# Tests of Alternate Asset Pricing Models for Misspecification Using The Hansen-Jagannathan Distance

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**Abstract**

I compare the performance of several important linear asset pricing models using a common set of test assets over the same time period. I evaluate the relative performance of models using the Hansen-Jagannathan distance criterion. I find that the Fama-French five-factor statistically outperforms return-based models (CAPM (1964), the Fama-French three- (1992) and five-factor (1993), the Chen, Roll and Ross five-factor (1986) and the Chen, Novy-Marx and Zhang three-factor (2010)) explaining equity assets; the Chen, Roll and Ross five-factor statistically performs well than other return-based models to price equities and bonds; Chen, Roll and Ross macro-factor (1986) statistically outperforms other consumption-based models, i.e. Yogo non-durable-durable CCAPM (2006) and Piazzesi, Schneider and Tuzel housing CCAPM (2007) to price equity assets, so does the Santos and Veronesi CCAPM with labor income (2006) comparing with scaled models i.e. Lettau and Ludvigson scaled CCAPM (2001), Lustig and Van Nieuwerburgh housing-collateral CCAPM (2005) and Piazzesi, Schneider and Tuzel housing-scaled CCAPM (2007).

*JEL Classification:* G12

*Keywords:* Asset Pricing, Hansen-Jagannathan Distance

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# 1 Introduction and Motivation

The purpose of this paper is to evaluate the comparative performance of several linear asset pricing models. I use different data set of test assets and various comparison measures to evaluate linear factor asset pricing models' performances. Particularly, those linear factor asset pricing models will be ranked according to the normalized size of their pricing errors. Since all asset pricing models can be viewed as approximations of reality and are likely to be misspecified, one has to take a stand on what measure of model misspecification to use. While there are many possible choices, Hansen and Jagannathan (1997) propose an interesting measure of model misspecification.

The first hypothesis of the paper concerns the distinction performances between CAPM or APT and other well-known linear factor asset pricing models or newer consumption-based models and the traditional consumption CAPM in order to understand cross-section returns on assets. Both in academic and industry there still exists debates on whether or not CAPM or APT is out of time and on macroeconomic models or macro-variables can catch the characteristics of assets better than pure finance models (return-based models).

Evidences that, while the literature highly recommends Fama-French three- and five-factor models, in industry, practitioners often use the CAPM or APT may be inferred by review of the more popular MBA level corporate finance textbooks [e.g. Brealey and Myers(2000)] and texts written by practitioners [e.g. Grinold and Kahn (1995)]. More directly, Graham and Harvey (2000) report that 73.5 percent of respondents (392 CFOs), always or almost always use the CAPM when estimating the cost of equity capital with the majority of the remaining participants using either the historical average stock return or multi-factor models. Da, Guo and Jagannathan (2009), in their academic paper, argue that the CAPM may be a reasonable model for estimating the cost of capital for projects in spite of increasing criticisms in the empirical asset pricing literature.

Furthermore, Lewellen, Nagel and Shanken (2010) take a skeptical view of the asset pricing tests of a number of macroeconomic factor models found in several papers, finding that none of the many proposed macroeconomic models of the SDF performs well in explaining a cross-section of average stock returns. But economic theory implies that the true sources of systematic risk must be macroeconomic in nature. Therefore, if economic theory is correct and systematic risk is macroeconomic in nature, we should expect a factor structure in macroeconomic data, and we should expect a variety of macroeconomic indicators to be correlated with return-based factors.

To compete against CAPM, well-known return-based linear factor candidates include the Chen, Roll and Ross (1986) five-factor model, in which factors are macroeconomic variables; the Fama-French (1992) three-factor model, in which the size and the value factors are added into CAPM; the Fama-French (1993) five-factor model which puts the maturity and the default factors into the Fama-French three-factor model; the Chen, Novy-Marx and Zhang (2010) three-factor model, in which the investment and the return on assets become the two other factors plus the market factor from CAPM.

To compare with the traditional consumption CAPM, Yogo (2006) nondurable and durable consumptions model and Piazzesi, Schneider and Tuzel (2007) non-housing and housing consumption model are chosen; the scaled consumption-based models comparison include the conditional consumption CAPM of Lettau and Ludvigson (2001), in which using the consumption-wealth ratio as a conditional variable; the consumption-housing CAPM of Piazzesi, Schneider and Tuzel (2007), in which the non-housing consumption expenditure share is used as a conditioning variable; the collateral-CCAPM of Lustig and Van Nieuwerburgh (2005), in which the housing collateral ratio is used as a conditioning variable; and the conditional CCAPM with the labor income of Santos and Veronesi (2006).

Another principal question needed this paper answer is as to whether the superior cross-section performance of such one model is maintained once the model explains various test portfolios via different measures, for the "best" model needs to explain the cross-section dispersion of risk across these test assets. Therefore, the second hypothesis is focus on the robustness of the best model among test portfolios and comparison criteria. In the paper, I apply HJ distance, modified HJ distance, unconstrained and constrained HJ distance measures against traditional adjusted $R^2$s to compare candidate models among Fama-French 25, 30 Industrial-sorted, 10 Deciles and Fama-French 25 combined government bonds portfolios.

My first contribution is I firstly compare almost all the linear factor asset pricing models through separating them as return-based, consumption-based and scaled consumption-based asset pricing models, because of data quality and model constructions. I find the Fama-French (1993) five-factor model behaves better than others in Fama-French 25 and 30 Industrial-sorted portfolios; the Chen, Roll and Ross (1986) five-factor can explain stocks and bonds combined portfolios well according to the HJ and the modified HJ distance measures. Using the unconstrained and the constrained HJ distance measures, the Fama-French five-factor model is ranked as the best model to explain Fama-French 25 size-value and 10 Deciles portfolios among candidate factor models; the Chen, Roll and Ross five-factor explain stocks and bonds combined portfolios well. Moreover, I result that when pricing Fama-French size-value and 30 Industrial-sorted assets: Chen, Roll and Ross macro-factor (1986) outperforms other consumption-based models, i.e. Yogo (2006) and Piazzesi, Schneider and Tuzel (2007); the Santos and Veronesi scaled CCAPM with labor income (2006) performs better than other scaled consumption-based models, i.e. Lettau and Ludvigson (2001), Lustig and Van Nieuwerburgh (2005) and Piazzesi, Schneider and Tuzel (2007). The second contribution states as I show all these results are statistically significantly robust among gross and excess returns via various distance measures.

The rest of the paper is organised as follows. Section 2 presents literature review. Section 3 introduces methodology on Hansen-Jagannathan distance. Section 4 describes candidate models. Section 5 describe the candidate models and data. Section 6 presents the empirical analysis. Section 7 extends to compare consumption-based and scaled consumption-based models. The final section summarises findings.

## 2 Literature Review

All asset pricing models are at best approximations of the reality, but none can price portfolios perfectly in general. Therefore, it is important for us to construct a measure to compare and evaluate the performance of different models. For this purpose, Hansen and Jagannathan (1997) develop the Hansen-Jagannathan distance. This measure is the quadratic form of the pricing errors weighted by the inverse of the second moment matrix of returns. Intuitively, the HJ distance equals the minimum pricing errors generated by a model for portfolios with unit second moment.

Most previous papers usually apply $R^2$ to tell their constructed models are better to explain cross-section returns on assets, for $R^2$ is a statistic that will give some information about the goodness of fit of a model. But values of $R^2$ outside the range 0 to 1 can occur in the case that the constant term is not regressed during the estimation. Moreover, adjusted $R^2$ can be negative, which implies that $R^2$ is not a good measure to compare models.

The Akaike information criterion (AIC) is another measure of the goodness of fit of a statistical model. Given a data set, several candidate models may be ranked according to their AIC, with the model having the minimum AIC being the best. In the general case, the AIC is $2k - 2ln(L)$, where k is the number of parameters in the statistical model, and L is the maximized value of the likelihood function for the estimated model. The drawback in AIC lays on L the maximized value of the likelihood function, in which we have to assume the distributions in advanced in order to find the specific values for those parameters that produces the distribution most likely to have produced the observed results. For instance, the distribution of stocks' prices is almost following a random walk, the amount of information in the data increases indefinitely as the sample size increases. In this case, there exists the inconsistent estimator, which makes AIC weak.

Moreover, the Vuong closeness test (1989) is likelihood-ratio-based test for model selection using the Kullback-Leibler information criterion. This statistic makes probabilistic statements about two models. It tests the null hypothesis, that two models (nested, non-nested or overlapping) are as close to the actual model against the alternative that one model is closer. But it is quite difficult to compute this statistic in the overlapping and nested cases.

Jagannathan and Wang (1996) apply an easily computed distance measure to show the performance of their conditional CAPM. In general, they demonstrate that the empirical support for conditional CAPM specification is rather strong. Hansen and Jagannathan (1997) theoretically and empirically derive the HJ distance measure via GMM estimator, which does not need any assumptions and distributions in advanced.

