be specified in terms of a parameter space and the optimization can be viewed as an incremental search in the parameter space while satisfying all constraints. As a result, nonlinear optimal control theory can be considered as a variant of nonlinear programming. It is also discussed that

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[6] For a detailed explanation see Appendix B.

the differential equations arising from dynamic programming or the minimum principle are difficult to solve analytically; therefore, in most cases, numerical techniques are used. In this paper, due to the nature of the nonlinear system, we are dealing with $3(n-1)$ control variables in the optimization problem, so the solution of the proposed model is obtained numerically and is studied in a discrete and deterministic framework and a numerical solution will be obtained.

We also determined the direction of the control variables at each step by computing the gradient of the function with respect to the parameters, while limited to move in a direction tangent to the constraints. Furthermore, a termination condition (value of the fund equal to zero) is enforced as a constraint in the optimization in the symmetric case to avoid the system to accumulate reserves at all times. Following general ideas about optimal nonlinear programming and control theory in discrete time, we will use a gradient method for optimal control. The gradient method is an active set method that works with inequality constraints. These inequalities are modified to equality using a linear slack variable (see Venkataraman (2009) [16])

3 Application

This section shows the results of the evolution of the contribution rate, normal retirement age and indexation of pensions using control theory in a dynamic nonlinear example, representing a generic defined benefit PAYGO pension system. If the three control variables are used, the minimum value for the contribution rate, age of normal retirement and indexation of the pensions are respectively 15%, 65 and -2% and the maximum values are 20%, 68 and 2%. It is also assumed that the change in the contribution rate varies between 0.3% and 0.5%, the age of normal retirement between 1.5 and 3 months and the indexation of the pensions between between -1% and 1%. The accumulated fund increases at an annual rate of 3% while the annual salary growth is equal to 2.5%

As a case study, the optimal nonlinear programming has been applied using two different structures of the population and a same level of salaries and pensions over time. Figure 1 shows the two different structures of population that we are using in this numerical example [7]. The graph represents a mature population versus an ageing population to highlight the differences.

Figure 1: Mature population versus ageing population in year 0 and year 20

There are 2.96 contributors who finance each pensioner under the first population scenario, compared with 2.29 under the ageing population. This ratio worsens over time, reaching values of 1.97 and 1.56 respectively in 20 years.

The initial pension is assumed to be 60% of the total contribution base in a particular year. Figure 2 shows the salary and pension structure.

Figure 2: Initial salary and pension structure

One of the main characteristics of this model is that it allows the system to accumulate surpluses. In this sense, we could design both an asymmetric and

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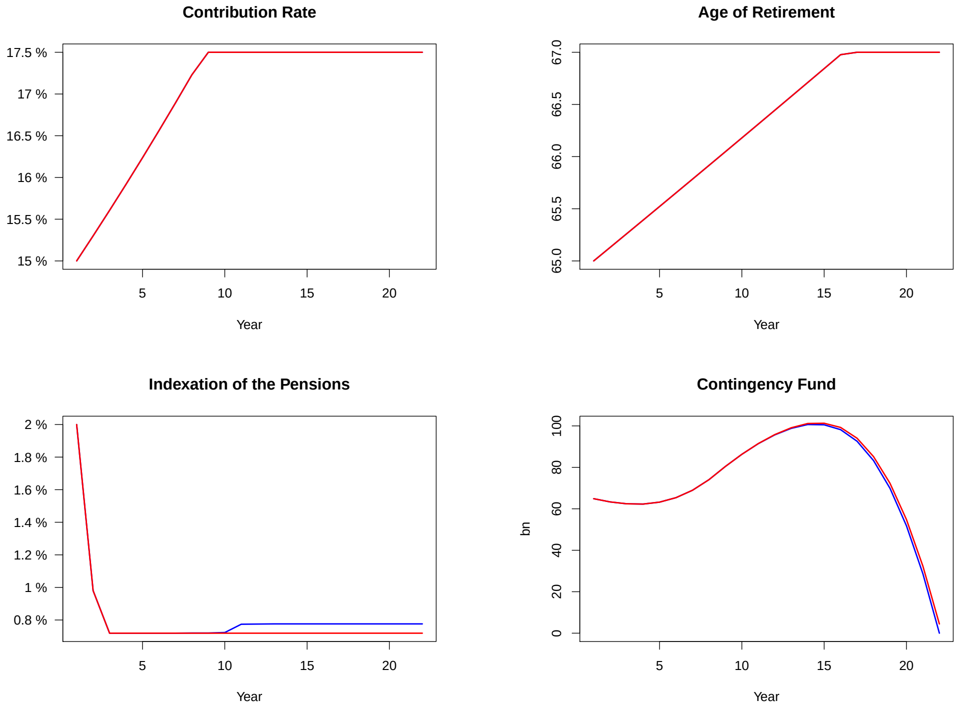
[7] For comparison purposes the population structure by ages has been normalised.

a symmetric design of the pension plan[8].

