Testing for Structural Breaks in Factor Loadings: An Application to International business cycle

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Abstract

This paper proposes the implementation of the Sup-Wald test of Andrews (1993) to detect structural breaks in the loadings of a static factor model. The procedure is illustrated by testing for structural breaks in the common factors of a sample of advanced countries from 1950 until 2006.

Key words: Factor loadings; Structural breaks; Sup-Wald test; Recursive estimation.
Code JEL: F41, F47, E32

1 Introduction

A powerful methodology employed to obtain a reference cycle from a large number of business cycle indicators is factor analysis (Forni et al, 2000; Stock and Watson, 2002). However, the relation of dependence between each of the observed series and the factor/s extracted can be affected by instability in the factor loadings. Stock and Watson (2008) and Banerjee et al. (2008) point out the possibility of changing factor loadings that could affect factor estimation and, thus, the forecasting ability of these models, up until now there are no papers that empirically contrast this possibility.

There are number of tests of structural change in the literature. This paper proposes the use of the Sup-Wald test of Andrews (1993) to detect structural breaks in the parameters –factor loadings or correlations- associated with the common factors. To illustrate this proposal, an application to international annual GDP series is performed. This will make it possible to discuss the approximation or distancing from each country regarding the factor or factors (cyclical convergence and divergence).

The rest of the paper is organized as follows: in section 2, the recursive sup-Wald test of Andrews (1993) is applied to the factor loadings of a factor model, section 3 presents the results of the empirical application of the test and section 4 presents some brief conclusions.

2 The model and the Sup-Wald test.

The well known results concerning the static factor model and its estimation by means of principal components are not presented here (See Stock and Watson, 2002). It should be noted that a linear relation can be established between each of the observed series and the factors. Thus the factor loadings can be recovered from a linear regression of each of the observed series on the factors previously obtained as in:

$$x_{j,t} = \beta_{1j}\hat{f}_{1,t} + \ldots + \beta_{mj}\hat{f}_{m,t} + v_{j,t} \qquad (1)$$

where $x_{j,t}$ is the *j-th* observed series –stationary transformed and standardized- and $\hat{f}_{i,t}$, the *i-th* estimated factor at *t*. β_{ij} corresponds to the factor loadings or correlations between each of the *j* series and the factor *i*. The error term, $v_{j,t}$, may include non-significant dependence on discarded factors or may be a specific variation of $x_{j,t}$. In both cases, $v_{j,t}$ must be uncorrelated with the regressors –retained factors- for consistent OLS estimation of (1), therefore previous factor estimation must be subject to the appropiate orthogonality conditions. Nevertheless, it is assumed that the error term in (1) can generally show both heteroskedasticity and autocorrelation. Thus, consistent OLS standard error estimates of β_{ij} must be robust to both assumptions.

In (1), if the change point (date) of a possible structural change is known, a Chow (1960) test could be performed. However, when such a date is unknown, a recursive testing procedure following Andrews (1993), whose distribution is non-standard, must be employed. This procedure consists of estimating recursively equation (1) as:

$$x_{j,t} = \beta_{1j}(\tau)\hat{f}_{1,t} + \ldots + \beta_{mj}(\tau)\hat{f}_{m,t} + v_{j,t}(\tau) \quad (2)$$

where τ a possible moving break date that covers the sample period as $\tau = \tau_0, \tau_0 + 1, \ldots, \tau_1$, where $\tau_0 = \pi T$ and $\tau_1 = (T - \pi T)$ (both the integer parts), and π is a minimum sample percentage excluded both at the beginning and at the end of the sample (the *trimming*). The parameter stability is assessed in the central sample proportion. The general linear null hypothesis of absence of structural change is

$$H_0 : R\beta_j(\tau) = \beta_j \quad (3)$$

where $\beta_j(\tau)$ is the recursive parameters in (2), β_j the whole sample parameters in (1) and R the matrix of linear restrictions to be tested with full rank q.