This type of comparison has been adopted in many recent papers. Jagannathan and Wang (1998), Kan and Zhang (1999) and Campbell and Cochrane (2000) use the HJ distance in order to explain why the CAPM and its extensions are better at approximating asset pricing models than the standard consumption-based asset pricing theory. Lettau and Ludvigson (2001), Vassalou (2003), Jacobs and Wang (2004), Vassalou and Xing (2004) and Huang and Wu (2004) run the HJ distance to catch the consumption effect, the firm effect and the specifications of option pricing models to explain cross-section returns on stocks market. The closest papers to my study are Hodrick and Zhang (2001) and Parker and Julliard (2005) which evaluate the specification errors of several empirical asset pricing models that have been developed as potential improvements on the CAPM, i.e. CAPM, CCAPM, Jagannathan and Wang(1998), Fama-French three-factor and Fama-French five-factor. However, I study the rankings' robustness using not only various HJ distance measures, but also using different assets portfolios. This is the main contribution of the paper.

Kan and Robotti (2008) conclude that the misspecification measure is no longer affected by an affine transformation of the factors if we apply to the case of gross returns, although many of results are the same for gross returns and for excess returns. Thus, if the excess returns are used by test assets (zero-cost portfolios), we have to restrict the candidate SDF to have unit mean. Kan and Robotti (2010) propose a new methodology to test whether or not two competing linear asset pricing models have the same HJ distance. They show that there is little evidence that conditional and inter-temporal capital asset pricing model (CAPM)-type specifications outperform the simple unconditional CAPM. In this paper, I apply the HJ distance, the modified HJ distance, the unconstrained HJ distance and the constrained HJ distance measures to rank candidate factor models.

# 3 Test Methodology

This paper assumes throughout that the risk-free rate $R_t^f$ is observed. Let $M_{t+1}$ be the stochastic discount factor. Any tradable asset with return $R_{t+1}$ must satisfy

$$1 = E_t[M_{t+1}R_{t+1}] \tag{1}$$

where $E_t$ denotes the expectation conditional on the information known at time t. For the basic consumption-based model, the asset pricing equation (Euler equation) above is derived from the first-order condition for the optimal consumption choice of a representative agent, and $M_{t+1}$ is equal to the intertemporal marginal rate of substitution $\gamma \frac{u^{'}(C_{t+1})}{u^{'}(C_t)}$, where $u(C_t)$ is the instantaneous utility function and $u^{'}(C_t)$ is the marginal utility of consumption $C_t$, and $\gamma$ is the subjective discount factor.

It is assumed that the stochastic discount factor $M_{t+1}$ can be approximated as a linear function of factors:

$$M_{t+1} = a + \lambda^{'} f_{t+1} \tag{2}$$

## 3.1 Hansen-Jagannathan Distance

How to examine the pricing error on the portfolios that are most mispriced by a given model? Hansen and Jagannathan (1997) develop a measure of degree of misspecification of an asset pricing models. This measure is defined as:

$$min_{M^* \in \aleph} \|M^* - M\|$$

the least squares distance between the stochastic discount factor associated with and an asset pricing model and the family of stochastic discount factors that price all the assets correctly.

$$[Figure 1]$$

From Figure 1 we can see that the HJ distance is the least squared distance between any point along the admissible SDF line and the cross point between these two orthogonal lines (the payoffs line).

Also this measure is equal to the maximum pricing errors generated by a model on the portfolios whose second moments of returns are equal to one

$$max_{\|x=1\|} |\pi^*(x) - \pi(x)|$$

where $\pi^*(x)$ and $\pi(x)$ are the prices of x assigned by the true and the proposed SDF, respectively.

I define $R_t = [R_{1,t}, R_{2,t}, ..., R_{N,t}]'$ being the gross returns on assets, and let

$$\alpha_t(\lambda) = R_t M_t(\lambda) - I_N = R_t \lambda f_t' - I_N \tag{3}$$

where $\alpha_t(\lambda)$ is the vector of pricing errors. In unconditional models, the number of moment conditions is equal to $N$, the number of test assets. Hansen and Jagannathan (1997) show that the maximum pricing error per unit norm of any portfolio of these N assets (HJ distance) is given by

$$\delta = \sqrt{E[(\alpha_t(\lambda))'][E(R_t R_t')]^{-1} E[\alpha_t(\lambda)]} \tag{4}$$

It is equivalent to a GMM estimator with the moment condition $E[\alpha_t(\lambda)] = 0$ and the weighting matrix $[E(R_t R_t')]^{-1}$, which is different from the optimal matrix. One reason is that we cannot use $W_T = S^{-1}$ to assess specification error and compare models. Suppose we have two different SDFs and use GMM with optimal weighting to estimate and test each model on the same set of asset returns. Doing so, we find that the over-identification restrictions are not rejected for $M^1_{t+1}$ but are for $M^2_{t+1}$. May we conclude that the model with $M^1_{t+1}$ is superior? No. The reason is that Hansen's $J_T$ statistic depends on the model-specific S matrix (the Hansen optimal matrix). As a consequence, Model 1 can look better simply because the SDF and pricing errors $\alpha_T$ are more volatile than those of Model 2, not because its pricing errors are lower and its Euler equations less violated. Or a failure to reject in a specification test of a model could arise because the model is poorly estimated and subject to a high degree of sampling error, not because it does explains the return data well[1].

## 3.2 Modified Hansen-Jagannathan Distance

When applying the HJ distance, we do not put any constraint on the mean of the stochastic discount factor when pricing excess returns on assets, which means it is possible to obtain different risk prices or risk loadings for risk-free assets. At the worst case, the rankings of misspecification errors will be changed a lot only by an affine transformation of the SDF. Because when only excess returns are used to measure model misspecification, one cannot specify proposed SDF in a way such that it can be zero for some values of $\lambda$. Intuitively, the moment restriction when excess returns using does not separately the parameters $a$ and $\lambda$ in equation (2). Because the GMM errors for the parameter pair $(a, \lambda)$ are proportional to the GMM errors for the parameter pair $(ka, k\lambda)$, for any scalar k.

Kan and Robotti (2008) suggest defining the SDF as a linear function of the de-meaned factors to avoid this affine transformation problem. Therefore, a modification of the traditional Hansen-Jagannathan distance (HJ distance) is needed when we use the de-meaned factors. They define the modified HJ distance as:

$$\delta_m = \sqrt{min_\lambda g_T(\lambda)' V_{22T}^{-1} g_T(\lambda)} \tag{5}$$

where $g_T(\lambda) = \frac{1}{T}\sum_{t=1}^{T} \alpha_t(\lambda)$ is the average on pricing errors, $V_{22T}^{-1}$ is the covariance matrix of the test portfolios.

## 3.3 Constrained Hansen-Jagannathan Distance

Although constricted the average of SDFs to be constant, it is possible for an SDF to price all the test assets correctly and yet to take on negative values

[1]see Ludvigson, 2011, Advances in Consumption-Based Asset Pricing: Empirical Tests

with positive probability. This case happens when these exist arbitrage opportunities among test portfolios (e.g. derivatives on test assets) and it could be problematic to set this SDF to price payoffs. Therefore, it is necessary to constrict the admissible SDFs being non-negative.

Following Gospodinov, Kan and Robotti (2010)'s mechanism, I denote the vector of gross returns on N assets at the end of period t by $R_t$, the corresponding costs of these N assets at the end of period $t-1$ by $q_{t-1}$, which $E[q_{t-1}] \neq 0$. Because when $E[q_{t-1}] = 0_N$, the mean of SDF cannot be identified and we have to choose some normalization of the SDF[2]. Empirically, I can solve the constrained HJ distance as:

$$\hat{\delta_+^2} = min_{M_t^*, t=1,...,T} \frac{1}{T} \sum_{t=1}^{T} (M_t - M_t^*)^2 \tag{6}$$

$$s.t. \frac{1}{T} \sum_{t=1}^{T} M_t^* R_t = \bar{q} \tag{7}$$

$$M_t^* \geq 0, t = 1, ..., T \tag{8}$$

where $M_t$ denotes the candidate SDF and $M_t^*$ stands for admissible SDF in the set $\aleph_+$.

## 3.4 Testing for Multiple Comparisons

A limitation of the Hansen-Jagannathan (1997) approach is that it provides no method for comparing HJ distances statistically: $HJ^1$ may be less than $HJ^2$, but are they statistically different from one another once we account for sampling error?

Suppose we seek to compare the estimated HJ distance measures of several models. Let $\delta_{j,T}^2$ denote the squared HJ distance for model j. Taking a benchmark model, e.g., the model with smallest squared HJ distance among $j = 1, ..., K$ competing models, and denoting:

$$\delta_{1,T}^2 = min(d_{j,T}^2)_{j=1}^K$$

The null hypothesis is:

$$H_0 : d_{1,T}^2 - d_{2,T}^2 \leq 0$$

[2]see Kan and Robotti, 2008, The exact distribution of the Hansen-Jagannathan bound

where $d^2_{2,T}$ is the competing model with the next smallest squared distance. To apply White's reality check test, we define the test statistic as $T^W = max_{2,...,5}\sqrt{T}(d^2_{1,T} - d^2_{j,T})$, based on White (2000). The distribution of $T^W$ is computed via block bootstrap (Chen and Ludvigson, 2009). Need to mention, the justification for the bootstrap rests on the existence of a multivariate, joint, continues, limiting distribution for the set $(d^2_{j,T})^K_{j=1}$ under the null.