For the mature population, under both the symmetric and asymmetric design, the adjustment in the contribution rate increases from 15.0% to 17.5% after 9 years, remaining unchanged after that. Similarly, the age of normal retirement increases from 65 to 67 over a period of 17 years.

Under the symmetric scenario, the indexation of the pensions decreases at the beginning of the period and then increases after ten years in order to redistribute the surpluses that have been accumulated during the period. It can be shown that the optimal path of the indexation of the pensions stabilizes around the value of 0.77% after ten years. However, under the asymmetric design, the indexation decreases and stabilizes around the value of 0.72%.

Figure 3: Evolution of the control variables for the symmetric (blue line) and asymmetric (red line) with the mature population structure



Unsurprisingly, the fund presents an accumulation period during the first

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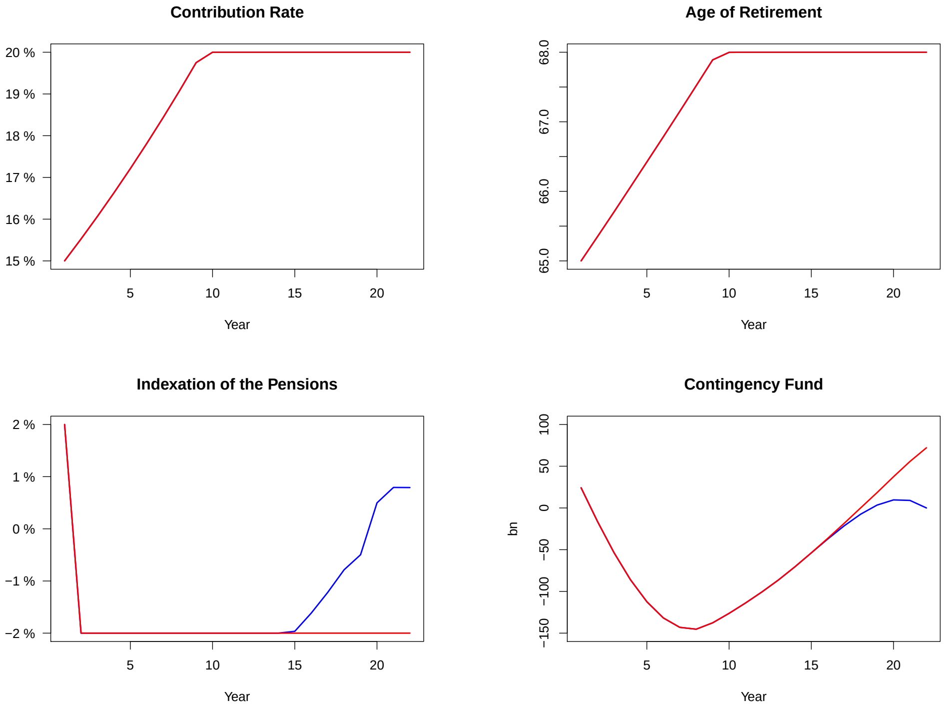
[8] In some cases both the asymmetric and symmetric designs provide the same results, see for example figure 5, 6 and 9.

15 years and a de-accumulation period over the latter years. The main reason for this is to absorb fluctuations in the population owing mainly to changes in longevity, fertility rates, life expectancy and salary growth. As expected, under the asymmetric design, the fund accumulated more funds than under the symmetric scenario. Figure 3 shows these results.

Since the Population 2 is older, the changes in the control variables are higher. For example, under the symmetric design the contribution rate needs to increase from 15 % to 20%, the age of normal retirement from 65 to 68, and the growth of the pensions needs to decrease by an annual 2% over a 15 year period. The fund is in deficit for the first 17 years. The difference with the asymmetric design is that the indexation decrease by 2% over all the period and the contingency accumulated a higher amount in the lasts years of the study. Results are shown in Figure 4.

