Endogenous timing of incentive contracts in mixed markets under Bertrand competition*

Juan Carlos Bárcena-Ruiz*

Departamento de Fundamentos del Análisis Económico I, Facultad de Ciencias Económicas y Empresariales, Universidad del País Vasco, Avenida Lehendakari Aguirre 83, 48015 Bilbao, Spain.
Phone: (34) 94 601 38 29; Fax: (34) 94 601 38 91; e-mail: juancarlos.barcena@ehu.es

Abstract

This paper analyzes whether owners of firms want to decide incentive contracts for their managers sequential or simultaneously under Bertrand competition. It is shown that in the private duopoly, if one firm is the leader in incentive contracts the other firm prefers to be the follower and thus in equilibrium firms' owners decide incentive contracts sequentially. However, in the mixed duopoly both the private and the public firm want to be the leader in incentive contracts and thus in equilibrium firms take decisions simultaneously.

Keywords: Mixed duopoly, Managerial incentive contracts, Endogenous timing, Bertrand competition
JEL Classification: L13, L32, D21

* Financial support from Departamento de Educación, Universidades e Investigación del Gobierno Vasco (IT-223-07) and Ministerio de Ciencia y Tecnología and FEDER (ECO2009-07939) is gratefully acknowledged.

1 Introduction

There are numerous papers that analyze markets in which private and public firms compete (see, for example, De Fraja and Delbono, 1989, 1990; Wilner, 2001; Bárcena-Ruiz and Garzón, 2005). However, the literature that analyzes these markets has dedicated little attention at investigating whether firms' owners want to decide incentive contracts for managers sequential or simultaneously. Thus, this paper analyzes this issue assuming Bertrand competition with heterogeneous goods and a mixed duopoly. This analysis is highly relevant since endogenous timing of decisions is an important issue to be analyzed since a sequential order of moves may give rise to significantly different results from those obtained in a simultaneous game.

The literature on industrial organization has analyzed whether the owner of firms take price or quantity decisions sequential or simultaneously when firms are privately-owned (see, for example, Gal-Or, 1985; Dowrick, 1986; Hamilton and Slutsky, 1990; Matsumura, 1999). This literature shows that in a private duopoly, if both firms have upward-sloping reaction functions and one firm prefers to be the leader, the other firm must prefer to be the follower; as a result, firms take decisions sequentially. If both firms have downward-sloping reaction functions, each firm prefers being the leader to being a follower, implying that firms take decisions simultaneously.

The above analysis has been extended to consider that the owners of the firms hire managers to who delegate price or quantity decisions (see, for example, Vickers, 1985; Fershtman and Judd, 1987; Sklivas, 1987). This literature analyzes the strategic value for shareholders of publicly observed and irreversible incentive contracts based on sale revenues and profits (not only profits).[1] In this framework, Lambertini (2000) analyzes whether firms' owners want that managers decide quantities (or prices) sequential or simultaneously. He shows that under Cournot competition quantities are chosen simultaneously and that under

[1] It is shown that under Cournot competition owners encourage their managers to produce beyond the profit maximization level. Under Bertrand competition with heterogeneous products, owners will encourage their managers to raise their prices above the prices set by profit maximizer firms.

Bertrand competition prices are chosen sequentially. Bárcena-Ruiz and Espinosa (1996) study, considering a duopoly model and a two production game, whether firms want to choose short-term contracts or a long-term contract for their managers. In the first case incentive parameters are chosen in each of the two periods; in the second case, incentive parameters for the two periods are chosen at the beginning of the first period. They show that under Cournot competition the owners of the firms choose long-term incentive contracts and thus incentive parameters for the two periods are chosen simultaneously. Under Bertrand competition one owner chooses a long-term contract and the other short-term contracts, which means that owners choose incentive parameters for the first period simultaneously while they choose incentive parameters for the second period sequentially.

The above cited papers have been extended to consider mixed markets since in many countries private firms compete in the product market not only with other private firms but also with public firms. Given that the objective of private firms is to maximize profits while the objective of public firms is to maximize social welfare, the order of moves chosen by firms in a mixed oligopoly differs from that in a private oligopoly. Pal (1998) shows that, under Cournot competition, the owners of the firms take production decisions sequentially while in a private oligopoly firms decide quantities simultaneously.[2] Under Bertrand competition, Bárcena-Ruiz (2007) shows that the owners of the firms decide prices simultaneously in a mixed duopoly.