By repeated sampling, the bootstrap estimates of the p-value:

$$\hat{p_W} = \frac{1}{B}\sum_{b=1}^{B} I_{(T^{W,b} > T^W)}$$

where B is the number of bootstrap samples and $T^{\hat{W},b}$ stands for White's original bootstrap test statistic. If null is true, the historical value of $T^W$ should not be unusually large, given sampling error. Given the distribution of $T^W$, reject the null if its historical value, $\hat{T^W}$, is greater than the 95th percentile of the distributions for $T^W$. At a 5 % level of significance, we reject the null if $\hat{p_W}$ is less than 0.05, but do not reject otherwise.

## 3.5 Chi-Squared Tests for Multiple Comparisons

Although Chen and Ludvigson (2009) develop an appropriate econometric method for comparing asset pricing models based on HJ distance, a general statistical procedure for model selection is still missing. Gospodinov, Kan and Robotti (2011) suggest that we should separate models into three categories: nested, strictly non-nested and overlapping. For non-nested and overlapping models they introduce a multivariate inequality test based on Wolak (1987,1989).

Let $\rho = (\rho_2, ..., \rho_{p+1})$, where $\rho_i = \delta^2_1 - \delta^2_i$. We set $\delta^2_1$ as the winner, and test $H_0 : \rho \leq 0_p$. We assume that

$$\sqrt{T}(\hat{\rho} - \rho) \xrightarrow{A} N(0_p, \Omega_{\hat{\rho}}) \tag{9}$$

Let $\widetilde{\rho}$ be the optimal solution in the following quadratic programming problem:

$$min_\rho (\hat{\rho} - \rho)' \hat{\Omega}^{-1}_{\hat{\rho}} (\hat{\rho} - \rho) \tag{10}$$

$$s.t. \rho \leq 0_r \tag{11}$$

where $\hat{\Omega}_{\hat{\rho}}^{-1}$ is a consistent estimator of $\Omega_{\hat{\rho}}^{-1}$. The likelihood ratio test of the null hypothesis is

$$LR = T(\hat{\rho} - \widetilde{\rho})' \hat{\Omega}_{\hat{\rho}}^{-1} (\hat{\rho} - \widetilde{\rho}) \quad (12)$$

Since the null hypothesis is composite, to construct a test with the desired size, they require the distribution of LR under the least favorable value of $\rho$, which is $\rho = 0_P$. Under this value, LR follows a 'chi-bar-squared distribution',

$$LR \xrightarrow{A} \sum_{i=0}^{p} w_i(\Omega_{\hat{\rho}}^{-1}) X_i \quad (13)$$

where the $X_i$ are independent $\chi^2$ random variables with i degrees of freedom and $\chi_0^2$ is simply defined as the constant zero. An explicit formula for the weights $w_i(\Omega_{\hat{\rho}}^{-1})$ is given in Kudo (1963).

For nested models, Gospodinov, Kan and Robotti (2011) suppose that $y_t^A(\lambda_1^*) = y_t^i(\lambda_i^*)$ can be written as a parametric restriction of the form $\varphi_i(\lambda_i^*) = 0_{k_i - k_1}$, where $\varphi(\cdot)$ is a twice continuously differentiable function in its argument. The null hypothesis for multiple model comparison can therefore be formulated as $H_0 : \varphi_2 = 0_{k_2-k_1}, ..., \varphi_{p+1}(\lambda_{p+1}^*) = 0_{k_{p+1}-k_1}$. The comparison test statistic follows Wald test with the degree of freedom $(\sum_{i=2}^{p+1} k_i - pk_1)$.

# 4 Description of the Candidate Models

I try to avoid models that researchers would never consider in practice. Thus I narrow down the focus on to a few high profile sets of factors that are most likely to be considered in applications, i.e. CAPM, the Fama-French three-factor, the Fama-French five-factor, the Chen, Roll and Ross five-factor and the Chen, Novy-Marx and Zhang three-factor models. Of course, it is always the case that empirical results are conditional on the researchers' prior. However, how to select the set of candidate models seems to be beyond the scope of any econometric methods.

Sharpe (1964) and Lintner (1965) develop the Capital Asset Pricing Model (CAPM). In general, the expected excess return on an asset equals the market risk $\lambda$ of the asset times the expected excess return on market portfolio, which can be expressed as

$$M_{t+1}^{CAPM} = a + \lambda R_{t+1}^{eM} \quad (14)$$

where $R^{eM}_{t+1}$ denotes excess returns on the market portfolios.

Fama and French (1992) (FF3) document the role of size and book/market in the cross-section of expected stock returns, and show that CAPM are not supported by the data.

$$M^{FF3}_{t+1} = a + \lambda_1 R^{eM}_{t+1} + \lambda_2 SMB_{t+1} + \lambda_3 HML_{t+1} \tag{15}$$

Fama and French (1993) (FF5) present a five-factor asset-pricing model to explain the cross-section returns on stocks and bonds. They state that this five-factor model can explain stocks and bonds better than three-factor model.

$$M^{FF5}_{t+1} = a + \lambda_1 R^{eM}_{t+1} + \lambda_2 SMB_{t+1} + \lambda_3 HML_{t+1} + \lambda_4 TERM_{t+1} + \lambda_5 DEF_{t+1} \tag{16}$$

where TERM and DEF stand for the maturity risk and the default risk factors.

Chen, Novy-Marx and Zhang (2010) (CNZ3) offer a new three-factor model from q-theory (e.g., Tobin (1969) and Cochrane (1991)) by outlining a two-period structure model. This q-theory factor model captures many patterns anomalous to the Fama-French model, and performs roughly as well as their model in explaining the portfolio returns which Fama and French (1996) show that their model is capable of explaining.

$$M^{CNZ3}_{t+1} = a + \lambda_1 R^{eM}_{t+1} + \lambda_2 IA_{t+1} + \lambda_3 ROA_{t+1} \tag{17}$$

Stephen Ross (1976) introduces the Arbitrage Pricing Theory (APT). APT holds that the expected return of a financial asset can be modeled as a linear function of various macro-economic factors. Chen, Roll, and Ross (1986) (CRR5) then develop a macroeconomic factor model in which the factor innovations are observed directly and the factor betas are estimated via time-series regression.

$$M^{CRR5}_{t+1} = a + \lambda_1 MP_{t+1} + \lambda_2 UI_{t+1} + \lambda_3 DEI_{t+1} + \lambda_4 UTS_{t+1} + \lambda_5 UPR_{t+1} \tag{18}$$

where MP is the growth rate of industrial production, UI is the unexpected inflation, DEI is defined as the change in expected inflation, the term premium UTS, and UPR the default premium.

# 5 Preliminary Analysis of Data

The data are value-weighted, from January in 1972 till June in 2007, as a common time period for all the tests.

## 5.1 Data Descriptions

This paper uses four test portfolios: Fama-French 25 portfolios sorted by firm size and book-to-market ratio; Industrial-sorted portfolios created by U.S. 30 industries; 10 Deciles portfolios formed on size (market capitalization); Fama-French 25 plus 7 government bonds with different maturities. Above portfolios include all NYSE, AMEX and NASDAQ stocks which are available and can be downloaded from Professor Kenneth French's webpage. The 7 different maturities government bonds data come from "The Monthly CRSP US Treasury Database". All macro-factor data come from FRED database at Federal Reserve Bank of St. Louis.

## 5.2 Empirical Specification

I identify the market portfolios excess return $R^{eM}$ by the difference between the market portfolios return $R^{M}$ to the risk-free rate $R_f$, which is the 30-Day Treasury Bill return. In the Fama-French three-factor model, SMB stands for the average return on the small portfolios minus the average return on the three big portfolios; HML denotes the average return on the value portfolios minus the average return on the growth portfolios. Fama-French five-factor model adds two extra factors: the difference between Long-Term Government Bond and one month T-bill rate, and the difference between Long-Term Corporate Bond minus Long-Term Government Bonds (Welch and Goyal, 2008).

The Chen, Novy-Marx and Zhang three-factor includes the market portfolios; the IA factor, the difference between the average of the returns on two low-IA portfolios and the average of the returns on the two high-IA portfolios; the ROA factor, the difference between the average of the returns on the two high-ROA portfolios and the average of the returns on the two low-ROA portfolios.

In the Chen, Roll and Ross five-factor model, I use the growth rate of industrial production which is defined as $MP_t = logIP_t - logIP_{t-1}$, where $IP_t$ is the index of industry production. Other factors include the unexpected inflation defining as $UI_t = I_t - E[I_t|t-1]$, where $I_t = logCPISA_t - logCPISA_{t-}1$; the change in expected inflation, $DEI_t = E[I_{t+1}|t] - E[I_t|t-1]$ where $E[I_t|t-1]$ is the expected inflation, and $E[I_t|t-1] = rf_t - E[RHO_t|t-1]$, here $rf_t$ is the one-month Treasury bill rate; the term premium is the yield difference between 20-year and 1-year yield, and the default premium is computed from BAA-AAA.

