Dynamics of the Implied Volatility Surface. Theory and Empirical Evidence

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Abstract

I perform a regression analysis to test two of the most famous heuristic rules existing in the literature about the behavior of the implied volatility surface. These rules are the *sticky delta* rule and the *sticky strike* rule. I present a new specification to test the sticky strike rule, which allows for dynamics in the implied volatility surface. In the empirical application I use monthly implied volatility surfaces corresponding to the IBEX 35 index. The estimation results show that the extended specification for the sticky strike rule presented in this article represents better the behavior of the implied volatility under this rule. Furthermore, there is not one rule which is the most appropriate at all times to explain the evolution of implied volatility surface. Depending on the market situation a rule may be more appropriate than another one. In particular, when the underlying asset displays trend, the sticky delta rule tends to prevail against the sticky strike rule. Conversely, when the underlying asset moves in range, then the sticky strike rule tends to predominate.

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1 Introduction

Options prices are usually quoted using implied volatilities obtained from the Black-Scholes (1973) option pricing formula. Let C^*_{KT-t} denote the market price of a European call with strike price K and time to maturity $T-t$, on an asset whose time t price is given by S_t. The Black-Scholes (1973) implied volatility Σ, is defined by:

$$C^*_{KT-t} = C^{BS}_{KT-t}(\Sigma)$$

where C^{BS}_{KT-t} is the option price obtained using the Black-Scholes (1973) formula. The implied volatility expressed as a function of the strike and the time to maturity is known as the time t implied volatility surface: $\Sigma_t(K, T-t)$. In absence of arbitrage opportunities, the put-call parity implies that the implied volatility of a European call coincides with the implied volatility of a European put with the same strike and time to maturity. Market implied volatilities are usually obtained using relatively out-of-the-money calls and puts. The reason is that these options display higher liquidity.

The assumptions of the Black-Scholes (1973) model imply:

$$\frac{\partial \Sigma_t}{\partial T} = \frac{\partial \Sigma_t}{\partial K} = 0 \tag{1}$$

$$\frac{\partial \Sigma_t}{\partial t} = \frac{\partial \Sigma_t}{\partial S_t} = 0 \tag{2}$$

Equation (1) means that the implied volatility surface should be flat, whereas equation (2) implies that this surface should be static. But since the stock market crash on October 1987, equity options markets have been characterized by a persistent negative dependence of implied volatility with respect to the strike price. This negative dependence is known as the implied *volatility skew* and it has been widely documented in the literature by Heynen

(1993), Derman and Kani (1994), Dupire (1994), Rubinstein (1994), Dumas, Fleming and Whaley (1997), Das and Sundaram (1999) and Derman (2003) among others. For foreign currencies out-of-the-money puts, as well as out-of-the-money calls usually exhibit higher implied volatilities than at-the-money options. In this case the relationship between the implied volatility and the strike price is known as the *volatility smile.* This feature has been documented by Rebonato (1999), Jex, Henderson and Wang (1999), Derman (2003), Hull (2006) and Daglish, Hull and Suo (2007).

On the other hand, the implied volatility surface displays term structure with different volatilities for options with different maturities. Stein (1989), Franks and Schwartz (1991), Heynen, Kemma and Vorst (1994), Heynen (1995), Avellaneda and Zhu (1997) or Härdle and Schmidt (2000) show evidence of this fact.

Furthermore, the implied volatility surface is not static. It varies stochastically through time generating vega risk. Examples of this feature have been documented, among others, by Franks and Schwartz (1991), Derman (1999), Bakshi, Cao and Chen (2000), Rama and da Fonseca (2001), Rama and da Fonseca (2002) and Daglish, Hull and Suo (2007).

From the previous evidence the following question arises: How does the implied volatility surface evolve through time? The correct answer to this question dictates the appropriate method for pricing and hedging derivatives.

There are a number of heuristic rules that attempt to describe the time evolution of the implied volatility surface. The *volatility-by-strike* or *sticky strike* rule assumes that the implied volatility corresponding to an option with a given maturity and strike price, is independent of the underlying asset price. The *volatility-by-moneyness* or *sticky delta* rule assumes that the implied volatility for a given maturity is a function exclusively of the moneyness, defined as the ratio of the underlying asset price to the strike price. In the literature there are several studies that try to determine which rule represents the evolution of the implied volatility surface best. Derman (1999) uses quoted options with maturity equal to three months on the Standard and Poor's 500 index. His study covers the period September 1997 to October 1998. He finds subperiods where each of the volatility rules appears to explain the data best. Daglish, Hull and Suo (2007) perform a regression analysis using monthly implied volatility surfaces during the period June 1998 to April 2002, corresponding to the same equity index. The difference of this study with respect to Derman's research is that these authors consider maturities ranging from six months to five years. Moreover, their data consists of consensus market implied volatility surfaces generated by market makers in the over-the-counter market. This fact increases the quality of the data. Daglish, Hull and Suo (2007) conclude that the sticky delta rule represents the behavior of the implied volatility surface better than the sticky strike rule[1].