The F type statistic (Wald type statistic) is obtained as

$$F_{\sup-wald} = \max\left[F(\tau_0), F(\tau_0 + 1), \ldots, F(\tau_1)\right] \quad (4)$$

where each of the $F(\tau)$ is defined as:

$$F(\tau)=\frac{[R\hat{\beta}_j(\tau)-\beta_j]'(R\hat{\Sigma}(\tau)^{-1}R')^{-1}[R\hat{\beta}_j(\tau)-\beta_j]}{q} \tag{5}$$

As pointed out, $\hat{\Sigma}(\tau)$ must be robust to heteroskedasticity and autocorrelation. For example, by employing the Newey-West (1987) procedure[1]

$$\hat{\Sigma}_{NW}^{-1}(\tau)=(\hat{\mathcal{F}}(\tau)'\hat{\mathcal{F}}(\tau))^{-1}\left[\frac{\tau}{\tau-m}\hat{S}_{NW}(\tau)\right](\hat{\mathcal{F}}(\tau)'\hat{\mathcal{F}}(\tau))^{-1} \tag{6}$$

where $\hat{\mathcal{F}}(\tau)$ is the estimated factor matrix conditioned to the sample proportion τ, that is,

$$\hat{\mathcal{F}}(\tau)=\left[\hat{f}_{1,t}(\tau), \quad \hat{f}_{2,t}(\tau), \quad \ldots, \quad \hat{f}_{m,t}(\tau)\right] \tag{7}$$

and the variance-covariance matrix is defined as:

$$\hat{S}_{NW}(\tau)=\hat{S}_{White}(\tau)+\sum_{g=1}^{h}\left\{\left(1-\frac{g}{h+1}\right)\sum_{t=g+1}^{\tau}\left(\hat{v}_{j,t}\hat{v}_{j,t-g}\left(\hat{\boldsymbol{f}}_t\hat{\boldsymbol{f}}_{t-g}'+\hat{\boldsymbol{f}}_{t-g}\hat{\boldsymbol{f}}_t'\right)\right)\right\} \tag{8}$$

where *h*, the maximum number of lags, can be established as $h=\text{int}(4(T/100)^{2/9})$ (int: "*largest integer not greater than x"*) or by means of some information criterion. $\hat{\boldsymbol{f}}_t=\left[\hat{f}_{1,t},\hat{f}_{2,t},\ldots,\hat{f}_{m,t}\right]'$ is the vector of estimated factors in *t*, and $\hat{S}_{white}(\tau)$ denotes the White (1980) heteroskedasticity consistent variance-covariance matrix estimator:

$$\hat{S}_{White}(\tau)=\sum_{t=1}^{\tau}\hat{v}_{j,t}^2\hat{\boldsymbol{f}}_t\hat{\boldsymbol{f}}_t' \tag{9}$$

Asymptotic distribution of the $F_{\sup-wald}$ statistic is not standard because the break date appears only under the alternative hypothesis. To deal with this problem, Andrews (1993) obtain**s** asymptotic critical values that depend on the number of tested restrictions and on the sample proportion limited by τ_0 and τ_1. However, with no asymptotic sample sizes and heteroskedasticity and autocorrelation residuals in (2), critical values different from those of Andrews' ones are expected. In this work, empirical critical values have been calculated with a

[1] A description of other procedures for estimating the variance-covariance matrix in presence of heteroskedasticity and autocorrelation can be found in den Haan and Levin (1997).

Montecarlo simulation according to our data characteristics; that is, by assuming autocorrelation disturbances and for the same sample sizes available in practice. The details of the Montecarlo simulation procedure are explained in section 3.

An important aspect provided by this recursive test is that, not only does it allow for the detection of a possible break date, but also, a visual inspection of the recursive $F(\tau)$ can provide additional information on other break dates and on the smoothness of the regression function. This visual inspection can be completed with recursive estimates of $\beta_j(\tau)$ to obtain additional information of eventual parametric changing patterns.