## 5.3 Empirical Results

Because factors all seem to have some explanatory power for the cross-section of asset returns, a natural first step is to examine the extent to which they are correlated.

$$[Table1]$$

I find very little correlation among them, except for UTS and TERM (0.9368), but they are applied to different factor models (the Chen, Roll and Ross five-factor and the Fama-French five-factor models). It suggests that they do not all proxy for the same aggregate shocks.

It is well documented in the literature[3] that the CAPM fails to explain small growth portfolios[4]. In this paper, I choose the standard CAPM as the benchmark model.

There is one point worth mentioning regarding the models chosen for the purpose of comparison. A direct comparison of the empirical performance of macroeconomic factor models with models that have pure financial factors is inappropriate, because there is measurement error in the macro factors that is not present for the financial factors. The reason is that returns are far better measured than consumption data, so pricing errors for return-based models that use the mimicking portfolio for marginal utility will be smaller than the underlying consumption-based model. For this reason, the macro-factor model I consider here, the Chen, Roll and Ross five-factor model, is also used only as a rough benchmark for performance.

Because most papers use $R^2$ in order to prove the better performance of their models over CAPM, therefore Table 2 reports adjusted $R^2$s from the cross-section regression-based test.

$$[Table2]$$

The Fama-French five-factor model is able to explain cross-section stock returns on 25 size-value portfolios 90.12%. To price 30 Industrial-sorted and

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[3]see R. Merton (1973), R.Roll (1977), Banz (1981), Basu (1983), Reinganum (1981), Chan, Chen and Hsieh (1985), Bhandari (1988), Gibbons (1982), and Shanken (1985) and Fama and French (1992, 1993, 1995, 1996)

[4]see Jagannathan and McGrattan, 1995, "The CAPM Debate" Federal Reserve Bank of Minneapolis Quarterly Review, Vol. 19, No. 4, Fall 1995, pp. 2-17.

10 Deciles portfolios, the Fama-French five-factor almost outperforms others, except for the Chen, Roll and Ross five-factor model using GLS with the constant and the Fama-French three-factor using OLS with the constant respectively, 13.34% and 86.52%. Adjusted $R^2$s on the Fama-French five-factor, and the Chen, Novy-Marx and Zhang three-factor are outstanding in Fama-French 25 plus 7 government bonds portfolios with 97% and 37.87%. The Chen, Roll and Ross five-factor model also wins against others with -70.07% pricing 25 size-value portfolios plus government bonds.

It seems that adjusted $R^2$ can pick the best model comparing with CAPM and the macro-factor model. But there exists one drawback, adjusted $R^2$ is affected by the estimate with the constant term and without the constant term. It is also worth to mentioning that rankings are not robust both on estimation methods and on test portfolios.

In Table 3 and Table 4, I rank candidates via the HJ and the modified HJ distance measures. For the gross returns on test portfolios, the Fama-French five-factor model produces smaller HJ and modified HJ distance measures than CAPM and other three factor models among the three sample test portfolios; that is, its normalized pricing errors are overall smaller and robust among different estimate methods than those of CAPM, the Fama-French three-factor, the Chen, Roll and Ross five-factor and the Chen, Novy-Marx and Zhang three-factor models. CAPM has the smaller normalized pricing errors than the Chen, Roll and Ross five-factor model to explain gross returns on 10 Deciles portfolios, that is, the macro-factor model is not able to catch the size effect well than CAPM, though able to catch the industry effect. The Chen, Roll and Ross five-factor has the smaller normalized pricing error than others when it prices stocks and bonds portfolios.

While the Fama-French five-factor explain excess returns better than others in Fama-French 25 and 30 Industrial-sorted portfolios again, the Chen, Novy-Marx and Zhang three-factor model shows a smaller magnitude of normalized pricing errors than the classic CAPM and others to price 10 Deciles portfolios. The Chen, Roll and Ross five-factor model maintains its advantage to explain excess returns on stocks and bonds portfolios. It illustrates that the macro-factor model can price different assets simultaneously well than other returns-factor models.[5].

Overall, the Fama-French five-factor model performs best in terms of the

[5]Campbell 1996, Campbell and Cochrane 1999, Campbell 2001 and Cochrane 2006

normalized pricing errors when catching the size, the value and the industry effects on stocks portfolios, that is, it shows a smaller magnitude of pricing errors than the classic CAPM, the Fama-French three-factor, the Chen, Novy-Marx and Zhang three-factor and the marco-factor models. On the other hand, the macro-factor model, Chen, Roll and Ross five-factor, is able to price stocks and bonds combined portfolios. All the distance measures are statistically significantly different from zeros at the 5% significance level. It is not surprised for the Fama-French factor models (the three- and five- factor) to perform well in Fama-French size-value portfolios and 10 size portfolios, because of their "factor-structure"[6].

Constraining that the stochastic discount factor is non-negative, in Table 5 and Table 6, it is clear to see that the Fama-French five-factor can explain gross returns on two test assets well: Fama-French 25 size-value and 10 Deciles, so can it explain excess returns on these test portfolios. The rankings on candidate models stay the same in these two test portfolios. In particular, the result of the unconstrained and the constrained normalized pricing errors support the findings via the HJ and the modified HJ distance measures. For testing, all these unconstrained and constrained HJ distance measures are significantly different from zero no matter the model is correctly specified or not, except for the unconstrained HJ distance of the Fama-French five-factor model in 10 Deciles portfolios.

While pricing gross returns on 30 Industrial-sorted portfolios, the macro-factor model catches the industry effect well than others. However, both distance measures are not statistically different from zeros at 5% significance level. Surprisingly, the Chen, Novy-Marx and Zhang three-factor obtains the bigger normalized pricing errors than the standard CAPM, which contradicts the result in their original paper. The rankings change a lot when candidates price the excess returns on 30 Industrial-sorted portfolios, that is, the Fama-French five-factor model outperforms others, and the macro-factor model cannot catch the industry effect even comparing with CAPM.

For gross returns on stocks and bonds combined portfolios, I obtain the Chen, Roll and Ross five-factor shows the smaller normalized pricing errors then other candidates via the unconstrained HJ distance. Need to mention, unconstrained distance measures for both the macro-factor and the Fama-French five-factor models are not statistically different from zeros at the 5% significance level, which makes it hard to tell which one is the best. Turning

[6]Lewellen, Nagel and Shanken (2009) advocate this factor structure problem

to the constrained measure, the Fama-French five-factor performs well than the macro-factor model, which is different from the result via the HJ and the modified HJ distance measures. Pricing excess returns on stocks and bonds combined portfolios, the macro-factor model is better than the Fama-French five-factor because of the smaller normalized pricing errors via the constrained distance measure. Furthermore, it is still difficult for me to tell which one is the best among these two candidates, because of the statistically non-significant pricing errors.

Overall, the Chen, Roll and Ross five-factor and the Fama-French five-factor change their positions via unconstrained and constrained HJ distance measures to explain gross and excess returns on Industrial-sorted and stocks and bonds combined portfolios. There are two points which need to mention. Firstly, the difference between the unconstrained and the constrained HJ distance is that the constrained HJ experienced iterative computation to satisfy the first order condition. Therefore, after an iteration computation, the Fama-French five-factor model has the chance to outperform the Chen, Roll and Ross five-fator model to price stocks and bonds combined portfolios, although I find that the Chen, Roll and Ross five-factor behaves well via the HJ, the modified HJ and the unconstrained distance measures. Secondly, I use 30-Day Treasury Bill as the risk-free interest rate. Therefore in the case that if it were not the truly risk-free rate, we had to consider about the measurable error on the risk-free rate and this measurable error will go to the HJ distance directly from the error terms, which will lead to different results.

For multiple comparisons, Table 7 shows that all bootstrapping p-values for the rankings are greater than 0.05, which means that we cannot reject the null hypothesis: the best models we picking outperforms others via their relatively smaller normalized pricing errors. Especially, I test several rankings via the unconstrained and the constrained HJ distance measures which are statistically non-significantly different from zeros. I find that the bootstrapping p-values are 0.7130 and 0.6890 when the macro-factor model explains gross returns on 30 Industrial-sorted well via the unconstrained and constrained HJ distance measures. Moreover, the Fama-French five-factor obtains 0.6090 and 0.6230 p-values for its best positions via the unconstrained HJ distance measures when pricing gross and excess returns on 10 Deciles. Furthermore, the macro-factor model gets 0.6260 and 0.5390 p-values when pricing gross and excess returns on stocks-bonds combined portfolios, respectively.