The above analysis is applied by Barros (1995) to investigate the use of incentive contracts as strategic variables in a mixed duopoly.[3] White (2001) shows that when firms have the choice of whether or not to hire managers, in equilibrium only the private firms do so. In this

[2] Matsumura (2003) and Lu (2006) also obtain that production decisions are taken sequentially considering that the public firm competes with foreign private firms. Moreover, Bárcena-Ruiz and Garzón (2010) extend the above analysis considering a firm jointly owned by the public sector and private domestic shareholders (a semipublic firm) rather than a private firm; they obtain that there is an equilibrium in which the owners of the firms take production decisions simultaneously.

[3] She shows that the owner of the public firm encourages its manager to behave more aggressively than the owner of the private firm since only the first firm takes consumer surplus into account. As a consequence, the public firm obtains a larger market share and makes higher profits than the private firm.

equilibrium, only private firms produce output, while a public firm exists only to impose discipline on the private firms. Bárcena-Ruiz (2009) studies the decision of firms as to whether or not to hire managers in a mixed duopoly and shows that under Bertrand competition both firms hire managers. Moreover, Nakamura and Inoue (2009) extend the above analysis and show that when firms' owners hire managers, prices are chosen simultaneously.

One issue that remains to be analyzed is whether firms' owners want to decide incentive contracts sequential or simultaneously under Bertrand competition.[4] In order to analyze this issue, I assume two firms that produce a heterogeneous good with identical constant marginal cost of production. I consider two cases: first, firms are privately-owned (private duopoly); second, one firm is privately-owned and the other is publicly-owned (mixed duopoly). The owners of the firms hire managers to who delegate price decisions. The owner of the public firm maximizes social welfare, the owner of the private firm maximizes profits and the managers maximize a linear combination of profits and sale revenues. Finally, firms' owners decide whether to set incentive contracts sequential or simultaneously.

It is obtained in the paper that in the private duopoly the owners of the firms choose incentive contracts sequentially.[5] This result is due to the fact that both prices and incentive parameters are strategic complements. Thus, the leader firm chooses an incentive parameter greater than in the simultaneous case to reduce market competition since the follower firm will react by also setting an incentive parameter greater than in the simultaneous case. Therefore, the prices set by firms in the sequential game are greater than in the simultaneous one, and both firms obtain greater profits in the first case. As a result, in equilibrium firms decide incentive contracts sequentially.

[4] It is easy to see that, under Cournot competition, as only the private firm hires a manager (see White, 2001) and given that incentive parameters are chosen before taking production decisions, the private firm becomes the leader in incentive contracts.

[5] A similar result, for the incentive parameters of the second production period, is obtained by Bárcena-Ruiz and Espinosa (1996) assuming two production periods and that firms choose short-term contracts or a long-term contract for their managers.

In the mixed duopoly, and in contrast to the result obtained in the private duopoly, it is shown that the owners of the firms choose incentive contracts simultaneously since it is a dominant strategy for the owners of both firms to become leaders in incentive contracts.

When the public firm is the leader in incentive contracts, as this firm takes consumer surplus into account, it chooses a lower value for the incentive parameter than in the simultaneous case in order to increase market competition. As incentive parameters are strategic complements, the follower firm (the private firm) also chooses a lower incentive parameter than in the simultaneous case. This means that both firms set lower prices when the public firm is the leader, and thus the profit of the private firm is greater in the simultaneous case. Besides, the consumer surplus is greater and the producer surplus lower when the public firm is the leader. As the consumer surplus has a greater effect on social welfare than the producer surplus, social welfare is greater in the simultaneous case.

When the private firm is the leader in incentive parameters, as this firm maximizes profits, it chooses a greater value for the incentive parameter than in the simultaneous case in order to reduce market competition. As incentive parameters are strategic complements, the follower firm (the public firm) also chooses a greater incentive parameter than in the simultaneous case. This means that both firms set greater prices when the private firm is the leader, and thus the profit of the private firm is greater in the first case. Besides, the consumer surplus is greater and the producer surplus is lower in the simultaneous case. As the consumer surplus has a greater effect on social welfare than the producer surplus, social welfare is greater in the simultaneous case.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes whether firms choose incentive parameters sequential or simultaneously in a private duopoly. Section 4 analyzes whether firms choose incentive parameters sequential or simultaneously in a mixed duopoly. Conclusions are drawn in section 5.

