Sources of Fluctuations in Output and Inflation in the U.S.A between 1960 and 2008: A Sign Restriction Approach

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Abstract

In this paper we identify a monetary policy shock, a demand shock and a supply shock in a three dimensional VAR. The shocks are jointly estimated by imposing sign restrictions on the impulse functions. Thus, we develop Uhlig (2005) work to the case of three shocks. The endogenous variables in the VAR are: inflation rate, output gap and federal funds rate. We use two measures for inflation rate: current inflation rate and expected inflation rate. Since the analyzed period is long, it could present structural changes. Thus, we test the stability of the VAR using Qu and Perron (2007) recent work. We use the forecast error variance decomposition and the historical decomposition to evaluate the effects of each shock on current and expected inflation rates, federal funds rate and output gap fluctuations. We find that a contractionary monetary policy is not the main source of fluctuations neither in current and expected inflation rate nor in output gap. These are mainly due to supply shock and demand shock. The forecast error variance decomposition gives more importance to supply shock in current and expected inflation rates variation. While, it gives more importance to demand shock in explaning output gap variation. The historical decomposition of the shocks allows us to identify in which sub-periods supply shock, demand shock and monetary shock are more important in explaning current inflation rate, expected inflation rate and output gap volatility.

Key Words: unit root, structural breaks in a system of regression, SVAR, monetary policy, fluctuation

JEL Classification: C12, C13, C32, E10, E32, E58

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1 Introduction

In this paper, we use a VAR with sign restrictions on the impulse responses to deliver results consistent with the conventional view of monetary policy, demand and supply shocks. The conventional view that contractionary monetary policy leads to an increase in interest rate and a decrease in prices and output, is used to identify monetary policy shock by imposing sign restrictions on the impulse responses. Thus, we consider that the federal funds rate reponse is positive during the first k months following the shock. The response of inflation rate and output gap are negative during these k months. We consider that a positive demand shock increases output and prices, while a positive supply shock increases output and decreases prices for the first k months following the shock. Thus, the conventional view is supported by construction and is not a result. This identification says nothing about whether the conventional wisdom is correct.

This method has been used to check the response of some economic variables to certain structural shocks. Uhlig (2005) used this method to identify the effect of a contractionary monetary policy shock on real GDP. He finds that the latter has no clear effect on real output. Mountford and Uhlig (2008) investigated the effect of fiscal policy changes on output responses, private consumption and private non-residential and residential investment. They identify three benchmark fiscal policy shocks: deficit-financed fiscal expansion, balanced budget fiscal expansion, revenue shock in which government revenues go up, but spending remains unchanged. Rafiq and Mallik (2008) examined the effect of monetary policy shocks on output in Germany, France and Italy. They conclude that monetary policy innovations play a modest role in generating output fluctuations. For more applied papers, see for example Vargas-Silva (2008), Peersman (2005) and Scholl and Uhlig (2008).

The impact of monetary policy, demand and supply shocks on output gap and inflation rates (current and expected), are jointly identified. The questions asked in this paper are: what are the effects of monetary policy on real economic activity? Thus, our objective is to determine whether monetary policy disturbances actually have played an important role in U.S economic fluctuations. Some empirical evidence suggests that, monetary policy is not the main driven force in output and prices fluctuations, but rather is adjusted to the state of the economy. Are fluctuations only caused by supply shock as neo-classical and real business cycle suggest? Or, are they due to demand shock as Keynesian economists say? The neo-classical theory explains output fluctuations, for example, by the forecast errors that agents do when forecasting the behavior of central banks or government policies. The real business cycle theory goes further and denies any role for money in explaining fluctuations. While the Keynesians consider that, only aggregate demand determines output and unemployment. Thus, any demand shock fine-tunes the economic activity.

The tools that have been employed to answer these questions, have evolved over time as the result of development in time series analysis. Some papers based their studies on the correlation between inflation and the growth rate of the money supply, or the correlation between either inflation or money growth and the growth rate of real output. Others used an identified VARs to estimate the effects of some structural shocks. Among the excellent discussions of these methods see Leeper, Sims and Zha (1996) and Christiano, Eichenbaum and Evans (1999). Canova and De Nicolo (2003) employed an identification approach that imposes sign restrictions on the cross-correlation functions of the aggregate variables' responses to particular shocks. In this way, they assign structural interpretation to orthogonal innovations. Here we impose sign restrictions on impulse responses of the orthogonal shocks. This approach is initiated by Uhlig (2005). We plot the mutually orthogonal sign-restricted impulse vectors, the forecast error variance decompositions as well as the historical decompositions, to evaluate the effect of each shock on the variables in the system.

