On the Complementarities between Human Capital and Public Revenues: Consequences for Develompment*

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Abstract

Empirical evidence shows that while the return of education at micro level is very high in low income countries, the relationship between human capital and economic performance is weak. This paper proposes a new theory to explain this puzzle which is based on the allocation of human capital to public and private sector along the development process. We present a model in which human capital has four uses: to produce private goods (private sector), to produce public goods, to collect taxes by public bureaucrats and to provide public education. Countries with low per capita income are characterized by low levels of human capital with a high return. A small portion of this human capital is devoted to the public sector, which involves low tax collection and low public expenditure. The transition in these countries is characterized by a growing share of human capital devoted to bureaucrats and public education and a declining share devoted to the private sector. This may explain why the increase of human capital does not have the impact on the production that it would be expected from the high private return on human capital.

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1. Introduction

One of the most intriguing puzzles in the growth and development literature is the role of the human capital. From a theoretical point of view there is no doubt about the positive impact of human capital in economic growth. Growth theory recognizes the contribution of human capital in growth process since the seminal contributions of Lucas (1988) and Romer (1990). However, the empirical macroeconomic literature is surprisingly mixed and ambiguous. While there is clear empirical evidence about the significant returns to education at micro level (see Card, 1999, for a survey), the empirical macroeconomic literature finds not only a weak relationship between human capital and economic performance but also shows a negative impact of human capital. Both, cross-sectional studies (Kyriacou, 1991, Benhabib and Spiegel, 1994, Nonneman and Vanhoudt, 1996, Pritchett, 2001) as more recent panel data studies (Kumar, 2006, Bond, Hoeffler and Temple, 2001, Caselli, Esquivel and Lefort, 1996; Islam, 1995) report insignificant or negative impact of human capital on economic growth, constituting a puzzle which has attracted the interest of many researchers in Growth and Economic Development.

This empirical puzzle is revealed even more intriguing when it is considered jointly the observation that the return on education is not the same among countries. Psacharopoulos (1994), Psacharopoulos and Patrinos (2004) and Strauss and Duncan (1995) evidence that the return to education in developing countries is extremely high and it declines with per capita income, what would imply developing countries growing at faster speed than developed countries. Nevertheless, it is also well documented (see Easterly 1994, 2001), that such a high return to education do not always lead to a successful growth process. This suggests that the relationship between human capital and growth performance is even weaker in developing countries.

This paper offers a new explanation to understand why there exists a weak relationship between human capital and economic growth in developing countries. The theory that this paper proposes is based on three other stylized facts. First, tax revenues in developing countries are very low due to tax evasion. Easterly and Rebelo (1993.a, 1993.b) and Gordon and Li (2009) find that the important differences in per capita GDP tax revenues between developed and developing countries cannot be explained by differences in statutory tax rates. While the maximum statutory personal income tax rate on average in developed countries is 1.23 times than in developing countries, the ratio between personal income tax revenue and GDP is 5.3 larger. These number reveal the difficulty of governments to raise taxes in developing countries. Second, an important portion of workers with human capital engage the public sector (see Schündlen and Playforth, 2010, Schmitt, 2010 and Pritchett, 2001). Third, in spite of there exists the concern about some enrollment of educated bureaucrats in rent-seeking activities, there is also evidence of generating positive effects. A report by the World Bank (1994) shows that the most successful of developing countries have had strong and active governments and highly educated bureaucrats.

This paper proposes a model in which public revenues determine the amount of public expenditure on education, which is an important factor to produce human capital. In the model the human capital has four uses: to produce private and public goods, to collect taxes (bureaucracy) and to produce human capital throughout the public education system (teachers). The level of tax collection depends positively on the size of the bureaucracy. A portion of public revenues is used to produce the public good and the remaining portion is used to pay bureaucrats and to finance the educational system (teachers). Thus, a feedback process arises: a higher level of human capital implies more efficient bureaucrats, who collect more taxes that are used to finance a public expenditure on education which in turn promotes human capital. Insofar human capital is growing throughout the transition to the steady state, the amount of collected taxes, the public expenditure on education and the size of bureaucracy also increase. There is a certain level of human capital in which the bureaucracy reachs the point in which the tax collection is maximized, and the effective tax rate coincides with the statutory tax rate. After this level, the bureaucracy sector remains stationary and any increment in the human capital is devoted only to the provision of education and to produce goods.

Under certain conditions, the feed-back process described above may generate multiple steady state because of public revenues at the steady state increases with public expenditure on education, which fosters human capital accumulation and therefore increases the per capita income and public revenues. Thus, a poverty trap characterized by a "vicious circle" may exists: public revenues are low because human capital and income are low due to the scarcity of public expenditure on education, which can not be raised due to the low public revenues. Nevertheless we disregard this cases and concentrate in the transition to an unique steady state.

In this paper, a portion of human capital is devoted to the government bureaucracy, which is needed to collect taxes. The fact that not all the human capital is devoted to public education or to produce goods may explains why, at least temporally, the human capital shows a low impact in the economy. The model involve a structural change throughout the transition which involves a reallocation of human capital to different uses: production of private and public goods, bureaucracy to collect taxes and the public education. When the per capita human capital is low, the scarcity of the human capital implies high private returns on education, encouraging the human capital accumulation. Nevertheless, on the other hand, there is also a low level of tax collection which implies a low expenditure in public education (teachers) the economy. Given the fact that public education (teachers) is a significant input in the production function of human capital, the low provision of education implies low return of human capital. This second mechanism on the return of human capital reduces partially the high private incentive to accumulate human capital, decelerating the human capital accumulation. When the per capita human capital is low, the share of human capital devoted to bureaucracy (tax collection) and the public education increases throughout the transition. This paper suggest that the weak relationship between human capital and the economic performance may be due to this reallocation of human capital toward bureaucracy and the public education system, which are important and necessary, but detract resources from producing good and have a social returns that may be obtained with a great delay over time.

Other interesting result of the paper is the importance of the allocation of tax revenues in its different uses, like expenditure in public good or expenditure in the public educational system, and how this allocation do not only have temporally effects throughout the transition, but also in the long run. We have proved that if the share of public revenues which is devoted to provide the public good decreases and consequently, it increases the amount of public resources devoted to finance the public education, then the levels of consumption and human capital increase in the steady state. Consequently, countries with the same initial human capital level and identical characteristics could have different structural process and economic results depending on how much they decide invest in human capital. This observation is in the line of the argues of Temple (1999), which finds that the impact of human capital on the economy is different among countries.

The model also accounts for the well documented fact that the higher is the education of the bureaucracy, the better is the economic performance of the economy. However, it is worth to notice that the mechanism proposed in the paper does not excludes the possibility that some part of the bureaucrats may not be involved in unproductive or rent-seeking activities (see Blackburn, Bose, and Haque 2006, Mauro, 2004). In the context of the paper, this kind of corruption would imply less effective tax collection. We also have proved that an improvement in the technology of the bureaucracy, this is, a gain in the efficiency which implies that the level of tax collection increases with the same number of bureaucrats, produces a reallocation of human capital form bureaucracy to the public education system, an increase in the total amount of human capital and a higher level of consumption and per capita income. Therefore, according to the prediction of the model, the impact of human capital may be delayed by a low level of the bureaucracy efficiency.

Besides the technical and empirical problems in measuring the effects of the human capital argued by the literature (see among others Freire-Serén, 2002, Soto, 2006, Cohen and Soto, 2007), some researchers have attempted to provide explanations for the insignificant or negative impact of human capital on economic growth. They share the vision of Temple (1999), which argues that for solving this puzzle it is needed to accept that the impact of human capital on growth has not been the same among countries. One strand of the literature highlights that conventional factors of production such as physical capital, human capital and technology are not the only driving mechanisms behind the growth performance (Easterly and Levine, 2002, Acemoglu, 2009). From this strand, the importance of institutions and corruption are suggested as ones of the main causes of the economic growth (see Hall and Jones, 1999; Acemoglu et al., 2001 for the institutional approach and Mauro, 2004, Blackburn, Bose, and Haque 2006 for the corruption one). Other strand of the literature focuses on characteristics of the human capital sector. North (1990) points out the possibility of an allocation problem in human capital: if the demand for human capital comes in some extent from individually remunerative yet socially wasteful activities, in this case, the education could rise the wage of each individual (producing the micro evidence), even while increase in average education would cause aggregate output to stagnate or fall (producing the macro evidence). Princhett (2001) also argues that if schooling quality may be so low that it does not raise cognitive skills or productivity (producing the macro evidence), this result could even be consistent with higher private wages in the case of education serves as a signal to employers of some innate ability.

The rest of this paper is organized as follows. Section 2 presents the basic elements of a model where the human capital has three different uses: to produce goods, to produce education (teachers) and to produce bureaucrats. Section 3 analyses the behavior and decisions of agents in the economy. Section 4 characterizes the steady state of the economy. Section 5 describes the dynamics of the economy. Section 6 shows the results of several experiments and finally, section 7 summarizes. All the proofs are included in the Appendix.