In table 8, I am continuous to showing that the Fama-French five-factor can not statistically significantly be rejected by the null hypothesis at 10%

level, which states it outperforms to CAPM and Fama-French three-factor these two nested models. To compare non-nested and overlapping models at the statistical significance 10% level, the Fama-French five-factor performs well than others when pricing Fama-French 25, 30 Industrial-sorted and 10 Deciles portfolios; the Chen, Roll and Ross five-factor model outperforms others to explain Fama-French 25 and government bonds combined portfolios.

# 6 Extensions

## 6.1 Consumption-Based Linear Factor Asset Pricing Models

Why care about consumption-based models? After all, all above models of risk that are functions of asset prices themselves. This suggests that we might bypass consumption data altogether, and instead look directly at asset returns. By this approach, asset prices are derived endogenously from macroeconomic risk factors, which arise endogenously from the intertemporal marginal rate of substitution over a complicated nonlinear function of current, future and past consumption, and possibly of the cross-sectional distribution of consumption, among other variables. From these specifications, we may derive an equilibrium relation between macroeconomic risk factors and financial returns.

### 6.1.1 Description of the Candidate Models

In this part, I consider the classic CCAPM of Lucas (1978) and Breeden (1979) as a benchmark, where consumption growth is the single factor. Recently this model has been augmented to deal with non-separable preference between non-durable consumption and durable consumption and between non-housing consumption and housing consumption. Yogo (2006) find that the growth rates on non-durable and durable consumptions are able to explain the cross-sectional variation in expected stock returns. Moreover, Piazzesi, Schneider and Tuzel (2007) argue that the composition of the consumption bundle is a new risk factor. They show that under the assumption of constant elasticity of substitution (CES) utility, the composition risk factor can be represented as the growth of the ratio of the non-housing consumption expenditure share, which is the ratio of non-housing consumption to the overall consumption expenditure. Below I also treat Chen, Roll and Ross five-factor model (macro-factor) as the benchmark, in order to compare its performance with these consumption-based models.

**CCAPM:** the consumption CAPM

$$M_{t+1}^{CAPM} = a + \lambda_1 c_{t+1}^{ndur} \tag{19}$$

where $c_{t+1}^{ndur}$ is the growth rate of non-durable consumption.

**Yogo:** the durable consumption CAPM of Yogo (2006)

$$M_{t+1}^{YOGO} = a + \lambda_1 R_{t+1}^{eM} + \lambda_2 c_{t+1}^{ndur} + \lambda_3 c_{t+1}^{dur} \tag{20}$$

where $R_{t+1}^{eM}$ is the excess returns on market portfolios and $c_{t+1}^{dur}$ denotes the consumption growth rate of durable goods.

**PST:** the consumption-housing CAPM of Piazzesi, Schneider and Tuzel (2007)

$$M_{t+1}^{PST} = a + \lambda_1 c_{t+1}^{nh} + \lambda_2 s_{t+1} \tag{21}$$

where $c_{t+1}^{nh}$ is the growth rate of non-housing consumption and $s_{t+1}$ denotes the log non-housing consumption expenditure share.

### 6.1.2 Empirical Results

The data used for this study are quarterly and the full-sample period is 1959:Q1-2000:Q4. Quarterly consumption data are from the U.S. national accounts. Non-durable consumption is measured as the sum of real personal consumption expenditures on non-durable goods and services. Non-durable consumption includes food, clothing and shoes, housing, utilities, transportation, and medical care. Durable consumption consists of items such as motor vehicles, furniture and appliances, and jewelry and watches. To measure housing services, I rely on the National Income and Product Account (NIPA) following Piazzesi, Schneider and Tuzel (2006). I use the NIPA expenditure series on housing services each period. To measure non-housing consumption, I use aggregate consumption of non-durables and services from NIPA excluding shoes and clothing, which is different from Yogo (2006). However, I exclude housing services. All stocks are divided by population.

In Table 9, I find that the macro-factor model almost dominates others offering the smallest normalised pricing errors when pricing returns on Fama-French 25 size-value portfolios; the Yogo non-durable and durable model outperforms others to explain the industrial-sorted portfolios; the Yogo again performs well in obtaining the smaller misspecification errors in order to explain 10 size portfolios. However, all these results are not statistically significantly different from zeros, which imply that all winners have the smallest pricing errors comparing to others when explaining excess and gross returns on assets.

To further analyse my results, I separate candidates as nested models including the traditional consumption CAPM and the Yogo nondurable-durable consumption model; non-nested and overlapping models including the winner of nested models, Piazzesi, Schneider and Tuzel nonhousing-housing consumption and Chen, Roll and Ross five-factor models. To test a joint inequity hypothesis, Table 10 shows that the Chen, Roll and Ross five-factor model is the best model to price excess and groo returns on Fama-French 25 and 30 Industrial-sorted portfolios comparing to other consumption-based linear factor asset pricing models. When explaining returns on 10 Deciles assets, I can only tell that the macro-factor model is better than nonhousing-housing consumption model, and so is the Yogo nondurable-durable consumption model.

## 6.2 Scaled Consumption-Based Linear Factor Asset Pricing Models

Many newer consumption-based theories imply that the pricing kernel is approximately a linear function depending on the state of the economy. In this case we can explicitly model the dependence of parameters in the stochastic discount factor on current period information. This dependence can be specified by simply interacting, or scaling factors with instruments that summarize the state of the economy. Precisely, the fundamental factors (e.g. consumption, housing and so on) that price assets in traditional unscaled consumption-based models are assumed to price assets in this approach. These factors are expected only to conditionally price assets, leading to conditional rather than fixed linear factor models.

As before, if I define the discount factor $M_{t+1}$ can be approximated as a linear function of consumption growth, such as

$$M_{t+1} = a + \lambda \Delta c_{t+1} \tag{22}$$

where $c_{t+1} = log(C_{t+1})$, and $a$ and $\lambda$ are parameters. The standard CCAPM of Lucas (1978) and Breeden (1979) specifies these parameters as constant, with consumption growth as the single factor. Then the conditional versions of the CCAPM can be written to allow the coefficients to vary over time as

$$M_{t+1} = a_t + \lambda_t \Delta c_{t+1} \tag{23}$$

Following Cochrane (1996), these time-varying coefficients can be modeled as linear functions of the conditioning variables $z_t$ in the time t information set:

$$M_{t+1} = (a_0 + a_1 z_t) + (\lambda_0 + \lambda_1 z_t)\Delta c_{t+1} = a_0 + \lambda' f_{t+1} \tag{24}$$

where $\lambda = [a_1, \lambda_0, \lambda_1]'$ and $f_{t+1} = [z_t, \Delta c_{t+1}, \Delta c_{t+1} \cdot z_t]$.

### 6.2.1 Description of the Candidate Models

The candidate models include the conditional consumption CAPM of Lettau and Ludvigson (2001), in which using the consumption-wealth ratio as a conditional variable; the consumption-housing CAPM of Piazzesi, Schneider and Tuzel (2007), in which the non-housing consumption expenditure share is used as a conditioning variable; the collateral-CCAPM of Lustig and Van Nieuwerburgh (2005), in which the housing collateral ratio is used as a conditioning variable; and the conditional CCAPM with the labor income of Santos and Veronesi (2006).

**LL:** the conditional consumption CAPM of Lettau and Ludvigson (2001)

$$M_{t+1}^{LL} = a + \lambda_1 c_{t+1}^{ndur} + \lambda_2 cay_t + \lambda_3 c_{t+1}^{ndur} cay_t \tag{25}$$

where $c_{t+1}^{ndur}$ is the growth rate of non-durable consumption and $cay_{t-1}$ is the consumption-wealth ratio.

**SPST:** the scaled consumption-housing CAPM of Piazzesi, Schneider and Tuzel (2007)

$$M_t^{SPST} = a + \lambda_1 c_{t+1}^{ndur} + \lambda_2 s_t + \lambda_3 c_{t+1}^{ndur} s_t \tag{26}$$

where $s_t$ is the non-housing consumption expenditure share.

**LVN:** the scaled collateral-consumption CAPM of Lustig and Van Nieuwerburgh (2005)

$$M_{t+1}^{LVN} = a + \lambda_1 c_{t+1}^{ndur} + \lambda_2 my_t + \lambda_3 c_{t+1}^{ndur} my_t \tag{27}$$

where $my_t$ is the housing collateral ratio.

**SV:** the scaled consumption CAPM with the labor income of Santos and Veronesi (2006)

$$M_{t+1}^{SV} = a + \lambda_1 R_{t+1}^m + \lambda_2 (R_{t+1}^m \cdot s_t^w) + \lambda_3 R_{t+1}^W + \lambda_4 (R_{t+1}^W \cdot s_t^w) \tag{28}$$

where $R_{t+1}^m$ is the return on non-human, or financial wealth which is proxied by a market portfolio returns, $R_{t+1}^W$, is proxied by labor income growth, $s_t^w$ denotes the ratio of labor income to consumption as a conditioning variable.