Our contribution to the litterature is two-fold: we find that monetary disturbances are not the main source of variations in current inflation rate, expected inflation rate and output gap. It seems that demand and supply shocks cause output and inflation fluctuations, and Federal Reserve accomodates the federal funds rate to the state of the economy. We also determined, using the forecast error variance decomposition and the historical decomposition, in which periods each shock contributes the most to output gap, inflation rates and federal funds rate fluctuations.

This paper is organized as follows. The second section discusses the sign restriction methodology. In section 3, we test the stationarity of the variables and the stability of the VAR. In section 4, we expose the two models that we consider and the restrictions imposed. The estimation results of monetary, demand and supply shocks, are discussed in section 5. In this section we comment and discuss in details the impulse reponses and the forecast error decompositions obtained. Section 6 deals with the historical decomposition methodology and discusses the results obtained. The last section concludes.

2 Data analysis

Data is monthly frequency from January 1960 to December 2008 for current inflation, output gap and federal funds rate and from January 1978 to December 2008 for expected inflation rate. Data is available on the web site of the Fedetal Reserve Bank of Saint Louis. Federal funds rate (i_t), current inflation rate (π_t^c) and output gap $(\widetilde{y}_t)$ are seasonally adjusted. The graphs of these series are plot in appendix (B). Current inflation rate (π_t^c) is the CPI inflation over the previous twelve months.[1] Output gap $(\widetilde{y}_t)$ is defined as the deviation of the actual output from the potential output. Since GDP is a quarterly series, we measure the actual output by the industrial production index taken in log $(LIPI)$.[2] Potential output is unobserved and must be estimated. To measure the potential output Taylor (1993) simply used a linear trend of log real GDP, Clarida, Gali and Gertler (1998, 2000) used a quadratic trend. This method worked fairly well during the post-war period that was characterized by a relatively constant growth in output. When we have serial correlation and non-normality in the regression residuals, this leads to parameter instability. Nelson and Plosser (1982) found that growth in U.S. output and most of the macro-economic series in the U.S. economy were not adequately described as fluctuations around a deterministic trend, but rather as a random walk with drift. To account for this, Perron (1989) uses a segmented linear trend adding dummies for a slope shift and an intercept shift. He considered the break date known.

In this paper, we test the possibility of the presence of an endogenous break date in $LIPI$. To do so, we use Lee and Strazicich (2001) unit root test. This test allows one break under the

[1] $\frac{CPI_t - CPI_{(t-12)}}{CPI_{(t-12)}} \times 100$

[2] Following Bastien and Bec (2007), the logarithm of industrial production index is also multiplied by 100.

null hypothesis of stationarity and the alternative. We consider the two models proposed by Lee and Strazicich (2001). Model A only allow a change in the mean and model C allow a change in mean and the slope. To do this test we choose the minimum lags number that eliminates the residuals autocorrelation. Thus, we consider 11 lags. The test statistics are respectively -4.127 and -4.123 for model A and C. We reject the null hypothesis of unit root with one break only for model A. Thus, we conclude that Industrial Production Index taken in log is stationary with a break in the mean in November 1970. Hereafter, we measure output gap as the gap between Industrial Production Index taken in Log and a quadratic trend with a mean change in 1970:11.

Before going further we check the data order integration. For this purpose we perform Eliott, Rothenberg and Stock (1996) test, noted hereafter $\mathbf{ADF^{GLS}}$, test. The null hypothesis of this test is the presence of unit root and the alternative assumption is stationarity. In order to treat under the stationary alternative the relevant deterministic component we simply visualize the graphs. We consider that, in level, federal funds rate (i_t), inflation rates (π_t^c, π_t^e) and output gap ($\widetilde{y}_t$) present only a constant. In first difference we retain the same models. We choose the lags number that minimizes the Modified Akaike Information Critira proposed by Ng and Perron (2001). According to table (1), $\mathbf{ADF^{GLS}}$ test rejects the null hypothesis of unit root for π_t^c at 10% and for $\widetilde{y}_t$ and i_t at 5%.