2 The model

We consider a mixed market comprising one public firm and one private firm, denoted by 0 and 1, respectively. On the consumption side, there is a continuum of consumers of the same type and the representative consumer maximizes the difference between the utility obtained by consuming the goods and the total amount paid by them: $U(q_0, q_1) - p_0q_0 - p_1q_1$, where $q_i \geq 0$ is the amount of the good i and p_i is its price ($i = 0, 1$). The function $U(q_0, q_1)$ is assumed to be quadratic, strictly concave and symmetric in q_0 and q_1:

$$U(q_0, q_1) = a(q_0 + q_1) - \frac{1}{2}((q_0)^2 + 2bq_0q_1 + (q_1)^2),\ 1>b>0,^6$$

where parameter b measures the degree to which goods are substitutes. Solving the problem of the representative consumer the following inverse demand functions are obtained: $p_i = a - q_i - bq_j$, $i \neq j$; $i, j = 0, 1$. Then, demand functions are given by:

$$q_i = \frac{a(1-b) - p_i + bp_j}{1-b^2},\ i \neq j;\ i, j = 0, 1. \quad (1)$$

The marginal cost of production of the two firms is given by c, which means that both firms are equally efficient. The profit of firm i is given by:

$$\pi_i = (p_i - c)\, q_i,\ i = 0, 1, \quad (2)$$

where q_i is given by (1). As usual, the public firm maximizes social welfare and the private firm maximizes its profit. Social welfare is measured as the sum of consumer surplus, CS, and producer surplus, PS:

$$W = CS + PS, \quad (3)$$

where $PS = \pi_0 + \pi_1$ and consumer surplus is given by:

$$CS = U(q_0, q_1) - p_0\, q_0 - p_1\, q_1 = \frac{2a(1-b)(a - p_0 - p_1) + (p_0)^2 - 2bp_0\, p_1 + (p_1)^2}{2(1-b^2)}. \quad (4)$$

[6] We consider a simplified version of the model used by Vives (1984), considering that $b<1$ to ensure that the function $U(q_0, q_1)$ is strictly concave.

The owners of the firms delegate price decisions to their managers. The owners offer linear incentive schemes to their managers,[7] and the managers, who are risk neutral, are paid on the margin according to a linear combination of profits and sales revenue. Formally, the manager of firm *i* (manager *i*) has the following objective function:

$$O_i = \alpha_i \pi_i + (1 - \alpha_i) S_i, \; i = 0, 1, \tag{5}$$

where π_i and S_i are profits and sales revenue, respectively, and α_i is the incentive parameter chosen by the owner of firm *i* (owner *i*).[8]

Given that $\pi_i=(p_i-c)q_i$ and $S_i=p_iq_i$, the objective function of manager *i* can be written as: $O_i=(p_i-\alpha_i c)q_i$. Thus, manager *i* considers $\alpha_i c$ as the marginal cost of production when setting the price of firm *i*. Then, owner *i* can make his manager to set a lower (greater) price, i.e. be more (less) aggressive than a profit maximizer firm, by choosing an incentive parameter, α_i, lower (greater) than one. If parameter α_i is one, the manager *i* maximizes profits.

I propose a three stage game with the following timing. In the first stage, the owners of the firms decide whether to set incentive schemes at time t=1 or at time t=2. In the second stage, the owners of the firms choose the incentive parameter of their managers. If both owners choose incentives in the same period, incentive parameters are chosen simultaneously, otherwise they are chosen sequentially. Finally, in the third stage, the managers of the firms decide the firms' prices. To obtain a subgame perfect equilibrium, the game is solved backwards.

3 Private duopoly

[7] This is a standard assumption when assuming a mixed oligopoly in which the owners of firms delegate decisions to managers (see Barros, 1995; White, 2001; Bárcena-Ruiz, 2009).

[8] As in Fershtman and Judd (1987), each owner offers its manager "take it or leave it" incentive schemes. Manager *i* receives a payoff $A_i+\beta_i O_i$, where A_i and β_i are constant, $\beta_i>0$; thus, manager *i* maximizes O_i. The owner *i* chooses A_i and β_i so that the manager gets only his opportunity cost, which is normalized to zero.

We consider first, as a benchmark case, that both firms are privately owned. Given the symmetry of the model and that incentive parameters can be chosen either sequential or simultaneously, there are two cases. First, both owners decide incentive simultaneously at t=1 or at t=2. Secondly, one firm is the leader in incentive parameters and the other the follower; the leader chooses the incentive parameter at t=1 and the follower at t=2.