3 Data and Results

The data used for this study are taken from the electronic version of the *The Conference Board and Groningen Growth and Development Center (GGDC), Total Economy Database, January 2008,* http://www.conference-board.org/economics. In the analysis carried out we used the series of real GDP per capita of 36 advanced countries[2] on an annual basis from 1950 to 2006 that is presented at market prices based on 1990 purchasing power parities (PPP) U.S. dollars.

This section presents the results of the aplication of the Sup-Wald test to an international GDP growth series of the advanced countries in which two common factors are found[3]. The results of the recursive estimates of both the recursive parameters in (2) and the recursive estimates of the $F(\tau)$ statistic in (4) are shown in Figures 1 and 2. For the generation of the critical values of the $F_{\sup-wald}$ statistic of Table 1, the following process has been performed[4]:

1. Model (1) is estimated by OLS for each country. The residuals are obtained as:

$$\hat{v}_{j,t} = x_{j,t} - \hat{\beta}_{1j}\hat{f}_{1,t} - \ldots - \hat{\beta}_{mj}\hat{f}_{m,t} \tag{10}$$

   The autoregressive structure of these residuals is estimated by fitting the *AR(p)* model where *p* is selected automatically by an information criterion.

[2] We selected the countries that have a high level of development according to the Human Development Index of the United Nations (UN, 2007) and share or provide information to the common factor. See De Lucas et al, 2009 for further details.

[3] Though not reported here, any details of the static factor model metodolgy employed for the estimation of international business cycles and results obtained can be found in De Lucas et al., 2009.

[4] Some work related with this procedure is del Hoyo and Cendejas (2007).

2. 1000 random series $\hat{v}_{j,t}^{b}$ following the *AR(p)* previously estimated are simulated. With these simulated disturbances $\hat{v}_{j,t}^{b}$, "new" GDP growth series are calculated for each country according to:

$$\hat{x}_{j,t}^{b} = \hat{\beta}_{1j}\hat{f}_{1,t} + \ldots + \hat{\beta}_{mj}\hat{f}_{m,t} + \hat{v}_{j,t}^{b} \quad (11)$$

where the factors $\hat{f}_{i,t}$ and the full sample OLS estimations $\hat{\beta}_{ij}$ are employed. In this way, equation (11) provides "null hypothesis variation" in the observed series by assuming constant β_{ij}.

3. With the 1000 simulated series $\hat{x}_{j,t}^{b}$, $F_{\sup-wald}$ statistic is calculated according to (4). The *trimming* has been fixed as $\tau_0 = 0.20$ or as 0.30 in some cases because of the high statistical values of $F_{\sup-wald}$ at the beginning of the sample, and $\tau_1 = 0.85$. The null hypothesis has been referred to one or both factors as indicated in Table 1.
4. Once the empirical distribution of the $F_{\sup-wald}$ statistic has been tabulated, the quantiles 90, 95 and 99 are selected. The corresponding critical values are shown in Table 1.