In this study I replicate the results of Daglish, Hull and Suo (2007) using implied volatility surfaces corresponding to the IBEX 35 index. The novel contribution of this work is the use of a more flexible version to test the sticky strike rule. This specification allows for dynamics in the implied volatility and represents better the evolution of the implied volatility surface under this rule. The empirical results show that the explanatory power of this new version of the sticky strike rule is quite similar to the explanatory power of the sticky delta rule. Moreover, the specification for the sticky strike rule

[1]These authors also consider the *square root of time* rule. This rule is related to the extrapolation of implied volatilities for maturities and strike prices for which there is no market. Since the main objective of this article is to study the evolution of the implied volatility surface, I will focus on the sticky strike and the sticky delta rules.

presented in this article, which accounts for dynamics in the implied volatility, can explain the evolution of the volatility surface in the data better than the sticky delta rule for some subperiods.

The rest of the paper proceeds as follows. Section 2 presents the volatility rules. Section 3 analyzes the arbitrage opportunities under both rules. Section 4 explains the data for the empirical application. Section 5 shows the specifications used to test the performance of both heuristic rules, as well as the empirical results. Finally, section 6 offers concluding remarks.

2 The sticky delta and the sticky strike rules

Derman (1999) posits that the implied volatility reflects the market's view of several features. The evolution of the underlying asset: does the asset price move in range or does it exhibit any trend? The realized volatility: it is stable, increasing or decreasing? The risk premium due to illiquidity. Finally, the probability that the market assigns to sharp drops in the underlying asset price. The volatility rules incorporate these elements differently, to generate a pattern of behavior for the implied volatility surface.

2.1 The sticky delta or sticky moneyness rule

The sticky delta rule postulates that the implied volatility of an option with a given maturity depends only on the moneyness $m = K/S$. Mathematically we have:

$$\begin{aligned} \Sigma_t \left(S_{t;} K, T-t\right) &= \psi_t \left(m_{t,} T-t\right) \\ \frac{\partial \psi_t}{\partial m_t} &< 0 \end{aligned} \tag{3}$$

The reason why this rule is known as the sticky delta rule has to do with the fact that the Black-Scholes (1973) delta depends on the strike and the asset price through the moneyness. The intuition behind this rule is that the at-the-money volatility should remain constant when the underlying asset price moves.

Note that, under this rule, if the asset price increases (decreases), then the implied volatility is higher (lower) for all strike prices.

2.2 The sticky strike rule

The sticky strike rule assumes that the implied volatility corresponding to a given strike remains constant when the asset price moves. Therefore, this rule postulates that the implied volatility does not depend on the underlying asset price. But it is important to remark that, under this rule, the implied volatility can be a function of other stochastic variables. Mathematically the sticky strike rule is given by the following expression:

$$\Sigma_t\left(S_{t;}K, T-t\right) = \Sigma_t\left(K, T-t\right) \tag{4}$$
$$\frac{\partial \Sigma_t}{\partial S_t} = 0; \quad \frac{\partial \Sigma_t}{\partial K} < 0$$

Importantly, both rules have very different implications for the correct calculation of the sensitivities of option prices. In particular, the delta Δ of a European option O_t is given by:

$$\Delta \equiv \frac{dO_t}{dS_t} = \frac{\partial O_t}{\partial S_t} + \frac{\partial O_t}{\partial \Sigma_t}\frac{\partial \Sigma_t}{\partial S_t}$$
$$\Delta = \Delta_{BS} + \nu \frac{\partial \Sigma_t}{\partial S_t}$$

where Δ_{BS} and ν are respectively, the delta and the vega of the option in the Black-Scholes (1973) model. Under the sticky strike rule the delta of the option matches the Black-Scholes (1973) delta. But under the sticky moneyness rule, the delta of the option is higher than the Black-Scholes (1973) delta, for options with positive vega. This fact is especially relevant for the correct risk management of options. If we do not calculate the delta consistently with the evolution of the implied volatility surface, we will have an exposure to the price of the underlying asset, although under the assumptions of the model we have a delta-neutral position.

3 Volatility rules and arbitrage opportunities

Notice that the sticky strike rule and the sticky delta rule are heuristic patterns of behavior. Therefore, unlike theoretical models, these rules may not be arbitrage-free. We consider that an arbitrage opportunity exists when it is possible to set up a portfolio of zero value today which is of positive value in the future with positive probability and of negative value in the future with zero probability.

Assume that the underlying asset price does not exhibit jumps. In this case, it is theoretically possible to find arbitrage opportunities under both rules.