### 6.2.2 Empirical Results

The data used for this study are quarterly and the full-sample period is 1959:Q1-2000:Q4. To measure the consumption-wealth ratio, $cay_t$ is defined as a cointegrating residual between log consumption, log asset wealth, and log labor income measured in 1992 dollars. The consumption data pertain to nondurables and services excluding shoes and clothing in 1992 chain-weighted dollars. The asset wealth data are the household net worth series provided by the Board of Governors of the Federal Reserve System. Labor income is defined as wages and salaries plus transfer payments plus other labor income minus personal contributions for social insurance minus taxes. Taxes are defined as [wages and salaries/(wages and salaries + proprietors income with IVA and Ccadj + rental income + personal dividends + personal interest income)]*(personal tax and nontax payments), where IVA is inventory evaluation and Ccadj is capital consumption adjustments. Both the net worth variable and the labor income variable are deflated by the personal consumption expenditure chain-type price deflator. All variables are given in per capita terms. To measure the housing collateral ratio $my_t$, I follow Lustig and Van Nieuwerburgh (2005) defining it as the ratio of collateralizable housing wealth to non-collateralizable human wealth, where the nonstationary component of human wealth is well approximated by the nonstationary component of labor income. The measure of the housing collateral stock is defined as the value of outstanding home mortgages from Bureau of the Census. To get the real per household housing collateral, I construct them using the item CPI from the BLS, and the total number of households from

Bureau of the Census. Aggregate income is labor income plus net transfer income.

From Table 11, the Santos and Veronesi scaled CCAPM with labor income donimates other in explain excess and gross returns both on Fama-French 25 and 30 Industrial-sorted assets. The winner in 10 Deciles portfolio include the scaled CCAPM with labor income when explaining gross returns; the Lustig and Van Nieuwerburgh collateral-CCAPM when pricing excess returns. The Lettau and Ludvigson consumption-wealth-CCAPM is able to outperform others via the constrained HJ distance pricing excess and gross returns on assets. These results are not statistically significant, which means that every winner is specified.

Because these models non-nested or overlapping, I result that Santos and Veronesi scaled CCAPM with labor income is the best model when pricing excess and gross returns on Fama-French 25 and 30 Industrial-sorted assets.

# 7 Conclusions

Multifactor linear asset pricing models play an important role in evaluating portfolios performances and the cost-of-capital applications for practitioners. I use various HJ distance measures to better understand which factor models are robust to explain different cross-section data, and to seek an economic interpretation of the specifications that appears most promising.

I find that, pricing gross returns on all test assets, the Fama-French five-factor model is ranked top by the HJ and the modified HJ distance measures to explain Fama-French 25 size-value, 30 Industrial-sorted and 10 Deciles portfolios. The Chen, Roll and Ross five-factor model performs better than others to price Fama-French 25 plus 7 government bonds portfolios via these two distance measures. Explaining excess returns on all test portfolios, the Fama-French five-factor model is able to explain Fama-French 25 size-value, 10 Deciles portfolios and 30 Industrial-sorted via the unconstrained and the constrained HJ distance measures except that it prices stocks and bonds combined portfolios according to the constrained HJ distance. The Chen, Roll and Ross five-factor model outperforms others in Fama-French 25 plus 7 government bonds and gross returns on that according to the constrained HJ distance.

Moreover, I result that when pricing Fama-French size-value and 30 Industrial-sorted assets: Chen, Roll and Ross macro-factor (1986) outperforms other consumption-based models, i.e. Yogo (2006) and Piazzesi, Schneider and Tuzel (2007); the Santos and Veronesi scaled CCAPM with labor income (2006) performs better than other scaled consumption-based models, i.e. Lettau and Ludvigson (2001), Lustig and Van Nieuwerburgh (2005) and Piazzesi, Schneider and Tuzel (2007). The second contribution states as I show all these results are statistically significantly robust among gross and excess returns via various distance measures.

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Table 1: Summary Statistics of Factors

| | Rmrf | SMB | HML | TERM | DEF | MP | UI | DEI | UTS | UPR | ROA | IA |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Mean | 0.51 | 0.19 | 0.50 | 0.019 | 0.01 | 0.00 | 0.00 | 0 | 0.01 | 0.01 | 0.98 | 0.40 |
| Median | 0.86 | 0.07 | 0.46 | 0.02 | 0.01 | 0.00 | 0.00 | 0 | 0.01 | 0.01 | 1.01 | 0.39 |
| S.D. | 4.48 | 3.26 | 3.05 | 0.02 | 0.00 | 0.01 | 0.00 | 0.00 | 0.01 | 0.00 | 4.05 | 1.89 |

| | Rmrf | SMB | HML | TERM | DEF | MP | UI | DEI | UTS | UPR | ROA | IA |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Rmrf | 1 | 0.26 | -0.45 | 0.09 | 0.05 | -0.08 | -0.10 | -0.03 | 0.11 | 0.10 | -0.22 | -0.33 |
| SMB | 0.26 | 1 | -0.29 | 0.10 | -0.08 | -0.04 | -0.01 | 0.01 | 0.11 | 0.09 | -0.40 | -0.03 |
| HML | -0.45 | -0.29 | 1 | -0.01 | 0.06 | 0.04 | 0.06 | -0.01 | 0.02 | -0.00 | 0.23 | 0.45 |
| TERM | 0.09 | 0.10 | -0.01 | 1 | -0.09 | 0.13 | 0.10 | 0.02 | 0.94 | 0.03 | -0.04 | -0.03 |
| DEF | 0.05 | -0.08 | 0.06 | -0.09 | 1 | -0.21 | -0.09 | -0.17 | 0.05 | 0.09 | 0.12 | 0.08 |
| MP | -0.08 | -0.04 | 0.04 | 0.13 | -0.21 | 1 | 0.05 | 0.15 | 0.08 | -0.24 | 0.01 | 0.03 |
| UI | -0.10 | -0.01 | 0.06 | 0.10 | -0.09 | 0.05 | 1 | 0.66 | 0.09 | -0.08 | -0.08 | 0.09 |
| DEI | -0.03 | 0.01 | -0.01 | 0.02 | -0.17 | 0.15 | 0.66 | 1 | -0.03 | -0.13 | -0.00 | -0.01 |
| UTS | 0.11 | 0.11 | 0.02 | 0.94 | 0.05 | 0.08 | 0.09 | -0.03 | 1 | -0.05 | -0.06 | -0.02 |
| UPR | 0.10 | 0.09 | -0.00 | 0.03 | 0.09 | -0.24 | -0.08 | -0.13 | -0.05 | 1 | -0.01 | 0.05 |
| ROA | -0.22 | -0.40 | 0.23 | -0.04 | 0.12 | 0.01 | -0.08 | -0.00 | -0.06 | -0.01 | 1 | 0.07 |
| IA | -0.33 | -0.03 | 0.45 | -0.03 | 0.08 | 0.03 | 0.09 | -0.01 | -0.02 | 0.05 | 0.07 | 1 |

Table 2: Cross Sectional Adjusted $R^2$

| | OLS | OLS with con | GLS | GLS with con |
|---|---|---|---|---|
| | | FF25 | | |
| CAPM | -1.7322 | 0.3073 | -0.9629 | 0.0696 |
| FF3 | 0.5037 | 0.7387 | -0.558 | 0.1355 |
| FF5 | **0.8066** | **0.9012** | **0.1403** | **0.4142** |
| CRR5 | 0.073 | 0.747 | -0.4346 | 0.3188 |
| CNZ3 | 0.6671 | 0.7031 | -0.2873 | 0.1114 |
| | | IND30 | | |
| CAPM | -2.6253 | 0.0099 | -0.6499 | 0.0106 |
| FF3 | -0.6059 | 0.0759 | -0.2282 | 0.0547 |
| FF5 | **0.0622** | **0.3284** | **0.0349** | 0.0901 |
| CRR5 | -8.088 | 0.2292 | -1.9232 | **0.1334** |
| CNZ3 | -1.4594 | 0.0534 | -0.6035 | 0.0971 |
| | | DECILES10 | | |
| CAPM | 0.6234 | 0.6219 | 0.0311 | 0.0647 |
| FF3 | 0.67 | **0.8652** | 0.1993 | 0.3318 |
| FF5 | **0.8188** | 0.8197 | **0.4277** | **0.4323** |
| CRR5 | 0.0484 | 0.722 | -1.6685 | 0.3594 |
| CNZ3 | 0.8085 | 0.8127 | 0.05 | 0.4318 |
| | | FF25+7G | | |
| CAPM | 0.5592 | 0.5913 | -4.3873 | 0.0899 |
| FF3 | 0.9215 | 0.922 | -4.5293 | 0.2686 |
| FF5 | **0.9701** | **0.9702** | -2.6556 | 0.3652 |
| CRR5 | 0.8211 | 0.8331 | **-0.7007** | 0.0929 |
| CNZ3 | 0.9474 | 0.9473 | -4.1913 | **0.3787** |