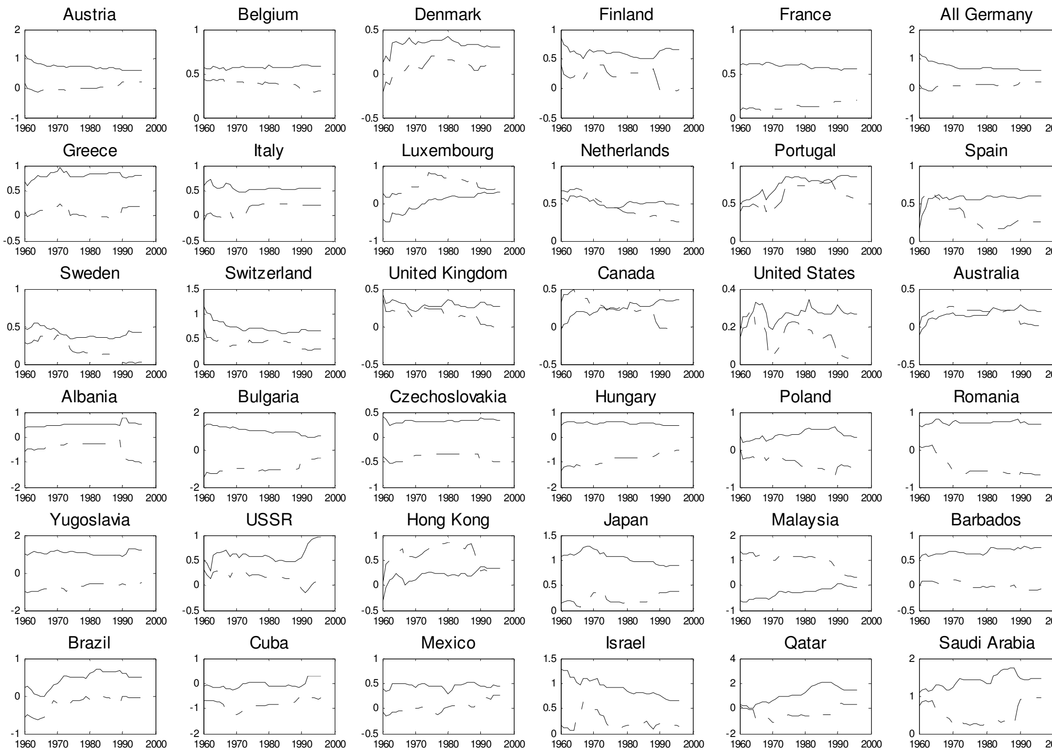
Figure 1 shows recursive factor loadings for common factor 1 and 2. This figure shows a clear dominance of the common factor 1 excepting Luxembourg, Hong Kong and Cuba, dominated by the behavior of the common factor 2. It is interesting to note that Eastern European countries exhibit an opposite behaviour in both recursive factor loadings on dates close to the late eighties or early nineties (see also Figure 2) when these economies change from a socialist system to a market system. The recursive test makes it possible to reject the null hypothesis of parameter stability according to simulated critical values for Albania and Hungary. The null stability hypothesis is also rejected for Malaysia, Brazil and Qatar. In general, the recursive estimates of Figure 1 confirm a declining importance of factor 2 for most of the countries.

The significativity of the dependence of each of the series respect to one or both factors has been taken into account in Figure 2 and in Table 1 where the test is performed for one or both factors. Although significant structural changes were not detected statistically, visual inspection of recursive estimates in Figure 1 would make it possible to verify eventual processes of convergence or divergence towards the common factors.