In the third stage, manager i chooses the value of p_i, that maximizes his objective function, given by (5). Solving this problem I obtain the reaction function in prices of firm i:

$$p_i = \frac{1}{2}(a(1-b) + c\alpha_i + bp_j),\ i \neq j;\ i, j = 0, 1. \tag{6}$$

It is easy to see from (6) that price decisions are strategic complements (i.e. reaction function in prices are upward sloping). From (6) I obtain:

$$p_i = \frac{a(2+b)(1-b) + 2c\alpha_i + bc\alpha_j}{4-b^2},\ i \neq j;\ i, j = 0, 1. \tag{7}$$

3.1 Owners decide incentive parameters simultaneously

In the second stage, the owners of the firms choose simultaneously at t=1 (denoted by superscript 11) or at t=2 (denoted by superscript 22) the incentive parameter that maximizes their profits. Solving these problems, I obtain the reaction function in incentive parameters of the firms:

$$\alpha_i = \frac{ab^2(2-b-b^2) + c(8-6b^2+b^4) + cb^3\alpha_j}{4(2-b^2)c},\ i \neq j,\ i,j=0, 1. \tag{8}$$

It can be seen from (8) that incentive parameters are strategic complements. From (8) I obtain:

$$\alpha^{ii} = 1 + \frac{b^2(1-b)(a-c)}{(4-2b-b^2)c},\ i = 1, 2. \tag{9}$$

As incentive parameters are strategic complements, the owner of each firm choose an incentive parameter greater than one ($\alpha^{ii} > 1$), which implies that managers consider a marginal cost of production greater than the real cost and set a higher price than a profit maximizing firm. Therefore, the owners encourage their managers to reduce market competition in comparison with the case in which firms maximize profits.

From (9), the following is obtained:

$$p^{ii} = \frac{2a(1-b)+c(2-b^2)}{4-2b-b^2},\ \pi^{ii} = \frac{2(1-b)(2-b^2)(a-c)^2}{(1+b)(4-2b+b^2)^2},\ i = 1, 2.$$

3.2 Owners decide incentive parameters sequentially

In the second stage, the owner of firm 0 (the leader) chooses the incentive parameter at *t*=1 and the owner of firm 1 (the follower) at *t*=2. I denote the leader (follower) firm by superscript 12 (21). I solve first the problem of the follower. The owner of firm 1 chooses the incentive parameter α_1 that maximizes its profit. Solving this problem, I obtain expression (8) for *i*=1 and *j*=0. The owner of the firm 0 chooses the incentive parameter α_0 that maximizes its profit taking into account (8) for *i*=1 and *j*=0. Solving I obtain:

$$\alpha^{12} = 1+\frac{b^2(4-2b-3b^2+b^3)(a-c)}{(16-16b^2+3b^4)c},\ \alpha^{21} = 1+\frac{(1-b)b^2(8+4b-4b^2-b^3)(a-c)}{2(16-16b^2+3b^4)c}. \quad (10)$$

As under simultaneous decisions, incentive parameters are greater than one. From (10), the following is obtained:

$$p^{12} = \frac{a(1-b)(4+2b-b^2)+c(1+b)(4-2b-b^2)}{2(4-3b^2)},$$

$$p^{21} = \frac{a(1-b)(8+4b-4b^2-b^3)+c(8+4b-8b^2-3b^3+2b^4)}{(2+b)(2-b)(4-3b^2)},$$

$$\pi^{12} = \frac{(1-b)(2-b^2)(8+4b-4b^2-b^3)^2(a-c)^2}{2(1+b)(2+b)(2-b)(4-3b^2)},\ \pi^{21} = \frac{(1-b)(4+2b-b^2)^2(a-c)^2}{4(1+b)(2+b)(2-b)(4-3b^2)}.$$

3.3 Owners' decisions as to whether to choose incentive parameters at *t*=1 or at *t*=2

It remains to solve the first stage of the game. In this stage, the owner of each firm decides whether to choose incentive parameters at *t*=1 or at *t*=2. From the results obtained in the two cases considered, the following is obtained.

Lemma 1. *When firms are privately owned, in equilibrium:*

i) $\alpha^{12} > \alpha^{21} > \alpha^{11} = \alpha^{22} > 1$*;*

ii) $p^{12} > p^{21} > p^{11} = p^{22}$.