For the sticky delta rule, I consider a portfolio of a long European call and a short European put with the same maturity. I assume the following relationship between the strike price of the call K_c and the strike price of the put K_p:

$$K_p < S < K_c$$

Moreover, I assume that the portfolio includes a position on the underlying asset. The quantity of asset held is continuously changed to maintain a delta-neutral position. This procedure is called dynamic hedging. Changing the number of assets held requires the continual purchase and/or sale of the stock. This is called rehedging or rebalancing the portfolio.

If the price of the underlying asset increases, there will be an increase of the implied volatility for all strike prices. Since the call will be more at-the-money than the put, the vega of the call will be higher than the vega of the put and therefore, we will make a profit. On the other hand, if the underlying asset price decreases, then the implied volatility will be lower for all strike prices. Since in this case the vega of the put will be higher than the vega of the call, it will be possible to make a profit again. This example shows that, in absence of jumps in the evolution of the underlying asset, it is possible to set up a delta-hedged portfolio of a short out-of-money put and a long out-of-the-money call, which allows for arbitrage opportunities under the sticky delta rule.

For the sticky strike rule, let $C_{tT}(K)$ denote the time t price of a European call with strike K and maturity T, on the underlying asset with spot price S_t. This option satisfies the Black-Scholes (1973) partial differential equation:

$$rC_{tT}(K) = \Theta + (r-q) S_t \Delta + \frac{1}{2}\Sigma^2 S_t^2 \Gamma \tag{5}$$

where $\Theta = \frac{\partial C_{tT}(K)}{\partial t}$ represents the theta; $\Delta = \frac{\partial C_{tT}(K)}{\partial S_t}$ is the delta; $\Gamma = \frac{\partial^2 C_{tT}(K)}{\partial S_t^2}$ denotes the gamma; r is the continuously compounded risk-free rate; q is the dividend yield and finally, Σ represents the implied volatility which matches the instantaneous constant volatility corresponding to the asset price process under the Black-Scholes (1973) model.

Let assume the following geometric Brownian motion process for the underlying asset price, under the real world probability measure P:

$$\frac{dS_t}{S_t} = \mu_t dt + \sigma_t dW_t^P$$

where μ_t is the drift term, σ_t is the instantaneous volatility, which may be stochastic and W_t^P is a Wiener process under the probability measure P. For simplicity, I assume that the continuously compounded risk-free rate r

and dividend yield q are zero. Let consider a portfolio of a long call and a position on the underlying asset, so that the portfolio maintains a delta-neutral position. Therefore, the changes in the value of the portfolio will be given by the gamma and the theta of the option[2]. Applying Ito's lemma to C_{tT} gives:

$$dC_{tT}\left(K\right) = \Theta_0 dt + \frac{1}{2}\Gamma_0 S_t^2 \sigma_t^2 dt \tag{6}$$

where Θ_0 and Γ_0 indicate that theta and gamma depend on the implied volatility Σ_0. Under the assumption of zero continuously compounded risk-free rate and dividend yield, it is possible to simplify equation (5) to obtain:

$$\Theta = -\frac{1}{2}\Sigma^2 S_t^2 \Gamma$$

Using the previous expression we can rewrite equation (6) as follows[3]:

$$dC_{tT}\left(K\right) = \frac{1}{2}\Gamma_0 S_t^2 \left[\sigma_t^2 - \Sigma_0^2\right] dt \tag{7}$$

Equation (7) shows a fundamental result. Even when we have a delta-neutral portfolio, there will be an additional profit or loss, which depends on the difference between the volatility of revaluation and the realized volatility. Notice that this result is weighted by the gamma of the option. Therefore, the impact of the discrepancy between the realized volatility and the implied volatility will be higher for options with strikes close to the underlying asset price. Importantly, the result of the equation (7) is independent of the drift corresponding to the underlying asset price process. This conclusion, which may seem surprising, is one of the most important consequences of the Black-Scholes (1973) model.

[2]I assume that the implied volatility is constant, so that there is no vega risk.

[3]See Carr (2002) for a similar result when the risk free rate and the dividend yield are different from zero.

From equation (7) it is possible to set up arbitrage strategies under the sticky strike rule. In particular, consider a delta-neutral portfolio Π_t, of positions in the underlying asset, as well as in calls with different strike prices:

$$\Pi_t = C_{tT}(K_1) - \frac{\Gamma_1}{\Gamma_2} C_{tT}(K_2) - \left(\Delta_1 - \frac{\Gamma_1}{\Gamma_2}\Delta_2\right) S_t$$
$$K_1 > K_2; \quad \Sigma_1 < \Sigma_2$$

Applying Ito's lemma to the previous expression gives:

$$d\Pi_t = \frac{1}{2}\Gamma_1 S_t^2 \left[\Sigma_2^2 - \Sigma_1^2\right] dt$$

Therefore, we have a strategy with positive theta and without gamma which leads to a sure profit. Nevertheless, the gamma changes as time passes and the underlying asset moves. Therefore, it would be necessary to rebalance the position in options. Given the bid-ask spreads in option prices, it can be difficult to set up the arbitrage in practice.