4 Conclusions

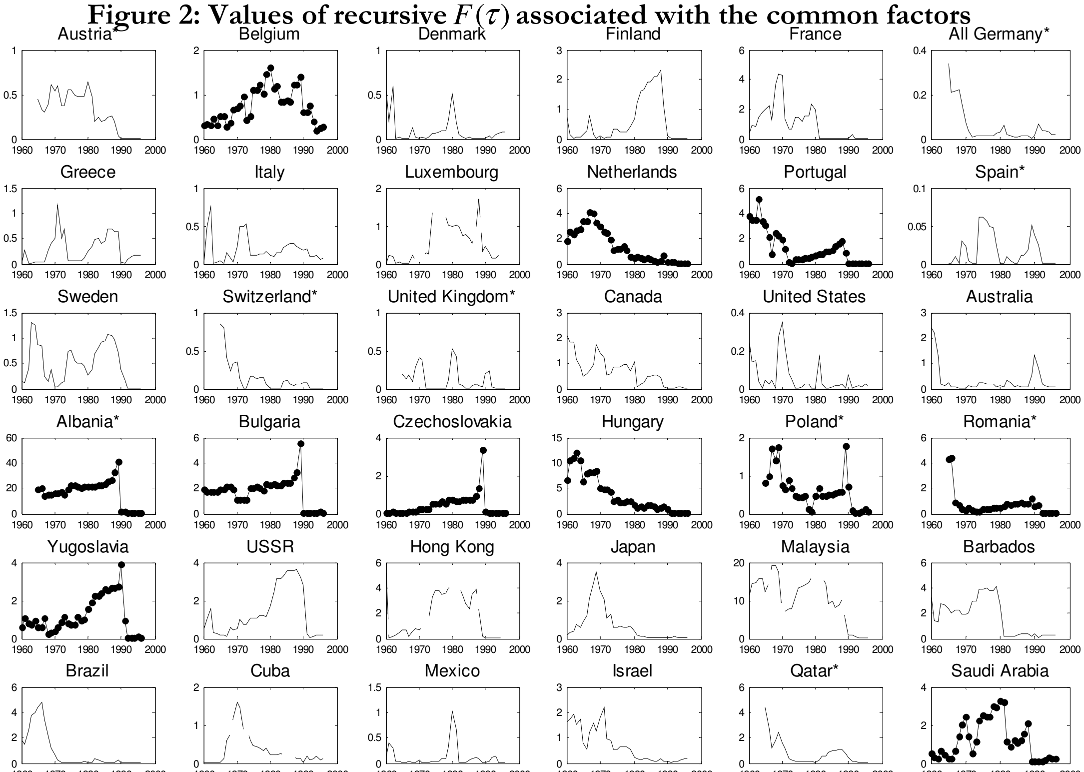
The recursive Sup-Wald test of Andrews (1993) has been applied to detect structural breaks in the factor loadings of a factor model. The application of this test to international GDP growth series in which common factors are found has served to locate some breaks whose interpretation is related to important changes of the openness to the global economy of countries affected by important transformation processes. The testing procedure can be extended to every factor model based on time series observations.

Figure 1: Recursive parameters associated with the common factors



Note: The recursive estimation for common factor 1 is show in continuous line and for common factor 2 in discontinuous line.

Figure 2: Values of recursive $F(\tau)$ associated with the common factors



Note: $F(\tau)$ has been computed for factor 1 (continuous line), for factor 2 (discontinuous line) or for both factors (dotted line). The trimming at he beginning of the sample has been 20% or 30% (denoted with *) and at the end of the sample 15%.