2. Model

2.1. Human capital

Human capital is the unique reproducible factor in the model. However, it has four different uses: it contributes to produce the private good and the public one, it produces public services which are used to produce human capital (teachers) and, it is used by the government to produce bureaucracy services which are needed to collect taxes. An intuitive way to think in this setting is to consider human capital as qualified workers who are allocated among the four different sectors, where some of them are hired to produce goods (private or public), others are hired by the government as teachers to produce more human capital and the rest are hired by the government as bureaucrats to collect taxes. Formally,

$$H = H_c + H_g + H_h + H_b \tag{2.1}$$

where H is the total amount of human capital in the economy, H_c denotes the amount of human capital that is devoted to the production of the private good, H_g denotes the amount of human capital that is devoted to the production of the public good, H_h the amount of human capital invested by the government to produce human capital and H_b the amount of human capital which is used by the government to generate the bureaucracy needed to collect taxes. The total amount of human capital devoted to production of private and public goods is denoted by $H_y = H_c + H_g$.

We adopt the assumption that the unique reproducible factor is the human capital in order to simplify. We want to focus our attention on the dynamics of the human capital throughout the transition to the steady state equilibrium of the economy and, specially, on the human capital reallocation among the different sectors of the economy. Consequently, this simplification assumption is perfectly justified, since the introduction of another reproducible factor would not alter at all the reallocation mechanisms of the human capital along the transition.[1].

[1] To this respect, Bucci and Segre (2011) consider the same assumption to study the effects of the interaction between cultural and human capital.

2.2. Technology in the production sector

Time is continuous with an infinite horizon. The economy is populated with many identical dynasties of homogeneous agents. Production of goods can be devoted to private and public consumption and to investment in human capital. The production technology of this good is given by the Cobb-Douglas production function:

$$Y(t) = AL(t)^{1-\alpha} H_y(t)^{\alpha} = C(t) + I(t) + G(t)$$

where $Y(t)$ denotes aggregate production of goods, $H_y(t)$ the human capital devoted to the production of goods and $L(t)$ the labor at t; $A \in R_{++}$ is the total factor productivity and $\alpha \in (0,1)$ is the share of the human capital, $C(t)$ denotes (aggregate) consumption, $I(t)$ denotes (aggregate) investment in human capital and $G(t)$ denotes (aggregate) public good provided by the government and consumed by households. It also can be rewritten in per capita terms as:

$$y(t) = Ah_y(t)^{\alpha} = c(t) + i(t) + g(t)$$

where $y(t)$ denotes the per capita income, $h_y(t)$ the per capita human capital devoted to produce goods, $c(t)$ the per capita consumption, $i(t)$ the per capita investment in human capital and $g(t)$ the per capita public good.

The accumulation equation of human capital is at follows:

$$\dot{h}(t) = h_h(t)^{\xi} i(t) - (\delta + n) h(t) \tag{2.2}$$

where $h(t)$ denotes the per capita human capital, $h_h(t)$ the per capita human capital hired by the government to produce human capital (teachers), $n > 0$ denotes the positive population growth rate, $\delta \in [0,1]$ is the depreciation rate and $\xi > 0$.

If $\xi + \alpha = 1$, then the model would become an endogenous growth model with permanent growth. Since we are focused on the evolution of human capital uses throughout the transition to the steady state, we prefer to concentrate in the neoclassical case which allows us to analyze such transition. Thus, we assume that $\xi + \alpha < 1$.

2.3. Household

There are many identical households, each of them with an infinite number of agents. The preferences of a household are given by a time separable utility function:

$$\int \left[u\left(c\left(t\right)\right)+v\left(g\left(t\right)\right)\right]e^{-(\rho-n)t}dt$$

where $c\left(t\right)$ denotes the household's per capita consumption at period t, $g\left(t\right)$ denotes the household's per capita public good at period t, $\rho \in R_{++}$ denotes the utility discount rate, $u\left(.\right)$ is continuous and twice differentiable in R_{++} and strictly concave. We assume that the function $v\left(.\right)$ is increasing. If the function $v\left(.\right)$ is strictly increasing, this means that the public good is valuable for households. We could also consider the case in which the function is a constant, which means that the public good is not valuable for household and consequently, is a waste of resources.

It is assumed that each member of the household has one unit of labor and households have $h\left(t\right)$ units of per capita human capital, where $h\left(t\right)$ denotes also the amount of per capita capital in the economy. This means that all the households are basically alike, that is, this is a representative agent model.

2.4. Government

The government provides a public good to the households and hires a certain amount of human capital as teachers to produce more human capital. In order to finance these expenditures, the government fix a "statutory" tax rate, $\overline{\tau}$, on the average earnings running from the human capital activities. However, in this model the government needs to hire bureaucrats to collect taxes. If there is no any bureaucrat to manage and control the tax collection, individuals would not pay any tax. Thus, the effective tax rate that individuals are paying depends positively on the bureaucrats that the government hires. There is a technology which translates the bureaucracy efforts in effective public revenues. In particular, the effective tax rate which is paid and so it becomes in public revenues in period t is as follows:

$$\begin{aligned}\tau\left(h_b\left(t\right)\right) &= \begin{cases}\Gamma h_b\left(t\right)^{\gamma} & \text{if } h_b\left(t\right)<\overline{h}_b\\ \overline{\tau} & \text{if } h_b\left(t\right)\geq\overline{h}_b\end{cases} \qquad \overline{h}_b \equiv \left(\frac{\overline{\tau}}{\Gamma}\right)^{\frac{1}{\gamma}}\\ \Leftrightarrow \tau\left(h_b\left(t\right)\right) &= \min\left\{\Gamma h_b\left(t\right)^{\gamma},\overline{\tau}\right\}\end{aligned}$$

where $\tau\left(h_b\left(t\right)\right)$ denotes the effective tax rate which is paid by individuals in period t, and $h_b\left(t\right)$ is the amount of per capita human capital devoted to the bureaucrat sector (number of bureaucrats), $\Gamma>0$ and $\gamma\in\left(0,1\right)$. It is assumed that the higher is the number of the bureaucrats assigned to manage the tax collection, the higher is the effective tax rate and so the higher is the amount of public revenues. There is a maximum number of bureaucrats, $\overline{h}_b$, that makes the effective tax rate, $\tau(h_b(t))$, equal to the statutory tax rate, $\overline{\tau}$.

The government budget constraint is as follows:

$$\tau(h_b)\,w_h\,(h_c + h_g + h_h + h_b) = g + w_h\,(h_h + h_b) \tag{2.3}$$

where w_h denotes the wage of human capital. The left side of the equation is the total public revenues of the government in per capita terms that come from the tax over the human capital income, such per capita public revenues are defined by the effective tax rate multiplied by the per capita human capital income. The right side of the equation are the tree expenditures of the government: i) the per capita public good provided to households, g; ii) the per capita expenditure in the public education system, $w_h h_h$, that is, the wages paid to teachers and; iii) the per capita wages paid to bureaucrats, $w_b h_b$. For simplicity, we assume that the government devote a fraction, $\lambda \in (0,1)$, of the public revenues for hiring human capital and a fraction, $(1-\lambda) \in (0,1)$ to provide the public good, this is,

$$\lambda\tau(h_b)\,w_h h = w_h\,(h_h + h_b) \tag{2.4}$$

$$(1-\lambda)\,\tau(h_b)\,w_h h = g \tag{2.5}$$

3. Agents' decisions

3.1. Firms

Firms behave competitively and hire the amount of workers and human capital that maximize its profits:

$$\max_{L(t),H_y(t)} AL(t)^{1-\alpha}\,H_y(t)^{\alpha} - w_h(t)\,H_y(t) - w(t)\,L(t) \tag{3.1}$$

where $L(t)$ and $H_y(t)$ denotes the amount of labor and human capital hired by the firm at period t and $w(t)$ denotes the wage of the labor at period t. The first order conditions of the above problem are:

$$\begin{aligned} \alpha A\left(\frac{L(t)}{H_y(t)}\right)^{1-\alpha} &= w_h(t) \\ (1-\alpha)\,A\left(\frac{H_y(t)}{L(t)}\right)^{\alpha} &= w(t) \end{aligned}$$

That is, firms hire a factor until the point in which the marginal productivity of such factor is equal to its price. Taking into account the fact that the per capita amount of labor is equal to one and the fact that all the labor is used in production, these first order conditions may be rewritten in per capita terms:

$$\alpha A \frac{1}{h_y(t)^{1-\alpha}} = w_h(t) \tag{3.2}$$

$$(1-\alpha) A h_y(t)^{\alpha} = w(t) \tag{3.3}$$

3.2. Government fiscal policy

In this economy, the government wishes collect taxes to provide a public good and teachers to the economy. However, to do that the government needs to hire bureaucrats in order to manage the tax collection. Therefore, the government maximizes the amount of public revenues minus the cost in bureaucrats needed to produce it. Once bureaucrats are hired and public resources are generated, the government can provide the public good and hire teachers to increase the private production of human capital. Thus, given the tax rate, $\overline{\tau}$, on the earnings running from the human capital factor, and the total amount of human capital, the government has to decide each period how many bureaucrats, $h_b(t)$, hires at current wages in order to obtain the maximum amount of public revenues. This is, the government has to decide how many bureaucrats hires in order to maximize the total amount of per capita taxes paid by individuals, $T(t)$.