This Lemma shows that the leader sets a greater incentive parameter than the follower who chooses an incentive parameter greater than in the simultaneous case. As incentive parameters are strategic complements firms choose incentive parameters greater than one to reduce market competition. Moreover, the leader firm chooses an incentive parameter greater than in the simultaneous case since the follower firm will react by also setting and incentive parameter greater than in the simultaneous case. A greater incentive parameter implies that the managers of the firms consider a greater production cost. This means that the manager of the leader firm sets a greater price than the manager of the follower firm, who sets a greater price than in the simultaneous case.

Proposition 1. *When firms are privately owned, in equilibrium one owner chooses incentive parameters at t=1 and the other at t=2.*

It is proved in the appendix that $\pi^{21} > \pi^{12} > \pi^{22} = \pi^{11}$. Therefore, both firms obtain greater profits if they set incentive parameters sequentially rather than simultaneously. As a result, in equilibrium firms take decisions sequentially. It has to be noted that there are two equilibria: in each of them one firm take decisions at *t*=1 and the other at *t*=2. However, as $\pi^{21} > \pi^{12}$ both firms prefer to be the follower rather than to be the leader. This result is due to the fact, as Lemma 1 shows, that when firms set incentive parameters sequentially prices are greater than when they set incentive parameters simultaneously. This reduces market competition increasing the profit of the two firms. But as the leader firm sets a greater price than the follower, it obtains lower profits.

4 Mixed duopoly

Given that incentive parameters can be chosen either sequential or simultaneously, there are three possible cases in the mixed duopoly. First, both owners decide incentive simultaneously at t=1 or at t=2. Second, the public firm is the leader in incentives. Finally, the private firm is the leader in incentives.

The third stage of the game is the same for the three cases. In this stage managers choose simultaneously the price that maximizes their objective functions, given by (5). Solving these problems I obtain expression (7). Next I solve the second stage of the game in the different cases considered.

4.1 Owners decide incentive parameters simultaneously[9]

In the second stage, the owner of the public firm chooses the incentive parameter α_0 that maximizes social welfare. Simultaneously, the owner of the private firm chooses the incentive parameter α_1 that maximizes its profit. Solving these problems, I obtain:

$$\alpha_0^{ii} = 1 - \frac{(1-b)(8-8b^2-2b^3+b^4)(a-c)}{(8-8b^2+b^4)c},\ \alpha_1^{ii} = 1 + \frac{b^2(1-b)(2-b^2)(a-c)}{(8-8b^2+b^4)c},\ i = 1, 2. \quad (11)$$

In the mixed duopoly, the owner of the private firm chooses an incentive parameter greater than one ($\alpha_1^{ii}>1$, i=1, 2) while the owner of the public firm provide incentives for its manager to produce more than a profit maximizer firm only if the degree to which products are substitutes is low enough ($\alpha_0^{ii}<1$ if and only if b<0.9429, i=1, 2). This is explained by two effects. First, as in the private duopoly, the private firm chooses an incentive parameter greater than one to reduce market competition. As prices are strategic complements, if the private firm rises its price the public firm does likewise. Secondly,

[9] This case is analyzed by Bárcena-Ruiz (2009).

given that the manager of the public firm does not take consumer surplus into account, the only way for the government to increase market competition is by choosing an incentive parameter lower than one for its manager. As prices are strategic complements, if the public firm lowers its price the private firm does likewise. These two effects influence prices in opposite directions and it remains to be seen which of the two effects dominates.

For the owner of the private firm the first effect dominates, which means that chooses an incentive parameter greater than one ($\alpha_1^{ii}>1$) to reduce market competition. The government chooses an incentive parameter lower than one for its manager if parameter *b* is low enough ($\alpha_0^{ii}<1$ if and only if $b<0.9429$). In this case, as the degree to which products are substitutes is low enough, the second effect dominates.[10] Thus the public firm chooses an incentive parameter lower than one to offset the reduction in market competition caused by the behavior of the private firm. When the degree to which products are substitutes is high enough (i.e. when $b\geq 0.9429$), as market competition is high, the first effect dominates and the public firm chooses an incentive parameter greater than one.

Finally, as the owner of the private firm tries to reduce market competition while the owner of the public firm seeks to increase it, the private firm chooses an incentive parameter greater than that chosen by the public firm ($\alpha_1^{ii}>\alpha_0^{ii}$). As a result, the public firm is more aggressive in the product market than the private firm.