Table 3: HJ and Modified HJ Distance on Gross Returns

| | Fama-French 25 Portfolios | | 30 Industrial Sorted Portfolios | |
|---|---|---|---|---|
| | HJ | Modif. HJ | HJ | Modif. HJ |
| **CAPM** | 0.1956 | 0.2078 | 0.2201 | 0.2231 |
| pv(HJ) | 0 | 0 | 0 | 0 |
| **FF3** | 0.1632 | 0.168 | 0.1877 | 0.1886 |
| pv(HJ) | 0 | 0 | 0 | 0 |
| **FF5** | **0.1298** | **0.1298** | **0.1637** | **0.1637** |
| pv(HJ) | 0 | 0 | 0 | 0 |
| **CRR5** | 0.1837 | 0.1837 | 0.1646 | 0.1647 |
| pv(HJ) | 0 | 0 | 0 | 0 |
| **CNZ3** | 0.1538 | 0.1549 | 0.2117 | 0.2137 |
| pv(HJ) | 0 | 0 | 0 | 0 |

| | 10 Deciles Portfolios | | Fama-French 25 plus 7 Gov. Bonds | |
|---|---|---|---|---|
| | HJ | Modif. HJ | HJ | Modif. HJ |
| **CAPM** | 0.1156 | 0.1157 | 164.26 | 246.19 |
| pv(HJ) | 0 | 0 | 0 | 0 |
| **FF3** | 0.102 | 0.1021 | 164.14 | 235.93 |
| pv(HJ) | 0 | 0 | 0 | 0 |
| **FF5** | **0.0758** | **0.0758** | 107.48 | 109.03 |
| pv(HJ) | 0 | 0 | 0 | 0 |
| **CRR5** | 0.1342 | 0.1342 | **60.07** | **60.07** |
| pv(HJ) | 0 | 0 | 0 | 0 |
| **CNZ3** | 0.1133 | 0.1134 | 161.75 | 203.67 |
| pv(HJ) | 0 | 0 | 0 | 0 |

Table 4: HJ and Modified HJ Distance on Excess Returns

| | Fama-French 25 Portfolios | | 30 Industrial Sorted Portfolios | |
|---|---|---|---|---|
| | HJ | Modif. HJ | HJ | Modif. HJ |
| **CAPM** | 0.4493 | 0.503 | 0.2225 | 0.2283 |
| pv(HJ) | 0 | 0 | 0 | 0 |
| **FF3** | 0.3996 | 0.4359 | 0.202 | 0.2063 |
| pv(HJ) | 0 | 0 | 0 | 0 |
| **FF5** | **0.288** | **0.3007** | **0.191** | **0.1946** |
| pv(HJ) | 0 | 0 | 0 | 0 |
| **CRR5** | 0.3234 | 0.3417 | 0.2502 | 0.2585 |
| pv(HJ) | 0 | 0 | 0 | 0 |
| **CNZ3** | 0.3758 | 0.4055 | 0.2141 | 0.2192 |
| pv(HJ) | 0 | 0 | 0 | 0 |

| | 10 Deciles Portfolios | | Fama-French 25 plus 7 Gov. Bonds | |
|---|---|---|---|---|
| | HJ | Modif. HJ | HJ | Modif. HJ |
| **CAPM** | 0.132 | 0.1332 | 0.8254 | 1.4619 |
| pv(HJ) | 0 | 0 | 0 | 0 |
| **FF3** | 0.1185 | 0.1194 | 0.8209 | 1.4375 |
| pv(HJ) | 0 | 0 | 0 | 0 |
| **FF5** | 0.1119 | 0.1126 | 0.7469 | 1.1232 |
| pv(HJ) | 0 | 0 | 0 | 0 |
| **CRR5** | 0.1169 | 0.1177 | **0.6018** | **0.7535** |
| pv(HJ) | 0 | 0 | 0 | 0 |
| **CNZ3** | **0.1099** | **0.1106** | 0.8129 | 1.396 |
| pv(HJ) | 0 | 0 | 0 | 0 |

Table 5: Unconstrained and Constrained HJ Distance on Gross Returns

| | Fama-French 25 Portfolios | | 30 Industrial Sorted Portfolios | |
|---|---|---|---|---|
| | Unconstr. HJ | Constr. HJ | Unconstr. HJ | Constr. HJ |
| **CAPM** | 0.15 | 0.1618 | 0.176 | 0.1763 |
| pv(HJ) | 0 | 0 | 0.05 | 0.09 |
| per($m < 0$) | 4.69 | 2.35 | 0.23 | 0.23 |
| **FF3** | 0.1356 | 0.1507 | 0.169 | 0.17 |
| pv(HJ) | 0 | 0 | 0 | 0 |
| per($m < 0$) | 6.81 | 3.05 | 1.17 | 0.23 |
| **FF5** | **0.1291** | **0.1434** | 0.1635 | 0.1659 |
| pv(HJ) | 0 | 0 | 0 | 0 |
| per($m < 0$) | 11.74 | 3.05 | 1.88 | 0.47 |
| **CRR5** | 0.1829 | 0.1981 | **0.1627** | **0.1652** |
| pv(HJ) | 0 | 0 | 0.3 | 0.27 |
| per($m < 0$) | 16.67 | 0 | 0 | 0 |
| **CNZ3** | 0.1443 | 0.1607 | 0.2149 | 0.224 |
| pv(HJ) | 0 | 0 | 0 | 0 |
| per($m < 0$) | 10.8 | 2.35 | 6.81 | 1.17 |

| | 10 Deciles Portfolios | | Fama-French 25 plus 7 Gov. Bonds | |
|---|---|---|---|---|
| | Unconstr. HJ | Constr. HJ | Unconstr. HJ | Constr. HJ |
| **CAPM** | 0.1084 | 0.1108 | 101.6269 | 106.495 |
| pv(HJ) | 0 | 0.02 | 0 | 0 |
| per($m < 0$) | 2.82 | 0.47 | 0 | 0 |
| **FF3** | 0.0951 | 0.0971 | 89.8155 | 95.295 |
| pv(HJ) | 0 | 0.02 | 0 | 0 |
| per($m < 0$) | 3.29 | 0.23 | 0.47 | 0.47 |
| **FF5** | **0.0699** | **0.0919** | 66.0429 | **84.4054** |
| pv(HJ) | 0.11 | 0 | 0.27 | 0.02 |
| per($m < 0$) | 41.31 | 0.94 | 27.46 | 5.16 |
| **CRR5** | 0.1321 | 0.1322 | **60.0283** | 90.049 |
| pv(HJ) | 0.04 | 0.03 | 0.28 | 0 |
| per($m < 0$) | 1.41 | 0 | 36.85 | 9.62 |
| **CNZ3** | 0.1067 | 0.1081 | 84.4238 | 92.1508 |
| pv(HJ) | 0 | 0.02 | 0 | 0 |
| per($m < 0$) | 1.88 | 0.7 | 6.81 | 2.58 |

Table 6: Unconstrained and Constrained HJ Distance on Excess Returns

| | Fama-French 25 Portfolios | | 30 Industrial Sorted Portfolios | |
|---|---|---|---|---|
| | Unconstr. HJ | Constr. HJ | Unconstr. HJ | Constr. HJ |
| **CAPM** | 0.1711 | 0.1915 | 0.2172 | 0.2249 |
| pv(HJ) | 0 | 0 | 0 | 0 |
| per($m < 0$) | 7.28 | 2.82 | 5.16 | 0.47 |
| **FF3** | 0.1522 | 0.1782 | 0.1922 | 0.2118 |
| pv(HJ) | 0 | 0 | 0 | 0 |
| per($m < 0$) | 11.74 | 3.76 | 31.92 | 1.41 |
| **FF5** | **0.1424** | **0.1708** | **0.1837** | **0.2053** |
| pv(HJ) | 0 | 0 | 0.04 | 0.02 |
| per($m < 0$) | 28.17 | 4.23 | 35.92 | 0 |
| **CRR5** | 0.2325 | 0.2515 | 0.2297 | 0.2359 |
| pv(HJ) | 0 | 0 | 0 | 0.06 |
| per($m < 0$) | 17.84 | 0 | 0.47 | 0 |
| **CNZ3** | 0.1591 | 0.1897 | 0.2149 | 0.224 |
| pv(HJ) | 0 | 0 | 0 | 0 |
| per($m < 0$) | 21.83 | 2.82 | 6.81 | 1.17 |

| | 10 Deciles Portfolios | | Fama-French 25 plus 7 Gov. Bonds | |
|---|---|---|---|---|
| | Unconstr. HJ | Constr. HJ | Unconstr. HJ | Constr. HJ |
| **CAPM** | 0.1161 | 0.1369 | 88.68 | 91.1745 |
| pv(HJ) | 0 | 0 | 0 | 0 |
| per($m < 0$) | 52.35 | 1.41 | 0 | 0 |
| **FF3** | 0.1033 | 0.1208 | 79.2368 | 82.0292 |
| pv(HJ) | 0 | 0 | 0 | 0 |
| per($m < 0$) | 45.31 | 0.94 | 0.47 | 0.47 |
| **FF5** | **0.0628** | **0.1135** | 56.305 | 69.6377 |
| pv(HJ) | 0.26 | 0 | 0.39 | 0.05 |
| per($m < 0$) | 52.35 | 2.58 | 23.47 | 5.16 |
| **CRR5** | 0.134 | 0.1896 | **47.3304** | **69.4786** |
| pv(HJ) | 0.81 | 0 | 0.5 | 0 |
| per($m < 0$) | 42.25 | 0.94 | 31.92 | 9.39 |
| **CNZ3** | 0.1141 | 0.1348 | 75.0134 | 79.169 |
| pv(HJ) | 0 | 0 | 0 | 0 |
| per($m < 0$) | 63.85 | 1.17 | 3.29 | 1.41 |