Therefore, the problem of the government in period t is formalized as follows:

$$\max_{h_b} T(t) - w_h(t) h_b(t)$$

where

$$T(t) = \tau(h_b(t)) w_h(t) h(t) = \min\{\Gamma h_b(t)^{\gamma}, \overline{\tau}\} w_h(t) h(t) \tag{3.4}$$

Notice that wages that government pays to bureaucrats and teachers are not controlled by it. Because of the human capital is assumed to be perfectly substitutable among sectors and there is perfect competition, then, wages equates among sectors and all of them are the same that the wages paid in the production sector. We denote by w_h the wage of the human capital and so, the problem of the government results as:

$$\max_{h_b} \tau(h_b(t)) h(t) - h_b(t) \tag{3.5}$$

The solution of the problem is the optimal amount of bureaucrats:

$$h_b^*(t) = \begin{cases} (\gamma\Gamma)^{\frac{1}{1-\gamma}} h(t)^{\frac{1}{1-\gamma}} & \text{if } h(t) < \overline{h} \\ \overline{h}_b & \text{if } h(t) \geq \overline{h} \end{cases}; \quad \overline{h} = \frac{\overline{\tau}^{\frac{1-\gamma}{\gamma}}}{\gamma\Gamma^{\frac{1}{\gamma}}} \tag{3.6}$$

The above optimal amount of bureaucrats implies that the shares of human capital in its four possible uses are as follows (see appendix):

$$\frac{h_b^*(t)}{h(t)} = \begin{cases} (\gamma\Gamma)^{\frac{1}{1-\gamma}} h(t)^{\frac{\gamma}{1-\gamma}} & \text{if } h(t) < \overline{h} \\ \frac{\overline{h}_b}{h(t)} & \text{if } h(t) \geq \overline{h} \end{cases} \tag{3.7}$$

$$\frac{h_h^*(t)}{h(t)} = \begin{cases} \left(\frac{\lambda-\gamma}{\gamma}\right)(\gamma\Gamma)^{\frac{1}{1-\gamma}} h(t)^{\frac{\gamma}{1-\gamma}} & \text{if } h(t) < \overline{h} \\ \overline{\tau}\lambda - \frac{\overline{h}_b}{h(t)} & \text{if } h(t) \geq \overline{h} \end{cases} \tag{3.8}$$

$$\frac{h_g^*(t)}{h(t)} = \begin{cases} \alpha\left(\frac{1-\lambda}{\gamma}\right)(\gamma\Gamma)^{\frac{1}{1-\gamma}} h(t)^{\frac{\gamma}{1-\gamma}} & \text{if } h(t) < \overline{h} \\ \alpha(1-\lambda)\overline{\tau} & \text{if } h(t) \geq \overline{h} \end{cases} \tag{3.9}$$

$$\frac{h_c^*(t)}{h(t)} = \begin{cases} 1 - \left[\frac{\lambda(1-\alpha)+\alpha}{\gamma}\right](\gamma\Gamma)^{\frac{1}{1-\gamma}} h(t)^{\frac{\gamma}{1-\gamma}} & \text{if } h(t) < \overline{h} \\ 1 - \overline{\tau}(\lambda + \alpha(1-\lambda)) & \text{if } h(t) \geq \overline{h} \end{cases} \tag{3.10}$$

The above equations are displayed in figure 1. The evolution of the shares of different uses of human capital depends on the evolution of the effective tax rate. The effective tax rate is an increasing function of the per capita human capital until reaching the threshold human capital level, $\overline{h}$, in which the effective tax rate coincides with the statutory tax rate $\overline{\tau}$. Beyond this threshold, the effective tax rate is constant and equal to the statutory tax rate. When the per capita human capital remains below the threshold level $\overline{h}$, the evolution of the effective tax rate implies that all the shares of human capital in the public sector (bureaucrats, teachers and human capital used in the production of the public good) rise with per capita human capital, while the share of human capital used to produce the private good (which is used for consumption and investment) declines with per capita human capital. This may be a possible explanation for the weak impact of human capital in production for countries with low per capita income: the significant reallocation of human capital from private to public sector may mitigate the impact of human capital in production. If the per capita human capital is beyond the threshold level $\overline{h}$, the share of the human capital used in the private sector (the production of the private consumption good and investment) and the share of human capital used in the production of the public good stabilize

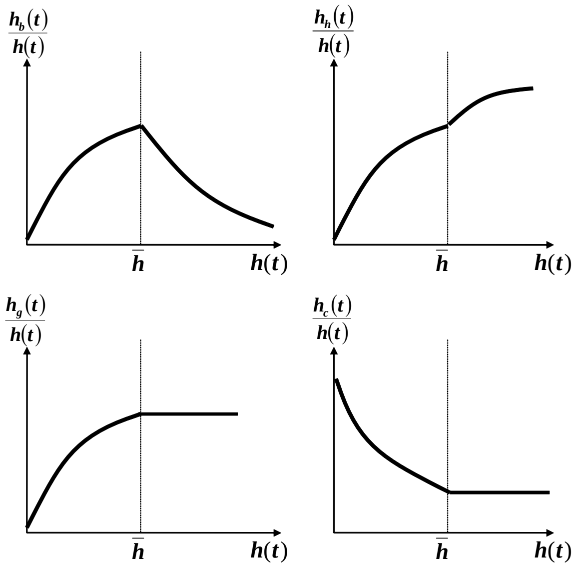

Figure 3.1: Shares of human capital uses

in a constant level, while the share of bureaucrats decreases and the share of the public education system (teachers) increases.

3.3. Households

The households' optimization problem is as follows:

$$\max_{\{c(t)\}_{t=0}^{+\infty}} \int \left[u\left(c\left(t\right)\right)+v\left(g\left(t\right)\right)\right] e^{-(\rho-n)t}dt \tag{3.11}$$

$$\begin{aligned} i\left(t\right) &= w_h\left(t\right)h\left(t\right)+w\left(t\right)-c\left(t\right)-T\left(t\right) \qquad (3.12)\\ \dot{h}\left(t\right) &= h_h\left(t\right)^{\xi}\, i\left(t\right)-\left(\delta+n\right)h\left(t\right)\\ h\left(0\right) &> 0 \end{aligned}$$

Households maximize their utility subject to: $i)$ their budget constraint: the expenditure in investment in human capital $i(t)$ and in consumption $c(t)$ should be equal to their disposable income that come from human capital $w_h\left(t\right)h\left(t\right)$

and labor $w(t)$, minus taxes $T(t)$ and; *ii*) the accumulation equation of human capital. The Euler Equation and the transversality condition associated to the households' optimization problem are:

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma(c(t))}\left[h_h(t)^{\xi} w_h(t) - \rho - \delta - \xi\frac{\dot{h}_h(t)}{h_h(t)}\right] \tag{3.13}$$

$$\lim_{t\to+\infty} u'(c(t))e^{-\rho t}h(t) = 0 \tag{3.14}$$

where $\sigma(c)$ is the elasticity of the marginal utility: $\sigma(c) = \frac{u"(c)c}{u'(c)}$. The first of the above conditions is the Euler equation. The speed at which consumption grows it displays the standard features: depends positively on the return of investment in human capital, $h_h(t)^{\xi} w_h(t) - \delta$ and negatively on the patient rate of the household, ρ. However, the higher is the growth rate of public provision of human capital (provision of teachers), the lower is the speed at which consumption grows. The reason behind is that the private cost of producing one unit of human capital decreases when the human capital devoted to the public education system, $h_h(t)$, rises, which encourages households to postpone the investment in human capital. Finally, the more concave the utility function (the higher $\sigma(c)$), the smoother the consumption path. The second equation is the standard transversality condition.

4. The Definition of Equilibrium

The equilibrium definition is standard: equilibrium occurs when agents maximize their objective functions and markets clear. Since all households and firms are alike, we may define equilibrium in per capita terms.

Definition 4.1. *Given the initial condition $h(0)$, a competitive equilibrium is an allocation $\{c(t), i(t), h(t), h_y(t), l_y(t), h_b(t), h_h(t), g(t)\}_{t=0}^{\infty}$ and a vector of prices $\{w(t), w_h(t)\}_{t=0}^{\infty}$ such that $\forall t$:*

- Households maximize their utility, that is, $\{c(t), i(t), h(t)\}_{t=0}^{\infty}$ is the solution of the household's maximization problem (3.11).
- Firms maximize profits, that is, $h_y(t), l_y(t)$ is the solution of the optimization problem of firms (3.1).

- The government chooses the amount of human capital devoted to bureaucracy, $h_h(t)$, which maximizes the public revenues net of bureaucratic expenditure (3.5) and chooses the amount of human capital devoted to the public educational system (teachers) $h_h(t)$ and the expenditure in the public good $g(t)$ according with the fiscal policies rules (2.4) and (2.5).
- Human capital market clears: $h(t) = h_h(t) + h_b(t) + h_y(t)$.
- Labor market clears: $l_y(t) = 1$.
- Good market clears: $Ak_y(t)^{\alpha} l_y(t)^{1-\alpha} = c(t) + i(t) + g(t)$

Definition 4.2. *Steady state equilibrium is an equilibrium in which both the allocation and the prices always remain constant over time.*

5. Dynamic Behavior

The dynamic behavior of this economy could be characterized by the dynamics of the consumption and the human capital variables. We now proceed to define the dynamic system of the economy.

5.1. Dynamic system

The dynamic system of this economy consists of the human capital accumulation equation (2.2), the Euler equation (3.13) and the transversality condition equation (3.14), this is,

$$\dot{h}(t) = h_h(t)^{\xi} i(t) - (\delta + n) h(t) \tag{5.1}$$

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma(c(t))}\left[r(t) - \rho - \delta - \xi \frac{\dot{h}_h(t)}{h_h(t)}\right] \tag{5.2}$$

$$\lim_{t\to+\infty} u'(c(t)) e^{-\rho t} h(t) = 0 \tag{5.3}$$

where $r(t) = h_h(t)^{\xi} w_h(t)$ denotes the "gross" return on the investment in human capital.