Series	ADF^{GLS}	Series	ADF^{GLS}
	Level		First difference
π_t^c	-1.810***(15)	$\Delta\pi_t^c$	-2.839*(17)
i_t	-2.263**(13)	Δi_t	-4.523*(15)
π_t^e	-0.645(13)	$\Delta\pi_t^e$	-2.558*(13)
$\widetilde{y}_t$	-2.361**(5)	$\Delta\widetilde{y}_t$	-2.750*(5)

NB: *,** and *** present respectively the rejection of the null assumption at 1% 5% and 10%. Between brackets the lags number.

Table 1: $\mathbf{ADF^{GLS}}$ **test**

We difference the non stationary variables and we consider the two following VAR models. Thus we difference expected inflation rate (π_t^e). The endogenous variables for the first VAR are π_t^c, $\widetilde{y}_t$ and i_t. These variables are taken in level. We name this VAR model 1. The endogenous variables for the second one are $\Delta\pi_t^e$, $\widetilde{y}_t$ and i_t. Only π_t^e is taken in first difference. We name this VAR model 2. There is a strong reason to beleive that american monetary policy may present at least one structural change between 1960-2008.

Hereafter, we apply the new test of multiple structural changes in a system of regressions recently developped by Qu and Perron (2007). We use this test to determine whether there are any breaks in the mean or the dynamic structure of the VAR. This is basically an LR procedure, that tests the equality of regressors across subsamples. We suppose that the break date $T_m \in [T_a; T_b]$ such as $T_a = \lambda \times T$ and $T_b = (1-\lambda) \times T$ with $\lambda = 0.15$. We do not allow consectutive breaks that is why we impose $T_m - T_{m-1} \geq \epsilon T$ with $\epsilon = 0.15$. We consider that the break dates are simultaneous in the three equations. We choose the lags number that eliminates autocorrelation from the VAR's residuals, from order one to 6 at 1%. The lags number is calculated before considering break dates. For model 1, it is set to 5. The residuals of the three dimensional VAR with $(\Delta\pi_t^e, \widetilde{y}, i_t)$ are not autocorrelatd to order 6 only with 46 lags. We did Qu-Perron test for a VAR(5) and a VAR (46), we found the same break date 1980M1. We think that the stability test results are robust and not affected by residuals autocorrelation. For that reason, the high lags number does not seem meaningfull and hereafter we retain 5 lags. Critical values if the total number of coefficients that are subject to change is less than 10 can be found in Bai and Perron (1998, 2003*a*,*b*). In the pure structural change model 48 coefficients are allowed to change. In the two partial structural change models 3 and 45 coefficients are allowed to change. In order to compute critical values, we introduce some modifications to the Gauss code provided by Perron and Qu (2006). To compute the critical values for the $Seq(l+1|l)$ test we use theorem 6 of Qu and Perron (2007). The tests for a full structural change or a partial structural change, are reported in table (2). In the first part of this table we apply Qu-Perron test to model 1 where $(\pi^c, \widetilde{y}, i)$ are the regressors. In the second part we apply Qu-Perron test to model 2, where $(\Delta\pi^e, \widetilde{y}, i)$ are the regressors. As shown in table (2), wether we use current or expected inflation, we reject the null hypothesis of stability against the alternative of one break for the three models. Hence, for more than one break, we only consider the full structural model and we apply the $SeqLR$ test to determine the breaks number. From table (2) part one, we see that the sequential test statistic of the null of one break against the alternative of two breaks does not reject the null at the 5% level. The break point that maximizes the likelihood function is $1980M1$. This break date seems plausible, since it coincides with the second oil price shock and the changes in the Federal Reserve operating procedures conducted by Paul Volcker, Chairman of the Board of Governors of the Federal Reserve System at that time. If we replace current inflation by expected inflation we find only one break over the period 1978-2008. The $Seq(2/1)$ test does not reject the null of one break against the two-break alternative at the 5% level. The estimated break date is also $1980M1$.