Table 1: Results of $F_{\sup-wald}$ and simulated critical values

Countries	nº restrictions	proportion of the initial sample	critical value 10%	critical value 5%	critical value 1%	Fsup-statistic
'Austria'	1	0,3	3,20	4,70	9,20	0,60
'Belgium'	2	0,2	9,80	14,60	35,30	1,60
'Denmark'	1	0,2	5,10	8,20	15,70	0,60
'Finland'	1	0,2	5,10	7,50	14,70	2,30
'France'	1	0,2	6,10	8,80	17,90	4,40
'All Germany'	1	0,3	3,60	4,70	8,70	0,30
'Greece'	1	0,2	6,10	8,60	17,00	1,20
'Italy'	1	0,2	4,10	5,90	12,60	0,80
'Luxembourg'	1	0,2	14,50	21,70	41,70	1,70
'Netherlands'	2	0,2	8,60	12,80	33,20	4,10
'Portugal'	2	0,2	1104	1195	1443	5,10
'Spain'	1	0,3	3,20	4,50	7,60	0,10
'Sweden'	1	0,2	5,80	8,00	15,30	1,30
'Switzerland'	1	0,3	3,30	4,50	9,60	0,90
'United Kingdom'	1	0,3	3,50	4,80	8,40	0,50
'Canada'	1	0,2	5,60	8,00	18,10	2,10
'United States'	1	0,2	5,20	7,60	14,80	0,30
'Australia'	1	0,2	5,80	8,80	18,10	2,40
'Albania'	2	0,3	6,70	9,00	16,20	41,30
'Bulgaria'	2	0,2	9,80	14,10	30,90	5,60
'Czechoslovakia'	2	0,2	10,00	14,50	30,60	3,30
'Hungary'	2	0,2	9,50	15,60	39,30	12,00
'Poland'	2	0,3	5,50	8,30	15,80	1,80
'Romania'	2	0,3	5,50	7,40	15,90	4,40
'Yugoslavia'	2	0,2	9,40	13,80	25,80	3,90
'USSR'	1	0,2	4,20	6,10	13,00	3,60
'Hong Kong'	1	0,2	10,70	16,40	39,40	5,10
'Japan'	1	0,2	4,50	5,90	10,40	3,50
'Malaysia'	1	0,2	11,50	16,70	33,40	19,10
'Barbados'	1	0,2	5,70	8,10	14,60	4,10
'Brazil'	1	0,2	4,50	6,60	12,80	4,80
'Cuba'	1	0,2	11,70	17,50	41,00	1,60
'Mexico'	1	0,2	5,40	7,40	14,20	1,00
'Israel'	1	0,2	5,40	9,20	15,30	2,20
'Qatar'	1	0,3	3,00	4,70	8,20	4,40
'Saudi Arabia'	2	0,2	7,00	10,00	21,50	3,30

Note: The number of restrictions refers to the number of tested parameters in the null hypothesis (3).

References

Andrews, W.K. (1993), "Tests for Parameter Instability and Structural Change with Unknown Change Point", *Econometrica*, vol. 61, No. 4, pp. 821-856.

Banerjee, A., M. Marcellino and I. Masten (2008), "Forecasting Macroeconomic Variables Using Diffusion Indexes in Short Samples with Structural Change", *European University Institute Working Papers*, Eco 2008, No. 17.

Chow, G.C. (1960), "Test of Equality between Sets of Coefficients in Two Linear Regressions", *Econometrica*, vol. 28, No. 3, pp. 591-605.

De Lucas, S., Álvarez, I. and Delgado, M.J. (2009), "Measuring international reference-cycles", Working Paper of Funcas, nº 4xx, pp. 1-30. (próxima aparición).

den Haan, W.J. and A. Levin (1997), "A Practitioner's Guide to Robust Covariance Matrix Estimation", in C.R. Rao and G.S. Maddala (eds.): *Handbook of Statistics*, vol. 15, pp. 291-341.

Del Hoyo, J. and J.L. Cendejas (2007), "Stability Analysis in ARMA and Unobserved Component Models", in G. L. Mazzi and G. Savio (eds.), *Growth and cycle in the Eurozone*, Palgrave Macmillan and EUROSTAT, pp. 291-302, New York.

Forni, M., M. Hallin, M. Lippi, and L. Reichlin (2000), "The Generalized Dynamic-Factor Model: Identification and Estimation", *The Review of Economics and Statistics*, vol. 82, No. 4, pp. 540–554.

Newey, W.K. and K.D. West (1987), "A Simple Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix", *Econometrica*, vol. 55, pp. 703–708.

Stock, J. H. and M. W. Watson (2002), "Forecasting Using Principal Components from a Large Number of Predictors", *Journal of the American Statistical Association*, vol. 97, No. 460, pp. 1167–79.

Stock, J. H. and M. W. Watson (2008), "Forecasting in Dynamic Factor Models Subject to Structural Instability", in J. Castle and N. Shephard (eds), *The Methodology and Practice of Econometrics, A Festschrift in Honor of Professor David F. Hendry*, Oxford University Press, Oxford.

White, H. (1980), "A Heteroskedasticity-Consistent Covariance Matrix Estimator And A Direct Test Of Heteroskedasticity", *Econometrica*, vol. 48, pp. 817-838.