Using the budget constraint of the household equation (3.12), the total amount of per capita taxes paid by individuals equation (3.4), wages equations (3.2 and

3.3), human capital devoted to produce final goods and to provide teachers and bureaucrats equations (9.4, 9.2 and 3.6) we rewrite the dynamic system as:

$$\dot{h}(t) = h_h^* (h(t))^{\xi} [w_h (h(t)) h(t) + w (h(t)) - c(t) - T (h(t))] - (\delta + n) h(t) \tag{5.4}$$

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma(c(t))} \left[r(h(t)) - \rho - \delta - \xi \frac{\dot{h}_h (h(t))}{h_h (h(t))} \right] \tag{5.5}$$

$$\lim_{t \to +\infty} u'(c(t)) e^{-\rho t} h(t) = 0 \tag{5.6}$$

where $r(h(t))$ is the function that relates per capita human capital with the gross return on the private investment in human capital. Such function is defined in the appendix. It follows from equation (5.5) that in order to analyze the dynamic of the economy, it is important to understand the way in which the net return on savings evolves with the human capital.

Lemma 5.1. *The function $r(h)$ is continuous in h. Furthermore, if $\xi + \alpha < 1 - \gamma (1 - \alpha)$, then:*

(*i*) $\lim_{h \to 0} r(h) = +\infty$ and $\lim_{h \to +\infty} r(h) = 0$

(*ii*) If $\overline{\tau} \leq \tau^* \equiv \frac{1}{\lambda} \left[1 - \frac{\gamma(1-\alpha)}{(1-\alpha-\xi)} \right]$ then $r(h)$ is strictly decreasing when $h \leq \overline{h}$. Otherwise, there is $\widetilde{h} \in \left(0, \overline{h}\right)$ such that $r(h)$ is strictly decreasing in the interval $\left(0, \widetilde{h}\right)$ and strictly increasing in $\left(\widetilde{h}, \overline{h}\right)$ and strictly decreasing in $\left(\overline{h}, +\infty\right)$

This proposition establishes that if the tax rate is large enough then, the rate of return is not monotonically decreasing in the human capital. The intuition behind involves two offsetting mechanisms. The first one is the standard neoclassical mechanism: when the human capital/labor rises then, the marginal productivity of the human capital declines and so, the rate of return of human capital. The second mechanism is related to the provision of public teachers: when the human capital rises, the amount of teachers increases as well, which reduces the private cost of producing human capital, increasing its return. Therefore, if the tax rate low enough (lower than τ^*) the provision of teachers is limited and the second mechanism is weak. Thus, the first mechanism prevails, implying a decreasing rate of return. On the contrary, if the tax rate is high (higher than τ^*) then, the second mechanism is stronger and the return on human capital becomes an increasing function of the per capita human capital in a certain range $\left(\widetilde{h}, \overline{h}\right)$.

We assume from now on that $\xi + \alpha < 1 - \gamma(1-\alpha)$ (see lemma 5.1). If this condition is not satisfied, there are multiple steady states and, more important, the trivial steady state (with human capital equal to zero) would be a saddle point. Obviously, these features would not be appealing for the goal of our paper.

Proposition 5.2. *If $\overline{\tau} \leq \tau^*$ then there is a unique steady state equilibrium. If $\overline{\tau} > \tau^*$, then there exists $\overline{\rho} > 0$ such that if $\rho < \overline{\rho}$ there is a unique steady state equilibrium and, if $\rho > \overline{\rho}$ there are three steady states equilibria.*

From the behavior of the rate of return described in lemma 5.1, it is straightforward to see that it could exist more than one steady state. However, since we are focused in analyzing the structural change throughout the transition, we will restrict ourself to analyze the case in which there exists a unique steady state. Thus, we assume from now on that either $\overline{\tau} \leq \tau^*$ or $\overline{\tau} > \tau^*$ and $\rho > \overline{\rho}$.

Proposition 5.3. *The steady state equilibrium is a saddle point.*

Phase diagram in figure 2 shows that the dynamic behavior of the economy is characterized by the typical saddle point dynamic. The saddle point dynamic implies there is a unique path which converges to the steady state. This means, that given the initial level of per capita human capital, there is a unique equilibrium path, which converges to the steady state. When the initial amount of per capita human capital is lower than the steady state level, the consumption grows throughout the equilibrium path, converging to its steady state level. When the amount of per capita human capital is larger than the steady state level the opposite happens.

The human capital behavior is more interesting. When the starting per capita human capital is below the steady state level and the human capital threshold, $\overline{h}$, a "structural change" throughout the transition arises: the public expenditure on bureaucracy increases, and consequently the effective tax rate rises as well. This allows the government to hire more human capital that will be devoted to the public education system (teachers) and to increase the public expenditure. Taking into account figure 1, it follows that throughout the transition the share of human capital devoted to collect taxes (bureaucrats) rises until the human capital threshold, $\overline{h}$, is reached. This implies that the effective tax rate rises

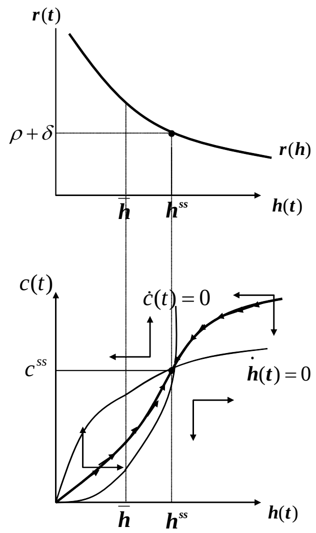

Figure 5.1: Phase Diagram

and consequently, shares of human capital used in the public education system (teachers) and in the production of the public good increase as well. However, once the bureaucratic sector reaches the critical size in which the effective tax rate equates to the statutory rate $\overline{\tau}$, at the threshold human capital $\overline{h}$, the government does not need to hire more bureaucrats and consequently, its share in the human capital uses declines. This allows the government to devote more resources to the public education system, with the consequent rise in the human capital share devoted to this sector, while the shares of human capital devoted to the production of the private and public good stabilize and stay constant.

6. Institutional Changes

6.1. The effect of an improvement in the technology of the bureaucracy

We analyze the effect of a technological improvement in the bureaucracy sector through an increase in the parameter Γ. In this context a technological improvement implies that for the same amount of bureaucrats (human capital devoted to

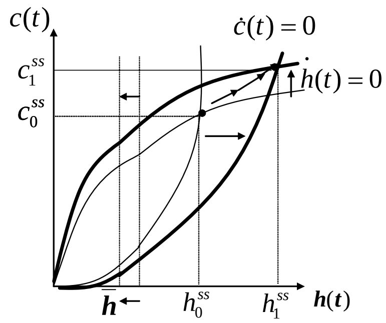

Figure 6.1: The effect of an increase in Γ

the bureaucracy sector), the tax collection increases, this is, the effective tax rate is closer to the legal tax rate.

An increase in Γ has a positive effect on the rate of return. Given that bureaucrats are now more efficient, a smaller amount of human capital is now needed to collect taxes. Thus, for a given amount of tax collection, the government may devote more human capital to the provision of teachers and a flow of human capital from the bureaucracy to the public education system arises. Consequently, the return on savings increases as well, encouraging the human capital accumulation which graphically renders in the movement of locus $\dot{c} = 0$ to the right. Similarly, an increase in Γ has a positive effect on the total amount of resources in the economy. Due to the fact that bureaucrats are more efficient and consequently, the government devotes more human capital to the provision of teachers, the productivity in the production sector increases and so, the total amount of resources in the economy (graphically the locus $\dot{h} = 0$ moves up). It is proved that the economy moves toward a new steady state with a higher level of consumption and a higher level of human capital.

Proposition 6.1. *If there is a technological improvement in the bureaucracy sector, measured as an increase of Γ, then the steady state consumption and human capital levels increase as well.*

Phase diagram in figure 3 shows that the dynamic behavior of the economy due to an increase in Γ. From the initial the initial level of per capita human capital,

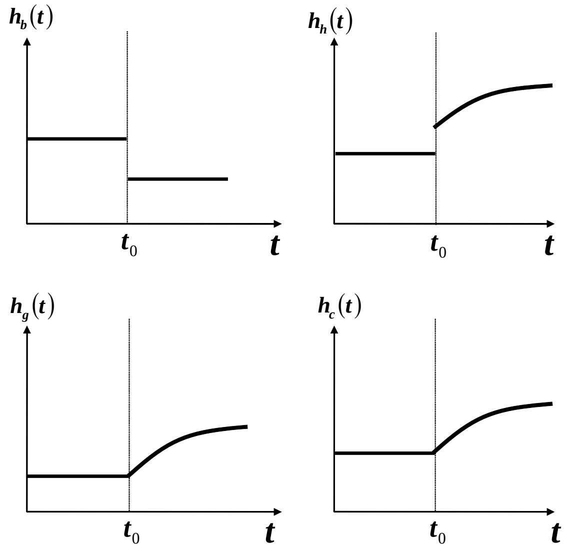

Figure 6.2:

there is a unique equilibrium path, which converges to the steady state with higher levels of consumption and human capital. Thus, a "structural change" throughout the transition occurs: the improvement in the technology of bureaucracy implies that a lower amount of human capital is required for bureaucracy which implies a reallocation of human capital from the bureaucratic sector to the public education system.