From (11), the following is obtained:

$$p_0^{ii}=\frac{2ab(1-b)+c(8-2b-6b^2+b^4)}{8-8b^2+b^4},\ p_1^{ii}=\frac{2a(1-b)(2-b^2)+c(4+4b-6b^2-2b^3+b^4)}{8-8b^2+b^4},$$

$$\pi_0^{ii}=\frac{2b(1-b)(8+2b-8b^2-2b^3+b^4)(a-c)^2}{(1+b)(8-8b^2+b^4)^2},\ \pi_1^{ii}=\frac{2(1-b)(2-b^2)^3(a-c)^2}{(1+b)(8-8b^2+b^4)^2},$$

$$CS^{ii}=\frac{(40+8b-78b^2-18b^3+50b^4+12b^5-12b^6-2b^7+b^8)(a-c)^2}{(1+b)(8-8b^2+b^4)^2},$$

[10] Parameter *b* can be interpreted as an imperfect measure of the degree of competition in the product market since the higher the value of parameter *b* is, the greater the competition is in the product market between firms.

$$PS^{ii} = \frac{2(1-b)(8-10b^2+2b^3+2b^4-b^5)(a-c)^2}{(8-8b^2+b^4)^2},$$

$$W^{ii} = \frac{(56+8b-114b^2-14b^3+74b^4+6b^5-16b^6+b^8)(a-c)^2}{(1+b)(8-8b^2+b^4)^2},\ i=1,\ 2.$$

In this case, as the public firm is more aggressive in the product market than the private firm, it sets a lower price ($p_0^{11} = p_0^{22} < p_1^{11} = p_1^{22}$) and produces more.

4.2 The public firm is the leader in incentive parameters

In the second stage, the owner of the public firm chooses the incentive parameter at *t*=1 and the owner of the private firm at *t*=2. We solve first the problem of the follower. The owner of the private firm chooses the incentive parameter α_1 that maximizes its profit. Solving this problem, I obtain expression (8) for *i*=1 and *j*=0. The owner of the public firm chooses the incentive parameter α_0 that maximizes social welfare taking into account (8) for *i*=1 and *j*=0. Solving I obtain:

$$\alpha_0^{12} = 1 - \frac{(1-b)(2-b)(8+4b-8b^2-5b^3)(a-c)}{(16-20b^2+5b^4)c},\ \alpha_1^{21} = 1 + \frac{(1-b)b^2(4-3b^2)(a-c)}{(16-20b^2+5b^4)c}. \quad (12)$$

It can be shown that the incentive parameter of the manager of the public firm is lower than one ($\alpha_0^{12}<1$) if and only if *b*<0.9617, and the incentive parameter of the manager of the private firm is greater than one ($\alpha_1^{21}>1$); finally, $\alpha_1^{21} > \alpha_0^{12}$.

From (12), the following is obtained:

$$p_0^{12} = \frac{2ab(1-b)(2-b^2)+c(16-4b-16b^2+2b^3+3b^4)}{16-20b^2+5b^4},$$

$$p_1^{21} = \frac{2a(1-b)(4-3b^2)+c(2-b)(4+6b-4b^2-5b^3)}{16-20b^2+5b^4},$$

$$\pi_0^{12} = \frac{2b(1-b)(2-b^2)(16+4b-20b^2-4b^3+5b^4)(a-c)^2}{(1+b)(16-20b^2+5b^4)^2},$$

$$\pi_1^{21}=\frac{2(1-b)(2-b^2)(4-3b^2)^2(a-c)^2}{(1+b)(16-20b^2+5b^4)^2},\ CS^{12}=\frac{(10+2b-12b^2-2b^3+3b^4)(a-c)^2}{(1+b)(16-20b^2+5b^4)},$$

$$PS^{12}=\frac{2(1-b)(2-b^2)(a-c)^2}{16-20b^2+5b^4},\ W^{12}=\frac{(14+2b-18b^2-2b^3+5b^4)(a-c)^2}{(1+b)(16-20b^2+5b^4)}.$$

In this case, as the public firm is more aggressive than the private firm, it sets a lower price ($p_0^{12}<p_1^{21}$) and produces more.