Figure 4: Evolution of the control variables for the symmetric (blue line) and asymmetric (red line) with the ageing population structure

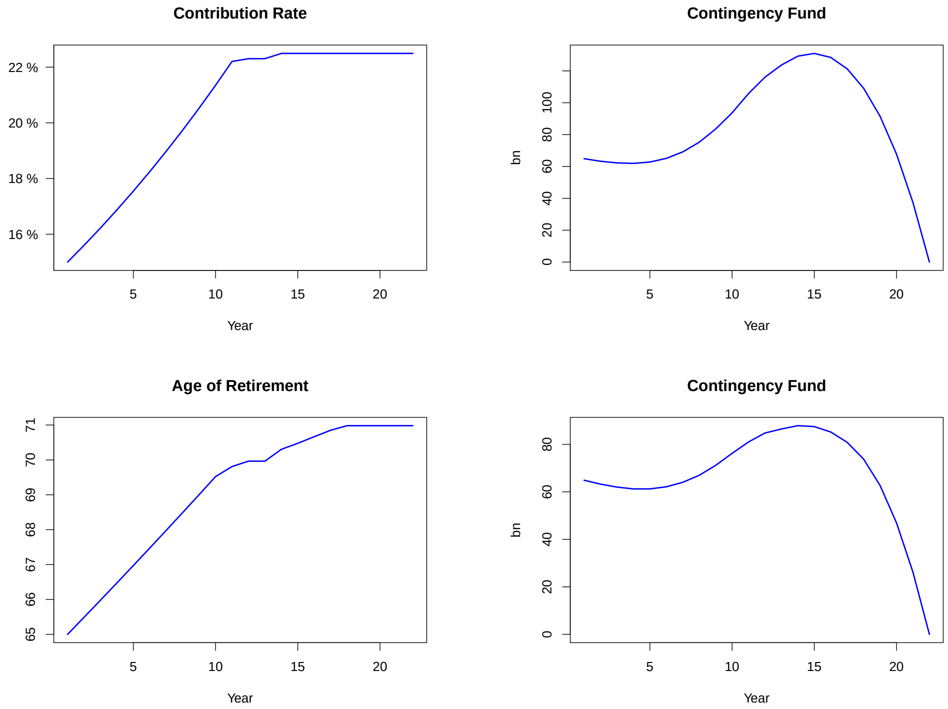


However, these results indicate that the control variables are increasing more due to the ageing population to make the system balance.

The results of controlling only one variable, for example, the age of normal retirement or the contribution rate for Population 1, are shown in Figure 5.

As expected, the increase in the control variable needs to be higher than before. The age of normal retirement would need to increase to 71 years whereas the contribution rate would increase to 22.5%.

Figure 5: Results after controlling only one variable with the mature population structure