Figure 4 shows the evolution of the of human capital devoted to the four sectors of the economy when Γ increases at time t_0. Given the fact that bureaucrats are more efficient collecting taxed, the government can obtain the same amount of public revenues with a less number of bureaucrats. Thus, the government reallocates human capital from the bureaucratic sector to the educational one, which implies a positive effect of the return on the human capital, encouraging the human capital accumulation. Insofar human capital is being accumulated, also increase the amount of it which is devoted to produce both private and public goods and to hire teachers.

6.2. The effect of an increase in share of public revenues devoted to hire human capital

We now study the effect of an increase in the share of public revenue which is devoted to hire human capital, λ, for a given tax rate. An increase in λ implies that for the same number or bureaucrats, and so the same amount of tax collection, the government spends a larger amount of public resources in hiring teachers at the expense of the expenditure in the public good. Thus, a flow of human capital from the production sector to public human capital arises, increasing the return on savings and so encouraging the human capital accumulation. Moreover, this flow of human capital increases the wage of the human capital which increases the benefits of investment in human capital furthermore and so, the incentives to accumulate more human capital (graphically renders in the movement of locus $\dot{c} = 0$ to the right).

Similarly, an increase in λ has also two opposite effects on the total amount of resources in the economy: first, in despite of households pay the same amount of taxes, the fact that the government hires more human capital at the expense of the human capital used in the production sector reduces the disposable amount of resources and so the level of consumption. Second, given that a larger fraction of the tax collection is devoted to increase the provision of public teachers, the productivity in the production sector and the amount of disposable resources increase. The combination of these two effects result ambiguous. In spite of this, it is proved that the economy moves toward a new steady state with a higher level of consumption and a higher level of human capital.

Proposition 6.2. *If there is an increase in share of public revenues devoted to hire human capital, λ, then the steady state consumption and human capital levels increase as well.*

The dynamic behavior of the economy due to an increase in λ could be represented by a phase diagram similar to figure 5, where figure 5.A shows the case in which the effect of λ on the disposable amount of resources is positive (locus $\dot{h} = 0$ goes up) and figure 5.B shows the opposite (locus $\dot{h} = 0$ goes down). From the initial the initial level of per capita human capital, there is a unique equilibrium path, which converges to the steady state with higher levels of consumption and human capital. Thus, a "structural change" throughout the transition occurs: the increase in share of public revenues devoted to hire human capital, implies a

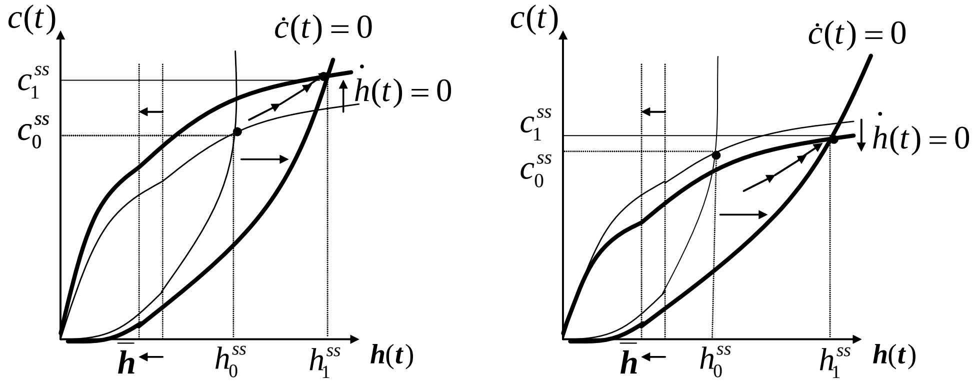

Figure 5.A **Figure 5.B**

Figure 6.3: The effect of an increase in λ

higher amount of public resources devoted to provide public human capital in the form of teachers, a stable number of bureaucrats, an increase in the total amount of human capital and a higher level of consumption and per capita income.

Figure 6 shows the evolution of the of human capital devoted to the four sectors of the economy when λ increases at time t_0. For the given amount of tax collection, the government decides now to devote a larger fraction to hire human capital. While the amount of human capital which is devoted to the bureaucratic sector remains stable, the government spends a higher amount of resources hiring more teachers at the expense of a drop in the public good. Since the public education system is more intensive in human capital than the production of the public good, the relative wage of human capital rises, involving a lower rate human capital/labor in the production of both the private and the public good. Thus, the human capital devoted to the production of the private good drops momentarily. However, given the fact that there is more public education (teachers) which increases the productivity of human capital, and so the return on it, a human capital accumulation process is generated. Insofar human capital is being accumulated, also the amount of it which is devoted to produce goods and to hire teachers increases throughout the transition to the new steady state.

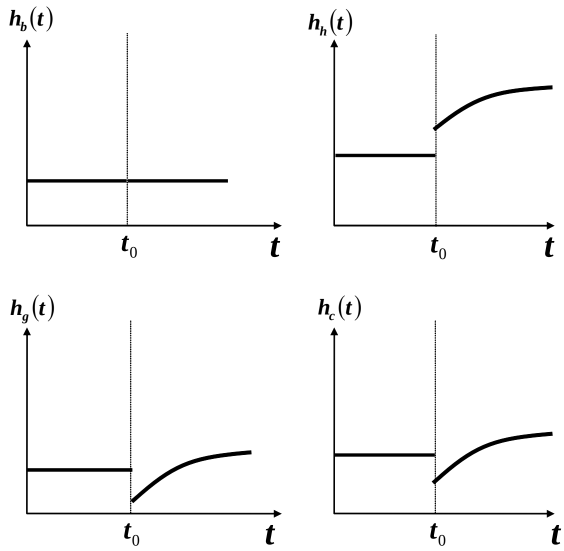

Figure 6.4:

7. Conclusion

The role of the human capital in economic growth and development is one of the most analyzed topics in the economic literature. One of the main reason behind is the mixed and ambiguous evidence that the literature offers about the impact of human capital in the economic performance. While, at the micro level, empirical literature reports significant returns to increases in education consistent with the theory, the macro analysis finds not only a weak relationship between human capital and economic performance but also shows a negative impact of human capital. This combination of strong positive effect of human capital at the micro level with an ambiguous effect at macro level constitutes a puzzle which has attracted the interest of many researchers in Growth and Economic Development.

Simultaneously, the high return on human capital in developing countries and the low growth performance they have followed suggest that the dichotomy between the effects of the human capital at the micro and the macro level look likes more pronounced in developing countries and moreover, are partially responsible of the stable convergence in income levels across countries (see Easterly, 2001 and Parente and Prescott, 1993).

This paper offers a new explanation to understand why there exists a weak relationship between human capital and economic growth and why the low impact of human capital on economic growth is steeper in developing countries. This paper suggests that the difficulties of government of owning a educated bureaucracy to collect taxes and, consequently, the fact that not all the human capital is devoted to public education or to produce goods is the reason that explains why, at least temporally, the human capital shows a low impact in the economy.

We build a model in which the public educational system is an important factor affecting the accumulation of human capital and where public revenues determine the amount of public expenditure on public education. The human capital is used to produce goods in the market, to create the government's bureaucracy and to constitute the government's provision of education in form of teachers. The level of tax collection depends positively on the size of the bureaucracy and a portion of public revenues is used to finance bureaucrats and to finance a public educational system (teachers) which is targeted to increase the human capital level. Thus, a feedback process arises: a higher level of human capital implies more efficient bureaucrats, who collect more taxes that are used to finance a public expenditure on education which in turn promotes human capital. Therefore, in the first stage, when the level of human capital is low, the scarcity of the human capital implies a high private returns of education, encouraging the human capital accumulation. Nevertheless, on the other hand, there is also a low level of tax collection which implies a low level of teachers in the economy. Given the fact that public education is a significant input in the production function of human capital, the low provision of education depress the return of human capital. This second mechanism on the return of human capital reduces partially the high private incentive to accumulate human capital, decelerating the human capital accumulation. Thus, when the level of human capital is low, income and tax collection are also low, implying low levels of public education and bureaucracy. Insofar human capital is growing throughout the transition to the steady state, the amount of collected taxes, the public expenditure on education and the size of bureaucracy also increase. There is a certain level of human capital in which the bureaucracy reach the point in which the tax collection is maximized, and the effective tax rate coincides with the statutory tax rate. After this level, the bureaucracy sector remains stationary and human capital is devoted only to the provision of the public education and to produce goods. The larger amount of human capital devoted to the educational sector will rise the productivity of the human capital and so, the accumulation of capital and the growth of the per capita income.

The paper also shows that the impact of the human capital in the economy depends crucially on the allocation of tax revenues to different uses, such as public educational system or the expenditure in public goods. This allocation of the public resources not only has a temporal effect throughout the transition, but it may also affects in the long run. If the share of public revenues which is devoted to provide the public good decreases then, a reallocation of public resources from expenditure in public goods to the public education system arises, producing higher levels of consumption and human capital in the steady state. Thus, countries with the same initial human capital level, apparent similar access to technologies and identical characteristics could have different structural process and economic results depending on how the public sector allocates resources to different uses like public education or expenditure in public goods.

Moreover, the model accounts for the well documented fact that the higher is the education of the bureaucracy, the better is the economic performance of the economy. However, it is worth to notice that the mechanism proposed in the paper does not excludes the possibility that some part of the bureaucrats may not be involved in unproductive or rent-seeking activities. On the contrary, it is proved that a more efficient bureaucracy produces a higher amount of public resources devoted to provide public human capital, an increase in the total amount of human capital and a higher level of consumption and per capita income. Therefore, according to the prediction of the model, the impact of human capital may be delayed by a low level of the bureaucracy efficiency.