4 Data

In the empirical study I use monthly implied volatility surfaces corresponding to the IBEX 35 index. The data base consists of 45 implied volatility surfaces during the period February 2004 to October 2007. On each month we have five maturities, ranging from six months to four years. Moreover, we have seven values of moneyness, ranging from 80% to 120%. Therefore, a total of 35 points on the implied volatility surface are provided each month and the total number of volatilities available is 1575. The data was part of the month-end pricing service operated by Markit Group Limited. This company collects monthly implied volatility data from a large number of market dealers and generates an estimate of the market implied volatility, for each maturity and strike price. Market participants consider implied volatilities provided by Markit, even more accurate than those generated by brokers. Daglish, Hull and Suo (2007) use the same data source in their study corresponding to the Standard and Poor's 500 index.

Figure 1 shows the implied volatility surface corresponding to the last month of the sample. The figure reveals the existence of negative volatility skew, which is most pronounced for near-term options.

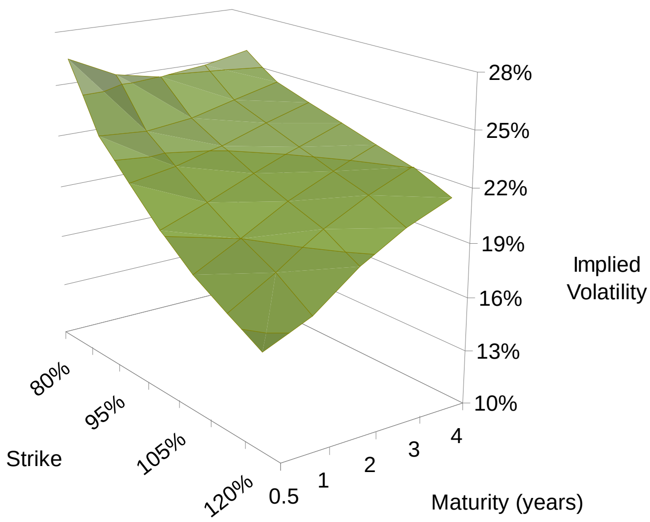

Figure 1: October 2007 implied volatility surface for the IBEX 35 index. Strike prices are expressed as a percentage of the asset price, while time is expressed in years.

5 Tests of the volatility rules

To carry out the tests of the volatility rules, I follow Daglish, Hull and Suo (2007) and consider a second order Taylor series expansion of the volatility function under each rule. Note that this approach is not consistent with the no-arbitrage conditions developed by Lee (2004) for the asymptotic behavior of the implied volatility at extreme strikes. Therefore, this approach is not valid to price options with an arbitrage-free parametric specification for the implied volatility surface. But it is still valid to test which volatility rule explains better the evolution of the implied volatility surface.

5.1 Sticky delta rule specification

I consider the following expression to test the sticky delta rule:

$$\begin{aligned}\Sigma_t\left(S_{t;}K,T-t\right)-\Sigma_t\left(S_{t;}S_t,T-t\right)&=\alpha_0+\alpha_1\ln\left(m_t\right)+\alpha_2\left[\ln\left(m_t\right)\right]^2 \\ &+\alpha_3\left(T-t\right)+\alpha_4\left(T-t\right)^2+\alpha_5\ln\left(m_t\right)\left(T-t\right)+\varepsilon_t\left(K,T-t\right)\end{aligned} \tag{8}$$

where $\Sigma_t\left(S_{t;}S_t,T-t\right)$ is the at-the-money volatility for options with time to maturity $T-t$ and where $m_t=\frac{K}{S_t}$ represents the moneyness. Under this specification we have:

$$E\left[\varepsilon_t\left(K,T-t\right)|\,m^t\right]=0;\qquad m^t=\left(m_{1,}\ldots m_t\right)'$$

The previous equation shows that the error term is mean-independent of the available moneyness observations. The specification of equation (8) is known as the relative sticky delta rule. This version of the sticky moneyness rule, allows the implied volatility to change through time, but when measured relative to the at-the-money volatility, the implied volatility depends only on m_t and $T-t$.