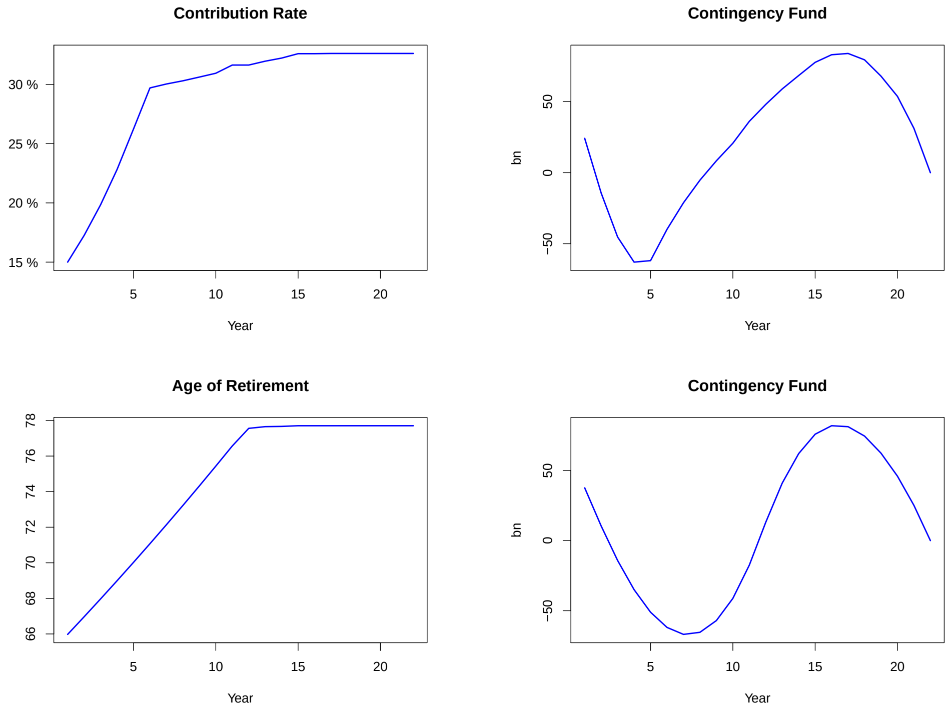


For Population 2 the results are even worse. As shown in figure 6, the age of normal retirement and contribution rates need to increase to 78 years and 32% respectively.

3.1 Results after expected and unexpected shocks

In this section, expected and unexpected shocks are incorporated into our analysis in order to see how the fund absorbs these effects.

Figure 6: Results after controlling only one variable with the ageing population structure



In the case of expected shocks, we assume that these are known at the start of the study.

As an example we consider a specific cohort with a lower life expectancy, what is translated into fewer pensioners at the end of our study period (figure 7).Figure 7 shows how, in the symmetric case, the surplus under this scenario is redistributed and the result of this is a decrease in the contribution rate and age of normal retirement, and an increase in the indexation of the pensions. However, under the asymmetric design, no surpluses are redistributed and as a result of this, the value of the fund is higher. Also, the contribution rate and the age of normal retirement remain in their maximum value and the indexation of the pension in the minimum.

Another expected change that is analysed is a change in the structure of the population (Figure 8). In this case, the two different structures of population are combined in this numerical example, so population scenario 1 is used from year 0 to year 10 and population scenario 2 is used after that. It can be seen how the fund is absorbing the changes in the number of contributors and the control variables are increasing in order to maintain the liquidity at the end of the analysis. Under the asymmetric design the indexation of pensions decreases and stabilizes around the value of -2%. As a result, the fund accumulates a higher value.

Figure 7: Expected shock for the symmetric (blue line) and asymmetric (red line): cohort with lower life expectancy