This paper provides an interesting setting for analyzing the role of the government and institutions in the human capital and growth literature. We think our model might be used for further research in this area. At theoretical level one may consider the possibility to endogenize the formation of institutions throughout the development process and to investigate at which extent institutions and human capital reinforce each other. Another possible extension of theoretical work is introducing corruption in the model and to analyze if there exists any feedback mechanism between the accumulation of human capital and the level of corruption and how it is evolving during the developing process. While at empirical level, there is need to look up to the point above which the returns of education are observable and have and may have positive and significant effect on growth and below which it has weak or negative effects on growth.

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9. Appendix

Equations (2.1), (3.5), (2.5), (2.4), (3.2)and (3.6) imply the following amount of tax revenues, and the following uses of the human capital:

$$T(t) = \begin{cases} w_h \Gamma^{\frac{1}{1-\gamma}} \gamma^{\frac{\gamma}{1-\gamma}} h(t)^{\frac{1}{1-\gamma}} & \text{if } h(t) < \overline{h} \\ \tau w_h h(t) & \text{if } h(t) \geq \overline{h} \end{cases} \tag{9.1}$$

$$h_h^*(t) = \begin{cases} \left(\frac{\lambda-\gamma}{\gamma}\right) (\gamma\Gamma)^{\frac{1}{1-\gamma}} h(t)^{\frac{1}{1-\gamma}} & \text{if } h(t) < \overline{h} \\ \overline{\tau}\lambda h(t) - \overline{h}_b & \text{if } h(t) \geq \overline{h} \end{cases} \tag{9.2}$$

$$h_g^*(t) = \begin{cases} \alpha\left(\frac{1-\lambda}{\gamma}\right) (\gamma\Gamma)^{\frac{1}{1-\gamma}} h(t)^{\frac{1}{1-\gamma}} & \text{if } h(t) < \overline{h} \\ \alpha(1-\lambda)\overline{\tau} h(t) & \text{if } h(t) \geq \overline{h} \end{cases} \tag{9.3}$$

$$h_c^*(t) = \begin{cases} h(t) - \left[\frac{\lambda(1-\alpha)+\alpha}{\gamma}\right] (\gamma\Gamma)^{\frac{1}{1-\gamma}} h(t)^{\frac{1}{1-\gamma}} & \text{if } h(t) < \overline{h} \\ \left[1 - \overline{\tau}\left(\lambda + \alpha(1-\lambda)\right)\right] h(t) & \text{if } h(t) \geq \overline{h} \end{cases} \tag{9.4}$$

$$h_y^*(t) = h_g^*(t) + h_c^*(t) = \begin{cases} \left[1 - \left[\frac{\lambda}{\gamma}\right] (\gamma\Gamma)^{\frac{1}{1-\gamma}} h(t)^{\frac{\gamma}{1-\gamma}}\right] h(t) & \text{if } h(t) < \overline{h} \\ \left[1 - \overline{\tau}\lambda\right] h(t) & \text{if } h(t) \geq \overline{h} \end{cases} \tag{9.5}$$

Proof lemma 5.1:

If we consider equations (9.2), (9.5) and (3.2) the return on the investment in human capital, $r(t)$, can be defined as:

$$r(h) = \begin{cases} \dfrac{\alpha A\left[(\lambda-\gamma)(\gamma^{\gamma}\Gamma)^{\frac{1}{1-\gamma}}\right]^{\xi}}{\left[1-\lambda(\gamma^{\gamma}\Gamma)^{\frac{1}{1-\gamma}} h^{\frac{\gamma}{1-\gamma}}\right]^{1-\alpha} h^{1-\alpha-\frac{\xi}{1-\gamma}}} & \text{if } h < \overline{h} \\ \dfrac{\alpha A\left[\overline{\tau}\lambda - \left(\frac{\overline{\tau}}{\Gamma}\right)^{\frac{1}{\gamma}} \frac{1}{h}\right]^{\xi}}{\left[1-\overline{\tau}\lambda\right]^{1-\alpha} h^{1-\alpha-\xi}} & \text{if } h \geq \overline{h} \end{cases} \tag{9.6}$$

The sign of the derivative of the above expression depend on:

$$\frac{\frac{\partial r(h)}{\partial h}h}{r\left(h\right)}=\left\{\begin{array}{ll}\left[-\left(1-\alpha-\frac{\xi}{1-\gamma}\right)+\left(1-\alpha\right)\frac{\gamma}{1-\gamma}\frac{\lambda(\gamma^{\gamma}\Gamma)^{\frac{1}{1-\gamma}}h^{\frac{\gamma}{1-\gamma}-}}{\left[1-\lambda(\gamma^{\gamma}\Gamma)^{\frac{1}{1-\gamma}}h^{\frac{\gamma}{1-\gamma}}\right]}\right] & \text{if } h<\overline{h} \\ \left[-\left(1-\alpha-\xi\right)+\xi\frac{\left(\frac{\overline{\tau}}{\Gamma}\right)^{\frac{1}{\gamma}}}{\left[\overline{\tau}\lambda h-\left(\frac{\overline{\tau}}{\Gamma}\right)^{\frac{1}{\gamma}}\right]}\right] & \text{if } h\geq\overline{h}\end{array}\right. \tag{9.7}$$

Since the first branch of the above function is increasing in h and negative when $h\to 0$ (under the assumption that $\xi+\alpha<1-\gamma\left(1-\alpha\right)$), then $r(h)$ is strictly decreasing when $h<\overline{h}$ if and only if $-\left(1-\alpha-\frac{\xi}{1-\gamma}\right)+(1-\alpha)\frac{\gamma}{1-\gamma}\frac{\lambda(\gamma^{\gamma}\Gamma)^{\frac{1}{1-\gamma}}\overline{h}^{\frac{\gamma}{1-\gamma}-}}{\left[1-\lambda(\gamma^{\gamma}\Gamma)^{\frac{1}{1-\gamma}}\overline{h}^{\frac{\gamma}{1-\gamma}}\right]}\geq 0$. Otherwise, $r(h)$ will be strictly decreasing in a certain interval $\left(0,\widetilde{h}\right)$ and strictly increasing in $\left(\widetilde{h},\overline{h}\right)$ where $\widetilde{h}\in\left(0,\overline{h}\right)$:

$$-\left(1-\alpha-\frac{\xi}{1-\gamma}\right)+\left(1-\alpha\right)\frac{\gamma}{1-\gamma}\frac{\lambda(\gamma^{\gamma}\Gamma)^{\frac{1}{1-\gamma}}\overline{h}^{\frac{\gamma}{1-\gamma}-}}{\left[1-\lambda(\gamma^{\gamma}\Gamma)^{\frac{1}{1-\gamma}}\overline{h}^{\frac{\gamma}{1-\gamma}}\right]}\geq 0\Leftrightarrow$$

$$\left(1-\alpha-\frac{\xi}{1-\gamma}\right)>\left(1-\alpha\right)\frac{\gamma}{1-\gamma}\frac{\lambda(\gamma^{\gamma}\Gamma)^{\frac{1}{1-\gamma}}\left[\frac{\overline{\tau}^{\frac{1-\gamma}{\gamma}}}{\gamma\Gamma^{\frac{1}{\gamma}}}\right]^{\frac{\gamma}{1-\gamma}-}}{\left[1-\lambda(\gamma^{\gamma}\Gamma)^{\frac{1}{1-\gamma}}\left[\frac{\overline{\tau}^{\frac{1-\gamma}{\gamma}}}{\gamma\Gamma^{\frac{1}{\gamma}}}\right]^{\frac{\gamma}{1-\gamma}}\right]}=\left(1-\alpha\right)\frac{\gamma}{1-\gamma}\frac{\lambda\overline{\tau}}{[1-\lambda\overline{\tau}]}\Leftrightarrow$$

$$\begin{aligned}\left(1-\alpha-\frac{\xi}{1-\gamma}\right) &> \left(1-\alpha\right)\left[\frac{\gamma}{1-\gamma}\frac{\lambda\overline{\tau}}{[1-\lambda\overline{\tau}]}\right] \\ \left(1-\alpha\right)\left[1-\frac{\gamma}{1-\gamma}\frac{\lambda\overline{\tau}}{[1-\lambda\overline{\tau}]}\right] &> \frac{\xi}{1-\gamma} \\ \left(1-\alpha\right)\left[\frac{\left(1-\gamma\right)\left[1-\lambda\overline{\tau}\right]-\gamma\lambda\overline{\tau}}{[1-\lambda\overline{\tau}]}\right] &> \xi \\ \left(1-\alpha\right)\left[1-\gamma-\lambda\overline{\tau}\right] &> \xi\left[1-\lambda\overline{\tau}\right] \\ \left(1-\alpha\right)\left(1-\gamma\right)-\xi &> (1-\alpha-\xi)\lambda\overline{\tau}\end{aligned}$$

Thus, if $\lambda\overline{\tau}\leq\frac{(1-\alpha)(1-\gamma)-\xi}{(1-\alpha-\xi)}$ then $r(h)$ is strictly decreasing when $h<\overline{h}$. Otherwise, $r(h)$ is strictly decreasing in a certain interval $\left(0,\widetilde{h}\right)$ and strictly increasing in $\left(\widetilde{h},\overline{h}\right)$ where $\widetilde{h}\in\left(0,\overline{h}\right)$. Notice that the second branch of function $\frac{\frac{\partial r(h)}{\partial h}h}{r(h)}$ (equation