5.2 Sticky strike rule specification

$$\begin{aligned}\Sigma_t\left(K,T-t\right) &= \gamma_0+\gamma_1\ln\left(K\right)+\gamma_2\left[\ln\left(K\right)\right]^2+\gamma_3\left(T-t\right) \\ &\quad+\gamma_4\left(T-t\right)^2+\gamma_5\ln\left(K\right)\left(T-t\right)+\varepsilon_t\left(K,T-t\right)\end{aligned} \tag{9}$$

Under this specification $E\left[\varepsilon_t\left(K,T-t\right)\right]=0$. Notice that in this case, the randomness of the implied volatility is given exclusively by the error term ε_t.

5.3 Extended sticky strike rule specification

Equations (8) and (9) are similar to those used by Daglish, Hull and Suo (2007) to test the rules of behavior corresponding to the implied volatility surface. Notice that the sticky strike rule specification of equation (9) assumes that the implied volatility is independent of the asset price. But it does not take into account that the implied volatility surface may evolve stochastically through time and may depend on other random variables. In this paper, I introduce a more flexible specification to test the sticky strike rule. I denote this version as the extended sticky strike rule and it is given by the following expression:

$$\begin{aligned}\Sigma_t(K, T-t) &= \gamma_0 + \delta_t + \gamma_1 \ln(K) + \gamma_2 [\ln(K)]^2 + \gamma_3 (T-t) \\ &\quad + \gamma_4 (T-t)^2 + \gamma_5 \ln(K)(T-t) + \varepsilon_t(K, T-t)\end{aligned} \tag{10}$$

where δ_t accounts for the effect of some random variables, such as news, which affect the time t implied volatility surface. Under this specification, the error term satisfies the following equation:

$$E\left[\varepsilon_t(K, T-t) \,|\, \delta^t\right] = 0; \qquad \delta^t = (\delta_{1,} \ldots \delta_t)'$$

Notice that the time effects variable δ_t allows for parallel shifts in the implied volatility surface through time, while preserving the term structure, as well as the volatility skew.

There are several possibilities to model the time effects variable. For example, it could be possible to express it as a function of some variables which account for temporal shocks affecting the implied volatility surface. In this case we would have $\delta_t = \delta' x_t$, being δ a vector of parameters and where the aggregate variables vector x_t could include economic indicators, consumer confidence indicators and variables which account for the evolution of other markets such as foreign currencies, credit, interest rates or commodities. Another possibility might be to consider a time series model for the time effects variable. For example, Avellaneda and Zhu (1997) use a time series model to characterize the implied volatility of foreign currency options.

But the fundamental objective of this study is not to model the time effects variable δ_t, but to establish a flexible specification to test the sticky strike rule. When the number of time series observations is small compared to the total number of observations, the realizations of δ_t that occur in the sample can be treated as unknown period-specific parameters to be estimated[4]. To this end I specify a set of time dummies[5] so that:

$$\delta_t = \begin{cases} 0 & \text{for } t = 1 \\ \delta' d_t & \text{for } t = 2, ..., T_N \end{cases}$$

where T_N is the total number of time observations, $\delta = (\delta_2, ..., \delta_{T_N})'$ is a vector of parameters and d_t is a vector which takes value one in the *t-th* position and zero elsewhere. The set of time dummies afford a robust control for common aggregate effects.

5.4 Estimation results

To carry out the estimation of the parameters corresponding to the different versions of the heuristic rules for the behavior of the implied volatility surface, I use the method of ordinary least squares and calculate standard errors robust to heteroskedasticity. Table 1 shows the estimation results corresponding to the sticky delta specification of equation (8), while table 2 displays the estimation results for the sticky strike rule of equation (9).

[4]See Arellano (2003).

[5]Since I include a constant term in the regression equation, the parameters δ_i for $i = 2, ..., T_N$, account for the differential aggregate effect corresponding to observation *i-th* with respect to the first observation.

Table.1: Sticky delta rule

Dependent variable: $\Sigma_t(S_{t;}K, T-t) - \Sigma_t(S_{t;}S_t, T-t)$
Sample period: February 2004 to October 2007
Number of observations: 1575
Degrees of freedom: 1569

Variable	Coefficient	Standard error	*p-value*
constant	0.0051	0.0004	0.000
$\ln(m_t)$	-0.2429	0.0025	0.000
$[\ln(m_t)]^2$	0.1006	0.0069	0.000
$(T-t)$	-0.0045	0.0004	0.000
$(T-t)^2$	0.0008	0.0001	0.000
$\ln(m_t)(T-t)$	0.0338	0.0008	0.000
		Wald test of joint significance:	43452.15 [5]
			(0.000)
		R^2	0.9769
		Corrected R^2	0.9768

Standard errors and test statistics are robust to heteroskedasticity. The Wald test is asymptotically χ^2 with *p-values* in parentheses and degrees of freedom reported in brackets.