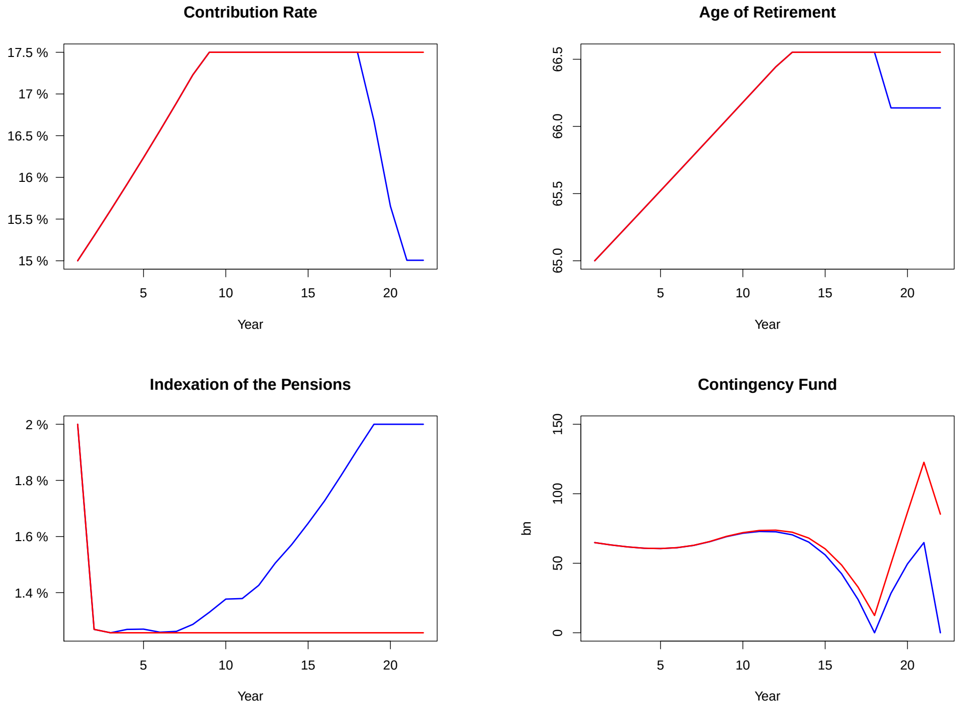
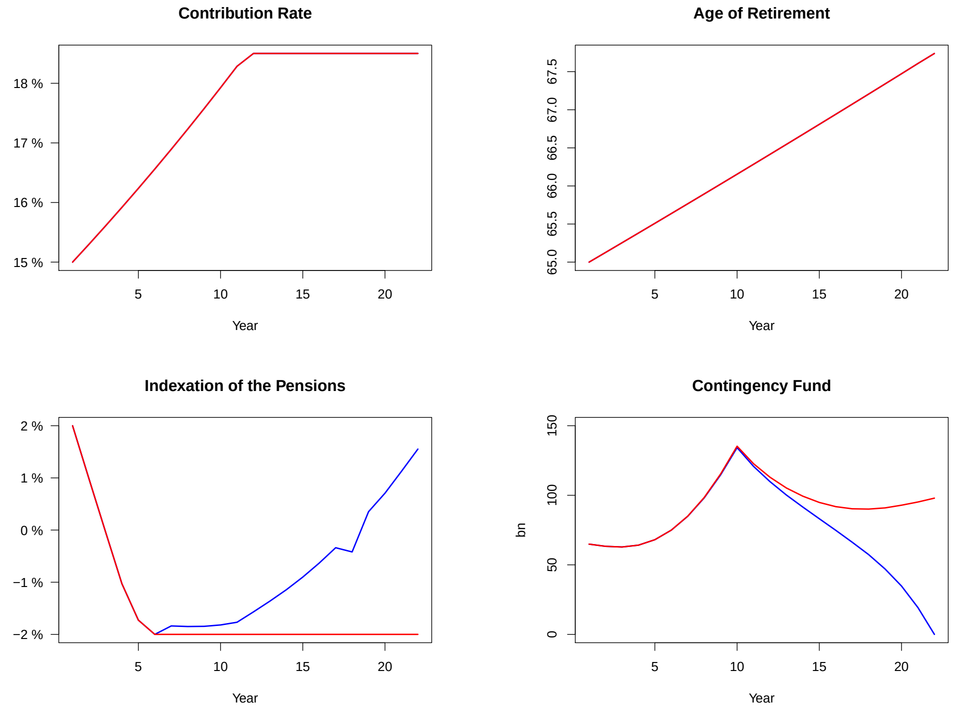
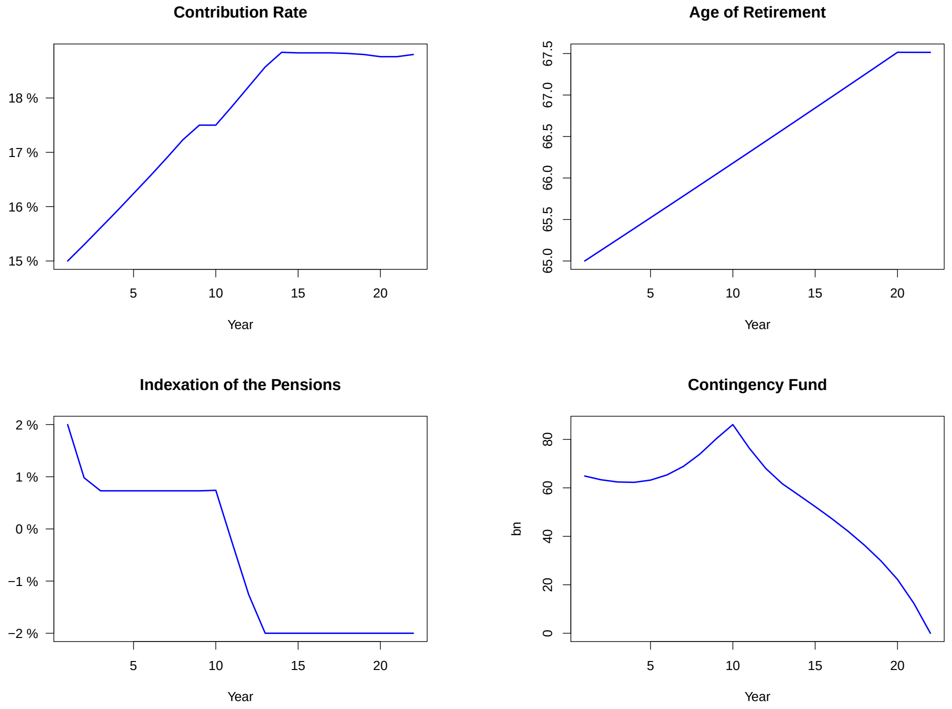