4.3 The private firm is the leader in incentive parameters

In the second stage, the owner of the private firm chooses the incentive parameter at *t*=1 and the owner of the public firm at *t*=2. We solve first the problem of the follower. The owner of the public firm chooses the incentive parameter α_0 that maximizes social welfare. Solving these problems, I obtain:

$$\alpha_0=\frac{a(2-b-b^2)^2+c(8-4b-6b^2+b^3+b^4)+cb^3\alpha_1}{(4-3b^2)c}. \tag{13}$$

The owner of the private firm chooses the incentive parameter α_1 that maximizes its profit taking into account (13). Solving we obtain:

$$\alpha_0^{21}=1-\frac{(4-4b^2-b^3)(a-c)}{4(1+b)c},\ \alpha_1^{12}=1+\frac{b^2(a-c)}{4(1+b)c}. \tag{14}$$

It can be shown that the incentive parameter of the manager of the public firm is lower than one ($\alpha_0^{21}<1$) if and only if $b<0.9032$, and the incentive parameter of the manager of the private firm is greater than one ($\alpha_1^{12}>1$); finally, $\alpha_1^{12}>\alpha_0^{21}$. From (14), the following is obtained:

$$p_0^{21}=\frac{ab+c(4+3b)}{4(1+b)},\ p_1^{12}=\frac{a(2-b^2)+c(2+4b+b^2)}{4(1+b)},\ \pi_0^{21}=\frac{b(4+b)(a-c)^2}{16(1+b)^2},$$

$$\pi_1^{12}=\frac{(2-b^2)(a-c)^2}{8(1+b)^2},\ CS^{21}=\frac{(20+24b+5b^2)(a-c)^2}{32(1+b)^2},$$

$$PS^{21}=\frac{(4+4b-b^2)(a-c)^2}{16(1+b)^2},\ W^{21}=\frac{(28+32b+3b^2)(a-c)^2}{32(1+b)^2}.$$

In this case, as the public firm is more aggressive than the private firm, it sets a lower price ($p_0^{21} < p_1^{12}$) and produces more.

4.4 Owners' decisions as to whether to choose incentive parameters at *t*=1 or at *t*=2

It remains only to solve the first stage of the game. In this stage, the owner of each firm decides whether choose incentive parameters at *t*=1 or at *t*=2. From the results obtained in the three cases considered, the following is obtained.

Lemma 2. *In the mixed duopoly, in equilibrium:*

i) $\alpha_0^{21} > \alpha_0^{11} = \alpha_0^{22} > \alpha_0^{12} > 1$ *and* $\alpha_1^{12} > \alpha_1^{11} = \alpha_1^{22} > \alpha_1^{21} > 1$*;*

ii) $p_0^{21} > p_0^{11} = p_0^{22} > p_0^{12}$ *and* $p_1^{12} > p_1^{11} = p_1^{22} > p_1^{21}$*;*

iii) $CS^{11} > CS^{21}$, $CS^{12} > CS^{22}$, $PS^{11} < PS^{21}$ *and* $PS^{12} < PS^{22}$.

When the public firm is the leader in incentive parameters, as this firm takes consumer surplus into account, it chooses a lower value for the incentive parameter than in the simultaneous case in order to increase market competition. As incentive parameters are strategic complements, the follower firm (the private firm) also chooses a lower incentive parameter than in the simultaneous case. Then: $\alpha_0^{11} = \alpha_0^{22} > \alpha_0^{12} > 1$ and $\alpha_1^{11} = \alpha_1^{22} > \alpha_1^{21} > 1$. Thus, both firms are more aggressive in the product market when the public firm is the leader than in the simultaneous case, which means that both firms set lower prices in the first case: $p_0^{11} = p_0^{22} > p_0^{12}$ and $p_1^{11} = p_1^{22} > p_1^{21}$. Finally, as market competition is greater when the public firm is the leader than in the simultaneous case, the consumer surplus is greater and the producer surplus is lower in the first case ($CS^{12} > CS^{22}$ and $PS^{12} < PS^{22}$).

When the private firm is the leader in incentive parameters, as this firm maximizes profits it chooses a greater incentive parameter than in the simultaneous case in order to reduce market competition. As incentive parameters are strategic complements, the follower firm (the public firm) also chooses a greater incentive parameter than in the simultaneous case.

Then: $\alpha_0^{21} > \alpha_0^{11} = \alpha_0^{22}$ and $\alpha_1^{12} > \alpha_1^{11} = \alpha_1^{22}$. Thus, both firms are less aggressive in the product market when the private firm is the leader than when incentive parameters are chosen simultaneously, which means that both firms set greater prices in the first case: $p_0^{21} > p_0^{11} = p_0^{22}$ and $p_1^{12} > p_1^{11} = p_1^{22}$. Finally, as market competition is lower when the private firm is the leader than in the simultaneous case, the consumer surplus is lower and the producer surplus is greater in the first case ($CS^{11} > CS^{21}$ and $PS^{11} < PS^{21}$).