Table.2: Sticky strike rule

Dependent variable: $\Sigma_t(K, T-t)$
Sample period: February 2004 to October 2007
Number of observations: 1575
Degrees of freedom: 1569

Variable	Coefficient	Standard error	*p-value*
constant	11.9878	0.7683	0.000
$\ln(K)$	-2.4985	0.1650	0.000
$[\ln(K)]^2$	0.1317	0.0089	0.000
$(T-t)$	-0.0590	0.0168	0.000
$(T-t)^2$	-0.0011	0.0005	0.046
$\ln(K)(T-t)$	0.0078	0.0018	0.000
		Wald test of joint significance:	733.83 [5]
			(0.000)
		R^2	0.2942
		Corrected R^2	0.2915

Standard errors and test statistics are robust to heteroskedasticity. The Wald test is asymptotically χ^2 with *p-values* in parentheses and degrees of freedom reported in brackets.

As shown in table 1 and table 2, both specifications are supported by the data. But the coefficient of determination R^2, indicates that the explanatory power of the sticky delta specification is much higher than the explanatory power corresponding to the simple version of the sticky strike rule of equation (9). The results are similar to those obtained by Daglish, Hull and Suo (2007). In particular, the coefficient of determination for their version of the sticky strike rule is 0.2672, whereas for the sticky delta rule they obtain a coefficient determination equal to 0.9493.

Table 3: Extended sticky strike rule

Dependent variable: $\Sigma_t (K, T-t)$
Sample period: February 2004 to October 2007
Number of observations: 1575
Degrees of freedom: 1525

Variable	Coefficient	Standard error	*p-value*
constant	3.4164	0.3460	0.000
$\ln(K)$	-0.5213	0.0746	0.000
$[\ln(K)]^2$	0.0177	0.0040	0.000
$(T-t)$	-0.0590	0.0054	0.000
$(T-t)^2$	-0.0011	0.0001	0.000
$\ln(K)(T-t)$	0.0078	0.0006	0.000
		Wald test of joint significance:	18108.16 [5] (0.000)
		Wald test of joint significance of time dummies:	11759.32 [44] (0.000)
		R^2	0.9521
		Corrected R^2	0.9505

Standard errors and test statistics are robust to heteroskedasticity. The Wald test is asymptotically χ^2 with *p-values* in parentheses and degrees of freedom reported in brackets.

As seen previously, the sticky strike rule postulates that the implied volatility is independent of the underlying asset price. But the implied volatility may vary stochastically through time and may depend on other random variables. Therefore, the specification of equation (9) may be a bit restrictive to test the explanatory power of this rule.

Equation (10) posits a more flexible specification to characterize the behavior of the implied volatility under the sticky strike rule. Table 3 displays the estimation results corresponding to the extended sticky strike version presented in this article. Comparing the corrected coefficient of determination of tables 1 and 3, we can see that the explanatory power of the extended sticky strike rule is very close to the explanatory power of the sticky delta rule.

The Wald test shows that time dummies are jointly significant. Note that, given the goodness of fit corresponding to the extended sticky strike rule specification, if we were able to forecast the evolution of δ_t, it would be possible to estimate the time evolution of the whole volatility surface quite accurately.

As said previously, the main objective is not to model the time effects variable δ_t but to posit an appropriate specification to characterize the evolution of the implied volatility under the sticky strike rule. Nevertheless, it might be interesting to devote some time to the interpretation of this variable. The time effects variable represents temporal shocks affecting the implied volatility surface in period t. It allows for parallel shifts in the implied volatility surface through time, while preserving the term structure and the volatility skew.

Figure 2 shows the evolution of the estimation corresponding to the time effects variable δ_t. On the other hand, figure 3 displays the time evolution of the at-the-money implied volatility for options with six months to maturity. As it is clear from the figures, both variables have a similar pattern of behavior. This is also true for options with longer-term maturities.