3 Sign restriction approach

In this section we concentrate on the key steps for the implementation of the sign restrictions. Technical details and terminology are provided in appendix (A).
We consider the following VAR in reduced form:

$$Y_t = c + \sum_{i=1}^{p} B_i Y_{t-i} + u_t \qquad \text{for} \qquad t = 1, \ldots, T \tag{1}$$

where $Y_t = (\pi_t, \widetilde{y}_t, i_t)'$ are $(n \times 1)$ vectors of endogenous variables, p is the lag length of the VAR, B_i are $(n \times n)$ coefficient matrices and u_t is the one step ahead prediction error. The variance-covariance matrix is $E(u_t u_t') = \Sigma$. We consider two measures for inflation rate, current inflation rate π_t^c and expected inflation rate π_t^e. $\widetilde{y}_t$ is output gap and i_t is the federal funds rate. The endogenous variables for model 1 are π_t^c, $\widetilde{y}_t$ and i_t. Those for model 2 are $\Delta\pi_t^e, \widetilde{y}_t$ and i_t.
Let $\widetilde{A}$ be the Σ Cholesky factor or any factorization of that form

$$\widetilde{A}\widetilde{A'} = \Sigma \tag{2}$$

Note that we are interested in the response to a monetary policy shock, a demand shock and a supply shock. There is therefore n fundamental innovations to identify. The idea consist in

	H_0^a	H_0^b	H_0^c				
m	$SupLR(m\|0)$	$SupLR(m\|0)$	$SupLR(m\|0)$	$\hat{T}_1$	$\hat{T}_2$	$\hat{T}_3$	$SeqLR(m+1\|m)$
part one: $Y = (\pi^c, \widetilde{y}, i)'$							
1	401.423*(105.416)	17.922*(13.98)	376.609*(100.30)	$1980M1$	-	-	142.029*(197.230)
2	-	-	-	$1980M1$	$1970M1$	-	
part two: $Y = (\Delta\pi^e, \widetilde{y}, i)'$							
1	299.363*(105.416)	35.019*(13.98)	297.906*(100.30)	$1980M1$	-	-	129.111*(197.230)
2	-	-	-	$1980M1$	$1990M6$	-	-

Note: * denotes rejection of the null at 5% level. Between brakets critical values at 5%. Respectively for $i = \{a, b, c\}$, H_0^i is the null hypothesis of structural changes in all the regressors, only the constant or only the lags.

Table 2: **Estimated break dates, $\mathbf{SupLR(m|0)}$ and $\mathbf{SeqLR(m+1|m)}$ tests**

finding the innovation corresponding to each shock. This amounts to identifying three column vectors $a \in \mathbb{R}^n$ of the matrix $\widetilde{A}$ in equation (2). Uhlig called the vector a an impulse vector. Any *impulse vector*, a^j, can be written as

$$a^j = \widetilde{A} q^j \tag{3}$$

q^j are the identifying weights which are to be determined such as $q^j = (q_1^j, q_2^j, \ldots, q_n^j)'$ have a length equal to one, $||q^j|| = 1$ and orthogonal to any other vector of matrix Q, $q^{j'}$. Given an impulse vector a, it is easy to calculate the appropriate impulse response.
Let $r_a(k)$ be the n-dimensional impulse response at horizon k to the impulse vector a^j. This can be written as:

$$r_a(k) = \sum_{i=1}^{n} q_i^j r_i(k) \tag{4}$$

see details in appendix (A). $r_i(k) \in \mathbb{R}^n$ is the vector response at horizon k to the *i-th* shock in the Cholesky decomposition represented in equation (2). The identification method searches over the space of possible impulse vectors to find those impulse responses that agree with the sign restrictions.
To simplify the exposition of the method, we consider for the moment just one shock; a monetary policy shock. The sign restrictions are imposed such that the responses of federal funds rate are positive, and the response of inflation rates and output gap are negative for short time. This means that the impulse response of output gap and inflation rate, are restricted to be negative for the first k months following a positive shock on federal funds rate. We suppose that $k = 5$. At long term, the effect of a contractionary monetary policy shock must be close to zero for output gap, current inflation rate and federal funds rate. Because expected inflation rate is not stationary in level, the shock will not dissipate at long term. The problem now consists in selecting the set of appropriate *impulse vectors*, a^j, that satisfies the above assumptions. Two methods have been used in the litterature. Sims and Zha (1998, 1999) and Uhlig (2005) considered a Bayesian approach. Uhlig (2005) considered also a simple "brut force" method. This agnostic identification is called the pure-sign restriction approach. Here, we opt for the latter. We follow the following steps:
First we compute the Cholesky decomposition of Σ using equation (2).
Second, we draw randomly an n-dimensional $\widetilde{q}^1$ vector from a standard normal distribution. To obtain a candidate draw for q^1, we divide $\widetilde{q}^1$ by its length. Hence, q^1 is a random vector with length equal to one. We multiply q^1 by $\widetilde{A}$ to obtain the *impulse vector*, a^1. The aim is to impose a set of inequality constraints on vector a^1; in such a way that a contractionary shock neither it leads to an increase in inflation rate and output gap nor it decreases federal funds rate. a^1 contains the contemporaneous responses of the endogenous variables to the primary shock. We change the sign of entries that violate the restrictions on the impulse responses for all relevant horizons, $k = \{0, \ldots, 5\}$. We generate 500000 candidate draws for a^1. The methodology checks wether $a^1 \in F(\hat{B}, \hat{\Sigma}, k)$, by considering the appropriate sign restrictions on the monetary impulse responses for all relevant intervals k. $\hat{B}$ and $\hat{\Sigma}$ are respectively the OLS estimators of the coefficients matrix and the variance-covariance matrix from the VAR. If restrictions are satisfied we keep vector a^1, if not we drop it. The set F represents an interval for the impulse responses to be calculated. Once the impulse vector is identified, impulse response functions corresponding to the monetary policy shock, are calculated using equation (4). These impulses should not be positive for the inflation rate and output gap and, nor negative for the federal

funds rate to horizon 6. We store and we plot the maximum, minimum and the mean of potential impulse responses for the a^1 that satisfies the restrictions. We also plot the mean and the bounds for 84% quantile and 16% quantile. We must note that this is a consistent, although slightly biased estimate of the confidence interval for impulse response. Bayesian estimation may provide a convenient framework to resolve this bias. This is beyond the purpose of this paper. Expected inflation rate is taken in first difference in the VAR, while sign restrictions are imposed on level. For this reason, we calculate for each draw the *accumulated responses* for current and expected inflation rate.

In our model, we jointly estimate three shocks. The computation of the second and the third shock is slightly different from the computation of the first one. Hereafter we only detail how to choose vectors a^2 and a^3 satisfying restrictions imposed in the case of a demand shock and a supply shock respectively.

The first step consists in computing a factorization of the covariance matrix Σ which controls a block of columns in the factorization itself. To do so we provide an $(n \times r)$ matrix A, where the columns of A are the vectors a^j, for $j \in \{1, \ldots, r\}$ and $r = n - 1$, already identified. $r = 1$ when we identify two shocks and $r = 2$ when we identify three shocks. Then we compute an $(r \times r)$ matrix Π so that Π is upper triangular and $A \times \Pi$ makes up the first r columns in every matrix F that factors Σ. Note that for every r we have a different matrix F. Now that we identified the monetary shock, we would like to identify the second shock. So we have two shocks jointly estimated. In this case $r = 1$ which mean that the first column of F is a scale multiple of A.

The second step consists in drawing randomly an $(n - 1)$-dimensional vector q^2 with a length equal to one. To compute the second impulse vector a^2 we use equation (3). We multiply an $(n \times n - 1)$ sub-matrix of F by vector q^2. The columns of the sub-matrix are the second and the third columns of matrix F. Once a^1 and a^2 are computed, we would like to compute a^3. For the three shocks jointly estimated $r = 2$. Because of the structure of Π, the first column of F is a scale multiple of the first column in A, and the second column in F will be a linear combination of the first two columns of A. Now that we identified F, we apply the second step. We draw randomly an $(n - 2)$-dimensional vector q^3 with a length equal to one. Such as in equation (3), we multiply an $(n \times n - 2)$ sub-matrix of F by q^3. Here the sub-matrix of F, is a vector column of the third column of F and q^3 is a scalar.