Proposition 2. *In the mixed duopoly, in equilibrium both owners choose incentive parameters simultaneously at t=1.*

It is proved in the appendix that $W^{11} > W^{21}$, $W^{12} > W^{22}$, $\pi_1^{11} > \pi_1^{21}$ and $\pi_1^{12} > \pi_1^{22}$. Therefore, it is a dominant strategy for the owners of both firms to choose incentives at *t*=1, which means that in equilibrium firms choose incentives simultaneously.

As we have seen in Lemma 2, when the public firm is the leader in incentive parameters competition in the product market is greater than in the simultaneous case. This implies that the profit of the private firm is greater in the last case ($\pi_1^{11} > \pi_1^{21}$). Besides, Lemma 2 shows that CS^{12} is greater than CS^{22} and PS^{12} is greater than PS^{22}; as the consumer surplus has a greater effect on social welfare than the producer surplus it is obtained that W^{12} is greater than W^{22}. On the other hand, as we have seen in Lemma 2, when the private firm is the leader in incentive parameters competition in the product market is lower than in the simultaneous case. This implies that the profit of the private firm is greater in the first case ($\pi_1^{12} > \pi_1^{22}$). Besides, Lemma 2 shows that CS^{21} is lower than CS^{11} and PS^{21} is greater than PS^{11}; as the consumer surplus has a greater effect on social welfare than the producer surplus it is obtained that W^{21} is lower than W^{11}.

5 Conclusions

The literature that analyzes mixed oligopolies has dedicated little attention at investigating whether firms' owners want to decide incentive contracts for their managers sequential or simultaneously. Thus, this paper analyzes this issue assuming Bertrand competition with heterogeneous goods and a mixed duopoly. Endogenous timing of decisions is an important issue to be analyzed since a sequential order of moves may give rise to significantly different results from those obtained in a simultaneous game.

The literature on industrial organization has analyzed whether the owners of the firms take decisions sequential or simultaneously assuming both a private duopoly and a mixed duopoly. This analysis has been extended to consider that firms' owners hire managers to who delegate price or quantity decisions. However, one issue that remains to be analyzed is whether firms' owners want to decide incentive contracts sequential or simultaneously under Bertrand competition. Thus, in this paper I assume two firms that produce a heterogeneous good with identical constant marginal cost of production. The owners of the firms hire managers in which delegate price decisions. Firms' owners decide whether to set incentive contracts sequential or simultaneously. It is obtained that in the private duopoly the owners of the firms choose incentive contracts sequentially. This is in contrast with the result obtained under Cournot competition where firms decide contracts simultaneously. In the mixed duopoly, and in contrast to the result obtained in the private duopoly, it is shown that the owners of the firms choose incentive contracts simultaneously. This is due to the fact that it is a dominant strategy for the owner of each firm to become the leader in incentive parameters.

Appendix

Proof of proposition 1. Given that $0<b<1$, then: $\pi^{11}-\pi^{21}=\frac{b^5(2-2b-b^2+b^3)(64-64b^2+12b^4+b^5)(a-c)^2}{2(1+b)(4-2b-b^2)^2(16-16b^2+3b^4)^2}>0$, $\pi_1^{21}-\pi_1^{12}=\frac{b^5(16-20b^2-b^3+5b^4)(a-c)^2}{4(1+b)(16-16b^2+3b^4)}>0$, $\pi_1^{12}-\pi_1^{22}=\frac{b^8(1-b)(a-c)^2}{4(1+b)(4-2b-b^2)^2(16-16b^2+3b^4)}>0$.

Proof of proposition 2. Given that $0<b<1$, then:

$$W^{11}-W^{21}=\frac{b^4(64-112b^2+52b^4-3b^6)(a-c)^2}{32(1+b)^2(8-8b^2+b^4)^2}>0, \qquad W^{12}-W^{22}=\frac{2b^6(1+b)(1-b)^3(a-c)^2}{(8-8b^2+b^4)^2(16-20b^2+5b^4)}>0,$$

$$\pi_1^{11}-\pi_1^{21}=\frac{8b^4(1-b)^2(64-144b^2+114b^4-37b^6+4b^8)(a-c)^2}{(8-8b^2+b^4)^2(16-20b^2+5b^4)^2}>0, \quad \pi_1^{12}-\pi_1^{22}=\frac{b^8(2-b^2)(a-c)^2}{8(1+b)^2(8-8b^2+b^4)^2}>0.$$

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