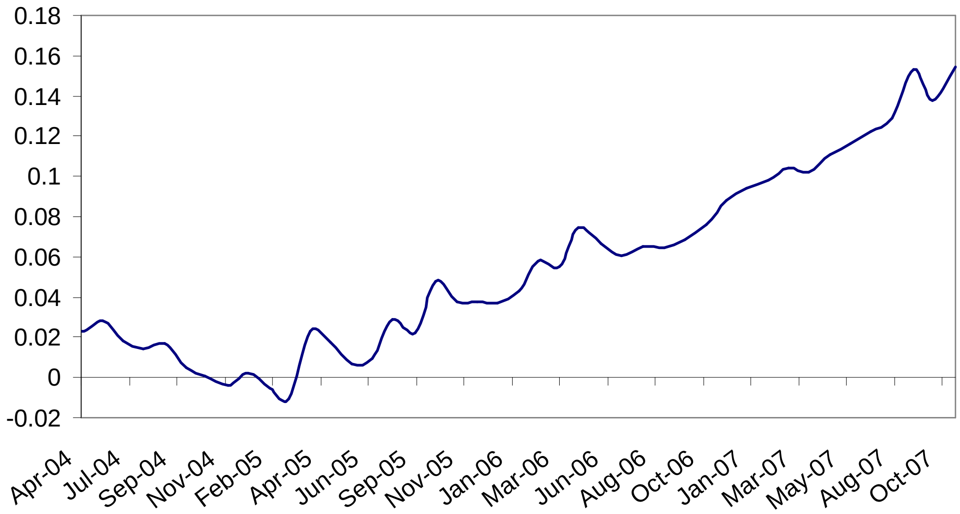

Figure 2: Estimation of the time effects variable δ_t in the equation (10).

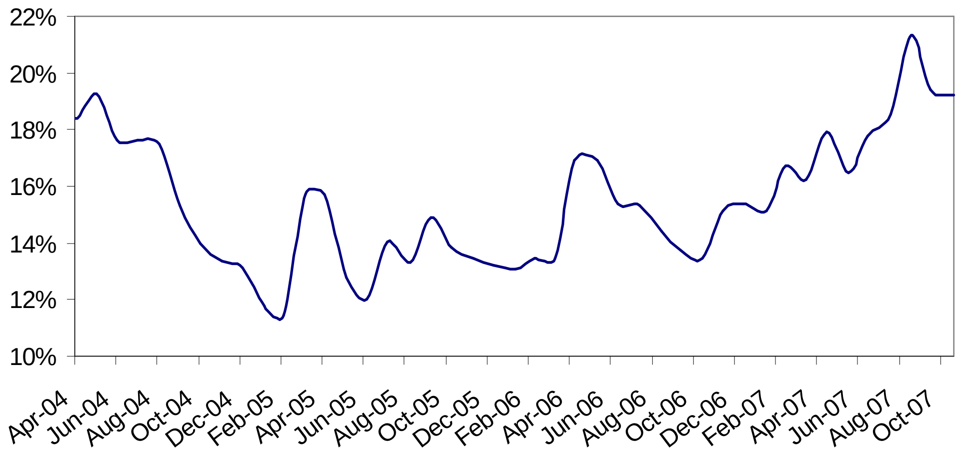

Figure 3: At-the-money implied volatility of options with six months to maturity corresponding to the IBEX 35 index.

If there were no disturbances in the implied volatility, then δ_t would not change whereas the at-the-money implied volatility would move along the length of the skew curve until the new at-the-money strike price. When there are temporal shocks affecting the implied volatility surface, the at-the-money implied volatility displays the effects of these shocks, as well as the movement along the implied volatility skew. Therefore, the difference between both variables would allow distinguishing, under the sticky-strike rule, the shocks effect and the movement along the skew effect in the evolution of the at-the-money implied volatility.

Derman (1999) posits that the correct choice of the volatility rule for each moment should depend on our perception of the market's situation. Suppose that the underlying asset exhibits trend, that is, it experiments a significant change in level while preserving the realized volatility. In this case, in the absence of a change in risk premium or an increased probability of jumps, the realized volatility will be the key element in the estimation of the at-the-money implied volatility. Therefore, as the underlying asset moves it may be appropriate to re-mark the current at-the-money implied volatility to the value of the previous at-the-money volatility, given that the realized volatility has not changed. Notice that, in this case, the implied volatility corresponding to options with a given moneyness remains constant. Therefore, this leads to the sticky delta rule.

On the other hand, if we assume that the underlying asset has not a clear trend, then it does not seem appropriate to increase (decrease) the implied volatility of all strikes when the asset price increases (decreases). Therefore, in this situation the sticky strike rule seems to be more appropriate.

Figure 4 displays the evolution of the IBEX 35 index during the sample period. This index has exhibited a growing trend during most of the period. However the uptrend breaks in 2007. As seen previously, it seems natural that the sticky delta rule prevails when the underlying asset presents trend. Therefore, the higher explanatory power corresponding to this rule can be motivated by the evolution of the index during the analyzed period. I now analyze what rule explains the evolution of the implied volatility surface better during the period December 2006 to October 2007, when the index does not show a clear trend.