Figure 8: Expected shock for the symmetric (blue line) and asymmetric (red line): two different structures of population are combined



In the case of unexpected shocks, we assume that the total number of contributors decreases suddenly in year 10. As we can see in Figure 9, the contribution rate increases after being stable for 2 years whereas the indexation of the pensions decreases from 0.74% to -2.0% and the age of normal retirement increases to the age of 67.5. In this case no surpluses arise that can be redistributed.

Figure 9: Unexpected shock: number of contributors suddenly decreases



4 Conclusions

This paper extends the ideas first proposed by Haberman and Zimbidis (2002) [5] and Pantelous and Zimbidis (2008) [11] and develops a model for the optimal nonlinear problem in a pension scheme including the indexation of the pensions as an additional control variable. The aim of this automatic balancing mechanism proposed is to re-establish the liquidity of PAYGO pension systems without the repeated intervention of the legislators.

The model presented in this paper could be an alternative to the traditional parametric reforms of the PAYGO systems around the world. We obtained, for a representative example, an optimal level of the contingency fund given an optimal growth of the contribution rate, age of normal retirement and indexation of the pensions based on an average interest rate of 3%. We show that the contribution rate, age of normal retirement and indexation stabilizes at the end of the period of analysis. The results are completely consistent with the theoretical analysis that we could expect. The contingency fund absorbs the fluctuation of the demographic patterns and the economic variables involved in the analysis.

When we first establish an ABM of this type, we need to set up the number of control variables to be included. The main advantages of a mechanism of this type is to guide the system back onto the road to long-term liquidity and at the same time to automate the measures to be taken, isolating them from the political arena, avoiding any delay and lack of time perspective. However, this ABM also allows some flexibility in the sense that the number of variables to be controlled can be changed to adapt the system to a specific situation. At the same time, it is also possible to impose more restrictions to the model to keep, for example, the normal retirement age or the contribution rate constant during some years, making the ABM more applicable in practice.

Appendices

A Control Theory

Control Theory is based on the idea that a good model of the object to be controlled is available and that one wants to somehow optimize the behaviour. The central tool here is the use of feedback in order to correct for deviations from the desired behavior.

In our model, the contribution rate, $c(t)$, the age of retirement, $x(t)$, and the indexation of the pensions, $\lambda(t)$, are controlled over time by an objective function, described in eq.(2), that determines the dynamic system where the fund, $F(t)$, is the state variable that fluctuates deliberately. As a result of the general ideas about optimal nonlinear programming and control theory in discrete-time, we will use a gradient method for optimal control.

With nonlinear constraint equations the model and constraint equations were expanded in a Taylor series, and only the first order terms were retained. Then with these linear equations, the constraint equations could be used to reduce the number of independent variables.

As was argued by LaValle [7] the differential equations arising from dynamic programming or the minimum principle are difficult to solve analytically; therefore, in most cases, numerical techniques are used. In the same vain as [7] we determined the direction in each step by computing the gradient of a cost functional with respect to the parameters while constrained to move in a direction tangent to the constraints. Furthermore, in the symmetric case a termination condition is enforced as a constraint in the optimization, no matter that this remove degrees of freedom from the optimization and more trajectory parameters are needed (Lavalle [7]).

B Modelling of the total contributions and benefits

The dynamics behind the total contribution base, $W(t)$, and the total expenditure on pensions, $B(t)$, are complex. For $W(t)$ we considered the dynamic of the population as an input and we model the total contribution base considering the total active workers at the age of normal retirement, $x_0(t)$, over the time. We assume an average wage for the population.

$$W(1) = \left(\sum_{x=20}^{x_0(t)-1} l_{x,1} * wage(x) \right) \tag{3}$$

For $t > 1$ we have:

$$W(t) = \left(\sum_{x=20}^{\lfloor x_0(t) \rfloor -1} (l_{x,t}) + (x_0(t) mod \lfloor x_0(t) \rfloor) l_{\lfloor x_0(t) \rfloor ,t} * wage(x) \right) * (1+g) \tag{4}$$

Where $l_{(x,t)}$ is the number of people alive at age x in time t and is distributed uniform over the year. g denotes the growth of the salaries. $\lfloor y \rfloor$ and $\lceil y \rceil$denotes the floor and ceiling functions that maps a real number to the largest previous or the smallest following integer, respectively. The *mod* (modulus or modulo) operation finds the remainder of division of one number by another.