??) is negative at $\overline{h}$:

$$\left[-(1-\alpha-\xi)+\xi\frac{\left(\frac{\overline{\tau}}{\Gamma}\right)^{\frac{1}{\gamma}}}{\left[\overline{\tau}\lambda\overline{h}-\left(\frac{\overline{\tau}}{\Gamma}\right)^{\frac{1}{\gamma}}\right]}\right]=-\left(1-\alpha-\xi\frac{\lambda}{\lambda-\gamma}\right)<-\left(1-\alpha-\frac{\xi}{1-\gamma}\right)<0$$

Thus, since the second branch of function $\frac{\frac{\partial r(h)}{\partial h}h}{r(h)}$ is decreasing in h, we conclude that function $r(h)$ is decreasing when $h>\overline{h}$. ■

Proof proposition 5.2:

It follows from the Euler equation (3.13) that at the steady state $r(t)=\delta+\rho$. Let's define $\overline{\rho}=r(\widetilde{h})-\delta$, where $\widetilde{h}$ was defined in lemma 5.1. It follows from lemma 1 that $\min\limits_{h\in\left(0,\overline{h}\right]} r(h)=\delta+\overline{\rho}$. Thus, if $\rho<\overline{\rho}=\min\limits_{h\in\left(0,\overline{h}\right]} r(h)-\delta$, then $r(h)$ is above $\delta+\rho$ in the interval $\left(0,\overline{h}\right]$. Thus, it follows from lemma 5.1 that there is a unique human capital $h^{ss}>\overline{h}$ such that $r(h^{ss})=\delta+\rho$ and consequently there is a unique steady state. If $\rho>\overline{\rho}=\min\limits_{h\in\left(0,\overline{h}\right]} r(h)-\delta$, then it follows from lemma 5.1 that there is three different levels of human capital such $r(h)=\delta+\rho$: the first in the interval $\left(0,\widetilde{h}\right)$, the second in the interval $\left(\widetilde{h},\overline{h}\right)$ and the third in the interval $\left(0,\overline{h}\right)$. Thus, there are three steady state equilibria. ■

Proof Proposition 4:

Locus $\dot{h}=0$ is described in equation (5.4) and simplifying, it can be rewritten as

$$c\left(t\right)=\begin{cases} Ah\left(t\right)^{\alpha}\left[1-\lambda\phi h\left(t\right)^{\frac{\gamma}{1-\gamma}}\right]^{\alpha}-\left(n+\delta\right)\frac{h(t)^{\frac{1-(\xi+\gamma)}{1-\gamma}}}{((1-\gamma)\lambda\phi)^{\xi}} & \text{if } \forall h\left(t\right)<\overline{h} \\ Ah\left(t\right)^{\alpha}\left(1-\lambda\overline{\tau}\right)^{\alpha}-\left(n+\delta\right)\frac{h(t)}{\left[\lambda\overline{\tau}h(t)-\frac{1}{\Gamma^{\frac{1}{\gamma}}}\right]^{\xi}} & \text{if } \forall h\left(t\right)\geq\overline{h} \end{cases}$$

where $\phi=\frac{(\gamma\Gamma\overline{\tau})^{\frac{1}{1-\gamma}}}{\gamma}$.

Locus $\dot{c}=0$ is described in equation (5.4) and simplifying, it can be rewritten as

$$c(t)=\begin{cases} \begin{array}{c} Ah(t)^{\alpha}\left[1-\lambda\phi h(t)^{\frac{\gamma}{1-\gamma}}\right]^{\alpha} * \\ *\left(1-\alpha-\frac{\alpha\left[\frac{1-(\xi+\gamma)}{\xi}+(\Gamma\overline{\tau})^{\frac{1}{1-\gamma}}\gamma^{\frac{\gamma}{1-\gamma}}h(t)^{\frac{\gamma}{1-\gamma}}\right]}{1-\lambda\phi h(t)^{\frac{\gamma}{1-\gamma}}}\right)+ \\ +\frac{1-\gamma}{\xi}\theta\frac{h(t)^{\frac{1-(\xi+\gamma)}{1-\gamma}}}{((1-\gamma)\lambda\phi)^{\xi}} \end{array} & \text{if } \forall h(t)<\overline{h} \\ \begin{array}{c} Ah(t)^{\alpha}(1-\lambda\overline{\tau})^{\alpha-1}\left[1-\overline{\tau}-\frac{\alpha\left(1-\frac{\gamma\Gamma^{\frac{1}{\gamma}}}{\lambda h(t)}\right)}{\xi}\right]+ \\ +\frac{\left(1-\frac{\gamma\Gamma^{\frac{1}{\gamma}}}{\lambda h(t)}\right)}{\xi}\theta\frac{h(t)}{\left[\lambda\overline{\tau}h(t)-\frac{1}{\Gamma^{\frac{1}{\gamma}}}\right]^{\xi}} \end{array} & \text{if } \forall h(t)\geq\overline{h} \end{cases}$$

where $\theta=(\rho+\delta)-\frac{\xi(n+\delta)}{1-\gamma}$.

It is straightforward that both of them are continuous functions. Thus because of the loci $\dot{h}=0$ and $\dot{c}=0$ are continuous functions and, once it is proved there exists a unique steady state, a sufficient condition to guarantee there is a saddle point dynamic is to prove that $\lim_{h\to 0}\left.\frac{\partial c(t)}{\partial h(t)}\right|_{\dot{h}=0}>\lim_{h\to 0}\left.\frac{\partial c(t)}{\partial h(t)}\right|_{\dot{c}=0}$.

(i) We first study locus $\dot{h}=0\ \forall h(t)<\overline{h}$. Thus,

$$\begin{aligned}\left.\frac{\partial c}{\partial h}\right|_{\dot{h}=0} &= \alpha Ah^{\alpha-1}\left[1-\lambda\phi h^{\frac{\gamma}{1-\gamma}}\right]^{\alpha-1}\left(1-\frac{\lambda\phi}{1-\gamma}h(t)^{\frac{\gamma}{1-\gamma}}\right)- \\ &\quad -(n+\delta)\left(\frac{1-(\xi+\gamma)}{1-\gamma}\right)\frac{h(t)^{\frac{-\xi}{1-\gamma}}}{((1-\gamma)\lambda\phi)^{\xi}}\end{aligned}$$

$$\begin{aligned}\left.\frac{\partial c}{\partial h}\right|_{\dot{h}=0} &= h^{\alpha-1}\left\{\alpha A\left[1-\lambda\phi h^{\frac{\gamma}{1-\gamma}}\right]^{\alpha-1}\left(1-\frac{\lambda\phi}{1-\gamma}h(t)^{\frac{\gamma}{1-\gamma}}\right)-\right. \\ &\quad \left.-(n+\delta)\left(\frac{1-(\xi+\gamma)}{1-\gamma}\right)\frac{h(t)^{1-\alpha-\frac{\xi}{1-\gamma}}}{((1-\gamma)\lambda\phi)^{\xi}}\right\}\end{aligned}$$

Therefore,

$$\lim_{h\to 0}\left.\frac{\partial c}{\partial h}\right|_{\dot{h}=0}=h^{\alpha-1}\{\alpha A\}$$

(ii) We proceed similarly with locus $\dot{c}=0\ \forall h(t)<\overline{h}$

$$\left.\frac{\partial c}{\partial h}\right|_{\dot{c}=0} =$$

$$\alpha A h^{\alpha-1}\left[1-\lambda\phi h^{\frac{\gamma}{1-\gamma}}\right]^{\alpha-1}\left(1-\frac{\lambda\phi}{1-\gamma}h(t)^{\frac{\gamma}{1-\gamma}}\right)*$$

$$*\left(1-\alpha-\frac{\alpha\left[\frac{1-(\xi+\gamma)}{\xi}+(\Gamma\overline{\tau})^{\frac{1}{1-\gamma}}\gamma^{\frac{\gamma}{1-\gamma}}h^{\frac{\gamma}{1-\gamma}}\right]}{1-\lambda\phi h^{\frac{\gamma}{1-\gamma}}}\right)+Ah^{\alpha}\left[1-\lambda\phi h^{\frac{\gamma}{1-\gamma}}\right]^{\alpha}*$$

$$*\left(-\alpha\left(\frac{\gamma}{1-\gamma}\right)h^{\frac{\gamma}{1-\gamma}-1}\frac{(\Gamma\overline{\tau})^{\frac{1}{1-\gamma}}\gamma^{\frac{\gamma}{1-\gamma}}\left[1-\lambda\phi h^{\frac{\gamma}{1-\gamma}}\right]+\lambda\phi\left[\frac{1-(\xi+\gamma)}{\xi}+(\Gamma\overline{\tau})^{\frac{1}{1-\gamma}}\gamma^{\frac{\gamma}{1-\gamma}}h^{\frac{\gamma}{1-\gamma}}\right]}{\left(1-\lambda\phi h^{\frac{\gamma}{1-\gamma}}\right)^{2}}\right)+$$

$$+\frac{1-\gamma}{\xi}\left(\frac{1-(\xi+\gamma)}{1-\gamma}\right)\theta\frac{h(t)^{\frac{-\xi}{1-\gamma}}}{((1-\gamma)\lambda\phi)^{\xi}}$$