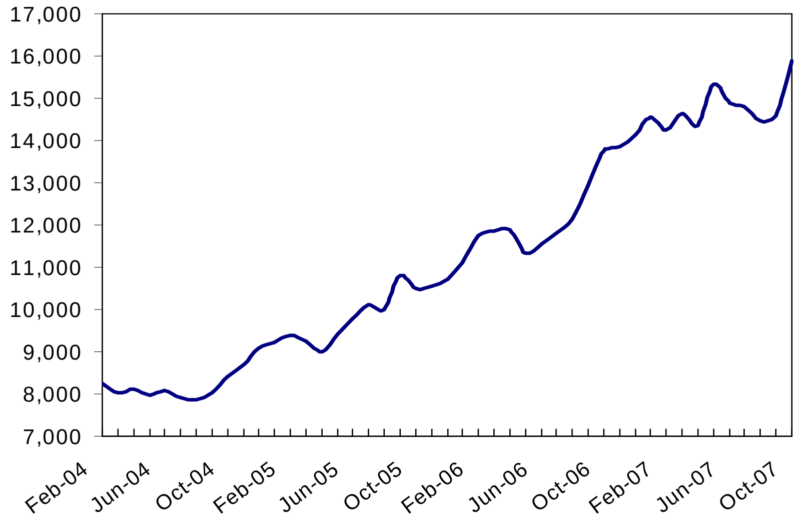

Figure 4: Evolution of the IBEX 35 index during the period February 2004 to October 2007. Data source: Bloomberg.

Table 4 shows the estimation results corresponding to the sticky delta rule, whereas table 5 displays the estimation of the parameter of the extended sticky strike rule. Both specifications can characterize the evolution of the implied volatility quite accurately. Nevertheless, the corrected coefficient of determination corresponding to the extended sticky strike rule is higher than the corrected coefficient of determination of the sticky delta rule. Therefore, when the underlying asset does not display a clear trend, the sticky strike rule seems to represents the evolution of the implied volatility surface of the IBEX 35 index better than the sticky delta rule.

Table 4: Sticky delta rule

Dependent variable: $\Sigma_t(S_{t;}K, T-t) - \Sigma_t(S_{t;}S_t, T-t)$
Sample period: December 2006 to October 2007
Number of observations: 385
Degrees of freedom: 379

Variable	Coefficient	Standard error	*p-value*
constant	0.0044	0.0006	0.000
$\ln(m_t)$	-0.2614	0.0029	0.000
$[\ln(m_t)]^2$	0.0705	0.0109	0.000
$(T-t)$	-0.0041	0.0007	0.000
$(T-t)^2$	0.0007	0.0001	0.000
$\ln(m_t)(T-t)$	0.0423	0.0011	0.000
		Wald test of joint significance:	9606.45 [5] (0.000)
		R^2	0.9766
		Corrected R^2	0.9763

Standard errors and test statistics are robust to heteroskedasticity. The Wald test is asymptotically χ^2 with *p-values* in parentheses and degrees of freedom reported in brackets.

Table 5: Extended sticky strike rule

Dependent variable: $\Sigma_t(K, T-t)$
Sample period: December 2006 to October 2007
Number of observations: 385
Degrees of freedom: 369

Variable	Coefficient	Standard error	*p-value*
constant	7.4680	1.2574	0.000
$\ln(K)$	-1.2662	0.2610	0.000
$[\ln(K)]^2$	0.0525	0.0135	0.000
$(T-t)$	-0.3849	0.0169	0.000
$(T-t)^2$	0.0003	0.0002	0.141
$\ln(K)(T-t)$	0.0408	0.0017	0.000
		Wald test of joint significance:	11996.47 [5] (0.000)
		Wald test of joint significance of time dummies:	8136.18 [10] (0.000)
		R^2	0.9823
		Corrected R^2	0.9816

Standard errors and test statistics are robust to heteroskedasticity. The Wald test is asymptotically χ^2 with *p-values* in parentheses and degrees of freedom reported in brackets.

6 Conclusion

It is well known that the implied volatility surface evolves stochastically through time leading to the existence of vega risk. From the previous fact, the following question arises: How should the implied volatility surface vary as markets move? The appropriate answer dictates the correct method for pricing and hedging derivatives. In this article I perform a regression analysis to test two of the most famous heuristic rules existing in the literature about the behavior of the implied volatility surface. Namely, the sticky delta rule and the sticky strike rule. Since both rules have very different implications in terms of the correct hedge ratios corresponding to option prices, it is very important to determine which rule explains better the evolution of the implied volatility surface. I present a new specification to test the sticky strike rule which allows for dynamics in the implied volatility. This extended version of the sticky strike rule allows the implied volatility surface to vary stochastically through time while preserving the term structure, as well as the volatility skew.

In the empirical study I use monthly implied volatility surfaces corresponding to the IBEX 35 index during the period February 2004 to October 2007, provided by Markit Group Limited. The estimation results show that the extended specification for the sticky strike rule presented in this article represents better the behavior of the implied volatility under this rule. Furthermore, there is not one rule which is the most appropriate at all times to explain the evolution of implied volatility surface. Depending on the market situation a rule may be more appropriate than another one. In particular, when the underlying asset displays trend, the sticky delta rule tends to prevail against the sticky strike rule. Conversely, when the underlying asset moves in range, then the sticky strike tends to predominate.

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