Modelling $B(t)$ is more complex as we are assuming that the indexation of the pensions, $\lambda(t)$ is dynamic over time. The expenditure on pensions at year 1 are modelling thus the observing average pension at year 1 and the projected number of individuals through projected mortality tables. Mathematically:

$$B(1) = P_{x_0(t),1} l_{x_0(t),1} + P_{x_0(t)+1,1} l_{x_0(t)+1,1} + P_{x_0(t)+2,1} l_{x_0(t)+2,1} + ... = \sum_{x=x_0(t)}^{X} P_{x,1} l_{x,1} \tag{5}$$

Where P is the average pension paid. X denotes the last year of survival.

For $t > 1$, the total expenditure on pensions is modelling as follows:

$$B(t) = \left(1 - (x_0(t) mod \lfloor x_0(t) \rfloor) l_{\lfloor x_0(t) \rfloor, t}\right) * P_{\lfloor x_0(t) \rfloor, t} + \sum_{x=\lceil x_0(t) \rceil}^{X} P_{x,t} l_{x,t} \tag{6}$$

If the indexation of pension, $\lambda(x)$, is included as a control variable we can rewrite $P_{x,t}$ as:

$$P_{x,t} = P_{x-1,t-1} * (1 + \lambda(t-1)) \tag{7}$$

With $P_{x_0(t),t} = P_{x_0(t),t} \mathbb{1}_{\{x_0(t),t=1\}} + P_{x_0(t),t} * (1+g)^t \mathbb{1}_{\{x_0(t),t>1\}}$.

Hence, the model proposed minimizes the effects caused by fluctuations in the variables involved in the PAYGO system by minimizing changes in the contribution rate, age of normal retirement and the indexation of the pensions over the time.

C Automatic balancing mechanisms

We briefly mention some of the principals countries that have an ABM. For a deep understanding see Boado-Penas 2009 [17], Tuner 2007 [13], Tuner 2009 [14], OECD 2012 [9] and OCDE 2013 [10]. The list is sorted alphabetically.

Country	Type	Main Characteristic
Canada	DB	Actuarial projections are carried every 3 years (time horizon of 75 years), if the plan is not financially sustainable a (semi-automatic) mechanism will be triggered by increasing the contribution rate by the amount necessary to cover 50% of the deficit, and the benefits are frozen for three years.
Finland	DB	The life expectancy coefficient automatically adjusts the amount of pensions in payment as longevity increases (or decreases).

Table 1 – continued from previous page

Country	Type	Main Characteristic
France	DB & points	Frances automatic adjustment mechanism operates maintaining constant the ratio between the duration of activity and the expected duration of retirement. However, the government has the right to not make these adjustments if labour market conditions do not support the extra years of work.
Germany	DB	The formula for revaluing pensions in payment includes a sustainability factor that takes into account the system's rate of dependence.
Italy	NDC	Italy uses a transformation coefficient. This coefficient is reviewed every three years in line with changes in mortality rates at different ages up to 2019 and every two years after that date. However, the adjustment is not completely automatic, because it requires legislative approval.
Japan	DB	A Modified Indexation it is applied to both the revaluation of the contribution bases, and the revaluation of pensions in payment. It takes into account both improvements in life expectancy and population increases (or decreases). It could be modify every five 5 years, due to the actuarial report.
Latvia	NDC	Latvia uses unisex life expectancy at retirement age to convert the NDC account balance to an annuity. It bases life expectancy on projected cohort life tables, which are adjusted annually.
Norway	NDC	The system has unisex life-expectancy indexing of benefits at retirement. The life expectancy divisors are determined for each cohort, based mainly on remaining life expectancy.
Poland	NDC	Poland uses an annuity divisor which is revised annually. It is based on average life expectancy at retirement age.
Portugal	DB	The pension reform includes an indexation to its social security benefits for improvements in life expectancy. The reduction of benefits is based directly on the percentage change in life expectancy.
Sweden	NDC	The ABM is triggered if a solvency ratio is less than one and it consists basically of reducing the growth in pension liability. The automatic balance mechanism only is used when the system reaches a certain point with respect to future solvency.

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