$$\left.\frac{\partial c}{\partial h}\right|_{\dot{c}=0} =$$

$$\alpha A h^{\alpha-1}\left[1-\lambda\phi h^{\frac{\gamma}{1-\gamma}}\right]^{\alpha-1}\left(1-\frac{\lambda\phi}{1-\gamma}h(t)^{\frac{\gamma}{1-\gamma}}\right)\left(1-\frac{\alpha\left[\frac{1-\gamma}{\xi}\right]}{1-\lambda\phi h^{\frac{\gamma}{1-\gamma}}}\right)-$$

$$-\alpha A h^{\alpha-1}\left[1-\lambda\phi h^{\frac{\gamma}{1-\gamma}}\right]^{\alpha-1}\left(h^{\frac{\gamma}{1-\gamma}}\left(\frac{\xi+\lambda(1-\xi-\gamma)}{\xi(1-\gamma)}\right)\frac{(\Gamma\overline{\tau}\gamma)^{\frac{1}{1-\gamma}}}{\left(1-\lambda\phi h^{\frac{\gamma}{1-\gamma}}\right)}\right)+$$

$$+\frac{1-\gamma}{\xi}\left(\frac{1-(\xi+\gamma)}{1-\gamma}\right)\theta\frac{h(t)^{\frac{-\xi}{1-\gamma}}}{((1-\gamma)\lambda\phi)^{\xi}}$$

$$\left.\frac{\partial c}{\partial h}\right|_{\dot{c}=0} =$$

$$h^{\alpha-1}\left\{\alpha A\left[1-\lambda\phi h^{\frac{\gamma}{1-\gamma}}\right]^{\alpha-1}\left(1-\frac{\lambda\phi}{1-\gamma}h\left(t\right)^{\frac{\gamma}{1-\gamma}}\right)\left(1-\frac{\alpha\left[\frac{1-\gamma}{\xi}\right]}{1-\lambda\phi h^{\frac{\gamma}{1-\gamma}}}\right)-\right.$$

$$-\alpha A\left[1-\phi h^{\frac{\gamma}{1-\gamma}}\right]^{\alpha-1}\left(h^{\frac{\gamma}{1-\gamma}}\left(\frac{\xi+\lambda\left(1-\xi-\gamma\right)}{\xi\left(1-\gamma\right)}\right)\frac{\left(\Gamma\overline{\tau}\gamma\right)^{\frac{1}{1-\gamma}}}{\left(1-\lambda\phi h^{\frac{\gamma}{1-\gamma}}\right)}\right)+$$

$$\left.+\frac{1-\gamma}{\xi}\left(\frac{1-\left(\xi+\gamma\right)}{1-\gamma}\right)\theta\frac{h\left(t\right)^{1-\alpha-\frac{\xi}{1-\gamma}}}{\left(\left(1-\gamma\right)\lambda\phi\right)^{\xi}}\right\}$$

Therefore,

$$\lim_{h\to 0}\left.\frac{\partial c}{\partial h}\right|_{\dot{c}=0} = h^{\alpha-1}\left\{\alpha A\left(1-\alpha\left[\frac{1-\gamma}{\xi}\right]\right)\right\}$$

(iii) Given that $\alpha\left[\frac{1-\gamma}{\xi}\right]<1$, it is straightforward to see that

$$\lim_{h\to 0}\left.\frac{\partial c\left(t\right)}{\partial h\left(t\right)}\right|_{\dot{h}=0} > \lim_{h\to 0}\left.\frac{\partial c\left(t\right)}{\partial h\left(t\right)}\right|_{\dot{c}=0}$$

and thus, the unique steady state displays a saddle point dynamics.

Proof Proposition 5:

We define the implicit function $F(.)$ besides the definition of the human capital level in the steady state (equation **??**) as follows:

$$F(.)\;=\;\alpha A\left(1-\lambda\overline{\tau}\right)^{\alpha-1}h^{\alpha-1}\left[\lambda\overline{\tau}h-\frac{1}{\Gamma^{\frac{1}{\gamma}}}\right]^{\xi}-\rho-\delta$$

(notice that we have removed superindex "ss" to easy calculations). Then, by the implicit function theorem we calculate $\frac{\partial h}{\partial\overline{\tau}}=-\frac{\partial F/\partial\Gamma}{\partial F/\partial h}$. We first obtain

$$\frac{\partial F}{\partial\Gamma}\;=\;\alpha A\left(1-\lambda\overline{\tau}\right)^{\alpha-1}h^{\alpha-1}\left[\lambda\overline{\tau}h-\frac{1}{\Gamma^{\frac{1}{\gamma}}}\right]^{\xi-1}\left(\frac{\Gamma^{-\frac{1}{\gamma}-1}}{\gamma}\right)\;>\;0$$

Continuously,

$$\frac{\partial F}{\partial h} = \alpha A\left(1-\lambda\overline{\tau}\right)^{\alpha-1} h^{\alpha-2}\left[\lambda\overline{\tau}h-\frac{1}{\Gamma^{\frac{1}{\gamma}}}\right]^{\xi-1} *$$

$$*\left(\xi\lambda\overline{\tau}h-(1-\alpha)\left[\lambda\overline{\tau}h-\frac{1}{\Gamma^{\frac{1}{\gamma}}}\right]\right) =$$

$$= \underbrace{\alpha A\left(1-\lambda\overline{\tau}\right)^{\alpha-1} h^{\alpha-2}\left[\lambda\overline{\tau}h-\frac{1}{\Gamma^{\frac{1}{\gamma}}}\right]^{\xi-1} *}_{\oplus}$$

$$\underbrace{*\left(\frac{1-\alpha}{\Gamma^{\frac{1}{\gamma}}}-\lambda\overline{\tau}h\left(1-\alpha-\xi\right)\right)}_{?}$$

$h^{\max} < \overline{h} \iff \frac{1-\alpha}{\Gamma^{\frac{1}{\gamma}}\lambda\overline{\tau}(1-\alpha-\xi)} < \frac{1}{\gamma\overline{\tau}\Gamma^{\frac{1}{\gamma}}} \iff \left(\frac{1-\alpha}{1-\alpha-\xi}\right)\gamma < \lambda$

We know that the human capital in the steady state verifies: $h > \overline{h} = \frac{1}{\gamma\overline{\tau}\Gamma^{\frac{1}{\gamma}}}$. Then, at the minimum level, $h = \overline{h}$, we get

$$\frac{1-\alpha}{\Gamma^{\frac{1}{\gamma}}}-\lambda\overline{\tau}\overline{h}\left(1-\alpha-\xi\right) =$$

$$\frac{1}{\gamma\Gamma^{\frac{1}{\gamma}}}\left((1-\alpha)-\frac{\lambda}{\gamma}\left(1-\alpha-\xi\right)\right) =$$

$$\frac{1}{\Gamma^{\frac{1}{\gamma}}\left(1-\alpha-\xi\right)}\left(\frac{\gamma\left(1-\alpha\right)}{\left(1-\alpha-\xi\right)}-\lambda\right) < 0$$

since we assumed $\left(\frac{1-\alpha}{1-\alpha-\xi}\right)\gamma < \lambda$ in proposition 1. Consequently,

$$\left(\frac{1-\alpha}{\Gamma^{\frac{1}{\gamma}}}-\lambda\overline{\tau}h\left(1-\alpha-\xi\right)\right) < 0 \;\; \forall h\left(t\right) > \overline{h}$$

which implies that $\frac{\partial F}{\partial h} < 0$ and thus,

$$\frac{\partial h}{\partial \Gamma} = -\frac{\partial F/\partial \Gamma}{\partial F/\partial h} > 0$$

Therefore, an increase in parameter Γ, it produces an increase in the steady state human capital level. Now, we turn to the definition of the consumption level in the steady state (equation **??**) and we calculate:

$$\frac{\partial c}{\partial \Gamma} = A\left(1-\lambda\overline{\tau}\right)^{\alpha-1}\alpha\left[1-\lambda\overline{\tau}-\alpha\left(\frac{\delta+n}{\rho+\delta}\right)\right]h^{\alpha-1}\frac{\partial h}{\partial \Gamma} > 0$$

since $\frac{\partial h}{\partial \Gamma} > 0$. Therefore, an increase in parameter Γ, it produces an increase in the steady state consumption level.

Proof Proposition 6:

We proceed as proposition 5. Taking into account function $F(.)$ defined in proposition 5, we first obtain

$$\begin{aligned}\frac{\partial F}{\partial \lambda} &= \alpha A\left(1-\lambda\overline{\tau}\right)^{\alpha-1}h^{\alpha-1}\left[\lambda\overline{\tau}h-\frac{1}{\Gamma^{\frac{1}{\gamma}}}\right]^{\xi} * \\ &* \left(\overline{\tau}\left(1-\alpha\right)\left[\lambda\overline{\tau}h-\frac{1}{\Gamma^{\frac{1}{\gamma}}}\right]+\overline{\tau}h\xi\left(1-\lambda\overline{\tau}\right)\right) > 0\end{aligned}$$

And from proposition 5 we know that $\frac{\partial F}{\partial h} < 0$. Therefore, by the implicit function theorem

$$\frac{\partial h}{\partial \lambda} = -\frac{\partial F/\partial \lambda}{\partial F/\partial h} > 0$$

Therefore, an increase in parameter λ, it produces an increase in the steady state human capital level. Now, we turn to the definition of the consumption level in the steady state (equation **??**). It is straightforward to observe that:

$$\frac{\partial c}{\partial \lambda} > 0$$

since the effect of λ is similar to the effect of $\overline{\tau}$ analyzed in proposition 6. Therefore, an increase in the fraction of public revenues which is devoted to hire human capital at the expense of spending in the public good, it produces an increase in the steady state consumption level.