Table 7: Testing for Multiple Return-Based AP Comparisons

Fama-French 25 Portfolios

| | HJ | Modified HJ | Unconstr. HJ | Constr. HJ |
|---|---|---|---|---|
| Gross Returns | $FF5^*$ | $FF5^*$ | $FF5^*$ | $FF5^*$ |
| Excess Returns | $FF5^*$ | $FF5^*$ | $FF5^*$ | $FF5^*$ |

30 Industrial Sorted Portfolios

| | HJ | Modified HJ | Unconstr. HJ | Constr. HJ |
|---|---|---|---|---|
| Gross Returns | $FF5^*$ | $FF5^*$ | $CRR5^*$ | $CRR5^*$ |
| Excess Returns | $FF5^*$ | $FF5^*$ | $FF5^*$ | $FF5^*$ |

10 Deciles Portfolios

| | HJ | Modified HJ | Unconstr. HJ | Constr. HJ |
|---|---|---|---|---|
| Gross Returns | $FF5^*$ | $FF5^*$ | $FF5^*$ | $FF5^*$ |
| Excess Returns | $CNZ3^*$ | $CNZ3^*$ | $FF5^*$ | $FF5^*$ |

Fama-French 25 plus 7 Gov. Bonds

| | HJ | Modified HJ | Unconstr. HJ | Constr. HJ |
|---|---|---|---|---|
| Gross Returns | $CRR5^*$ | $CRR5^*$ | $CRR5^*$ | $FF5^*$ |
| Excess Returns | $CRR5^*$ | $CRR5^*$ | $CRR5^*$ | $CRR5^*$ |

* denotes statistical significance at 5% level

Table 8: Testing for Multiple Return-Based AP Comparisons via Chi-Squared

| | **Nested** | **Non-nested&Overlapping** |
|---|---|---|
| | Fama-French 25 Portfolios | |
| Gross Returns | $FF5^{**}$ | $FF5^{**}$ |
| Excess Returns | $FF5^{**}$ | $FF5^{**}$ |
| | 30 Industrial Sorted Portfolios | |
| Gross Returns | $FF5^{**}$ | $FF5^{**}$ |
| Excess Returns | $FF5^{**}$ | $FF5^{**}$ |
| | 10 Deciles Portfolios | |
| Gross Returns | $FF5^{**}$ | $FF5^{**}$ |
| Excess Returns | $FF5^{**}$ | $FF5^{**}$ |
| | Fama-French 25 plus 7 Gov. Bonds | |
| Gross Returns | $FF5^{**}$ | $CRR5^{**}$ |
| Excess Returns | $FF5^{**}$ | $CRR5^{**}$ |

** denotes statistical significance at 10% level

Table 9: Testing for Multiple Consumption-Based AP Comparisons

Fama-French 25 Portfolios

| | HJ | Modified HJ | Unconstr. HJ | Constr. HJ |
|---|---|---|---|---|
| Gross Returns | CRR5 | CRR5 | CRR5 | CRR5 |
| Excess Returns | Yogo | Yogo | CRR5 | CRR5 |

30 Industrial Sorted Portfolios

| | HJ | Modified HJ | Unconstr. HJ | Constr. HJ |
|---|---|---|---|---|
| Gross Returns | $Yogo^{*}$ | Yogo | Yogo | Yogo |
| Excess Returns | Yogo | Yogo | Yogo | Yogo |

10 Deciles Portfolios

| | HJ | Modified HJ | Unconstr. HJ | Constr. HJ |
|---|---|---|---|---|
| Gross Returns | Piazzesi | Piazzesi | Yogo | Yogo |
| Excess Returns | Yogo | Yogo | Yogo | Yogo |

Fama-French 25 plus 7 Gov. Bonds

| | HJ | Modified HJ | Unconstr. HJ | Constr. HJ |
|---|---|---|---|---|
| Gross Returns | | | | |
| Excess Returns | | | | |

* denotes statistical significance at 5% level

Table 10: Testing for Multiple Consumption-Based AP Comparisons via Chi-Squared

| | **Nested** | **Non-nested&Overlapping** |
|---|---|---|
| | Fama-French 25 Portfolios | |
| Gross Returns | $Yogo^{**}$ | $CRR5^{**}$ |
| Excess Returns | $Yogo^{**}$ | $CRR5^{**}$ |
| | 30 Industrial Sorted Portfolios | |
| Gross Returns | $Yogo^{**}$ | $CRR5^{**}$ |
| Excess Returns | $Yogo^{**}$ | $CRR5^{**}$ |
| | 10 Deciles Portfolios | |
| Gross Returns | $Yogo^{**}$ | $CRR5\&Yogo$ |
| Excess Returns | $Yogo^{**}$ | $CRR5\&Yogo$ |
| | Fama-French 25 plus 7 Gov. Bonds | |
| Gross Returns | | |
| Excess Returns | | |

** denotes statistical significance at 10% level

Table 11: Testing for Multiple Scaled Consumption-Based AP Comparisons

Fama-French 25 Portfolios

| | HJ | Modified HJ | Unconstr. HJ | Constr. HJ |
|---|---|---|---|---|
| Gross Returns | SV | SV | SV | SV |
| Excess Returns | SV | SV | SV | SV |

30 Industrial Sorted Portfolios

| | HJ | Modified HJ | Unconstr. HJ | Constr. HJ |
|---|---|---|---|---|
| Gross Returns | SV | SV | SV | SV |
| Excess Returns | SV | SV | SV | SV |

10 Deciles Portfolios

| | HJ | Modified HJ | Unconstr. HJ | Constr. HJ |
|---|---|---|---|---|
| Gross Returns | SV | SV | LVN | LL |
| Excess Returns | LVN | LVN | LVN | LL |

Fama-French 25 plus 7 Gov. Bonds

| | HJ | Modified HJ | Unconstr. HJ | Constr. HJ |
|---|---|---|---|---|
| Gross Returns | | | | |
| Excess Returns | | | | |

* denotes statistical significance at 5% level

Table 12: Testing for Multiple Scaled Consumption-Based AP Comparisons via Chi-Squared

| | **Nested** | **Non-nested&Overlapping** |
|---|---|---|
| | Fama-French 25 Portfolios | |
| Gross Returns | | $SV^{**}$ |
| Excess Returns | | $SV^{**}$ |
| | 30 Industrial Sorted Portfolios | |
| Gross Returns | | $SV^{**}$ |
| Excess Returns | | $SV^{**}$ |
| | 10 Deciles Portfolios | |
| Gross Returns | | |
| Excess Returns | | |
| | Fama-French 25 plus 7 Gov. Bonds | |
| Gross Returns | | |
| Excess Returns | | |

** denotes statistical significance at 10% level

Distance/HJ Distance/HJandConstr.pdf

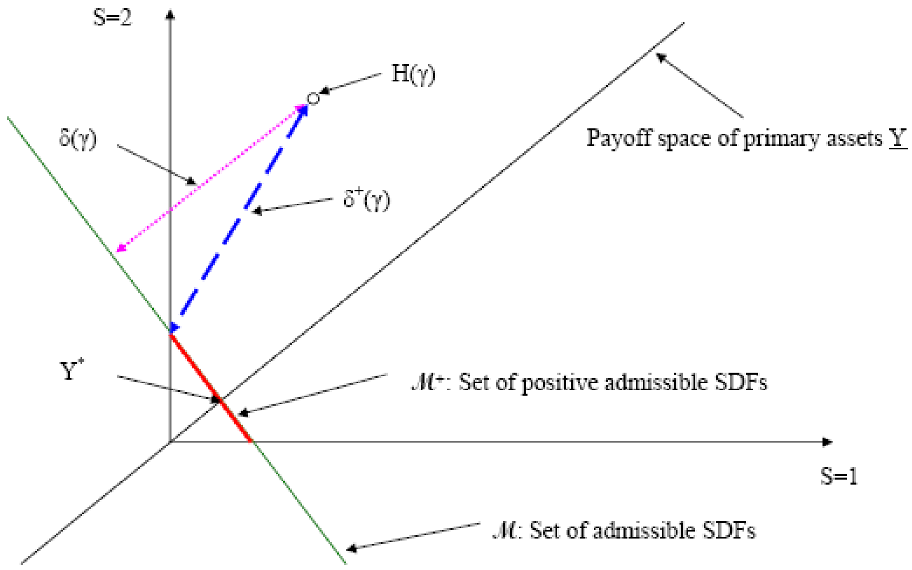


Figure 1: HJ Distance