4 The VAR and identifying restrictions

The stability analysis indicates that there is one break in January 1980 in the propagation mechanism for the two VAR models. Thus, we estimate the VARs over the different sub-periods. The lags number for each VAR, is the smallest one that eliminates autocorrelation from the residuals to order 6. We retain 3 lags for model 1 estimated over the following periods: 1960M1-1980M1, 1980M2-2008M12. We estimate model 2 over the period 1980M1-2008M12. We need 46 lags to eliminate residuals autocorrelation. This is not meaningful. To resolve this problem we eliminate some observations and we reduce the period to 1982M10-2008M12. For this period, the residulas of the VAR(5) are not autocorrelated. An overview for our identifying restrictions on impulse responses is provided in table (3). A "+" sign means that the impulse response of the variable in question is restricted to be positive for the first six

Shocks	$\widetilde{y}_t$	π_t^c	i_t	Shocks	$\widetilde{y}_t$	π_t^e	i_t
Monetary shock	-	-	+	Monetary shock	-	-	+
Demand shock	+	+		Demand shock	+	+	
Supply shock	+	-		Supply shock	+	-	

NB: The first part of this table contains the sign restrictions on the impulse responses for the first VAR's variables. The second part contains the sign restrictions on the impulse responses for the second VAR's variables.

Table 3: **Identifying sign restrictions**

months following the shock. A "-" sign indicates that the impulse reponse is restricted to be negative for the first six months. A blank entry indicates, that no restrictions have been imposed. The restrictions we employ are widely agreed by macroeconomic theory and shared by a number of empirical models.

Monetary policy shock: A contractionary monetary policy should raise the federal funds rate and lower prices. The identification scheme that we impose, accomplishes this. Thus, by construction we avoid the price puzzle. Some empirical results such as Uhlig (2005) suggest that there is little evidence that output will fall in reaction to a contractionary monetary policy shock. Here we do not focus on this. Since we are more interested in the sources of fluctuation movements in economic activity, we consider that conventional theoretical wisdom is true. We therefore impose that output gap fall following an increase in federal funds rate.

We use the Aggregate Demand-Aggregate Supply model to identify how prices and output respond to demand and supply shock.

Demand shock: A real demand shock (e.g. increases in government expenditures or open market operations) should generate a positive transitory responses in output and prices. To start, consider that aggregate supply do not move. When prices increase, the nominal demand money should increase too. Since money supply is fixed, interest rate should increase to incite consumer to reduce their money demand and thus, restore the equilibrium. Yet an increase in interest rate lowers aggregate demand which involves a decrease in the aggregate supply.

Supply shock: A temporary supply shock (e.g. a shock to labor supply or a technology shock) should generate positive transitory output responses and negative transitory responses in current inflation rate or even expected inflation rate. To illustrate the mechanism, consider that the determinants of the aggregate demand are constant. This means that aggregate demand does not move. At short term, aggregate output can be above or below his natural level. In this case, supply shock induces modifications in output and prices level until output converge to his natural level. If for instance output is below its natural level at time t, this means that current prices are lower than expected prices at time t. This involes a fall in expected prices at time $t+1$ which induces an increase in real money stock. An increase in the latter at time $t+1$ pushes interest rate downward. A fall in interest rate increases aggregate demand and aggregate supply at time $t+1$. The mechanism continues until output converges to its natural level.

5 The results

In the previous section we described the sign restrictions we impose on the impulse response functions, for the three shocks jointly identified. Here we analyse the impulse response function and the forecast error variance decomposition, that we obtain for each shock. Before describing the results in details, we would like to stress on the fact that the conventional view of the effects of monetary policy shock, demand shock and supply shock, is supported by construction. The approach applied in this way says nothing about whether the conventional wisdom is correct. As we mentioned in the introduction, in this paper we favor the conventional theoretical view. We consider that theoretical wisdom is applied and we look forward to see to what extent the three shocks explain the fluctuations in output gap and the two inflation rates.

5.1 Dynamic effect of monetary, demand and supply disturbances

The following graphs in figures (1) to (3) show the impulse responses for the monetary policy shock, the demand shock and the supply shock. The impulse reponses have been restricted to be of the appropriate sign described in table (3) for the first six months following the shock. By construction, puzzles in prices and output gap are avoided. We plot the mean and the bounds for 84% quantile and 16% quantile. As noted by Uhlig (2005) if the distribution was normal, these quantiles would correspond to a one standard deviation band. Many authors prefer a two standard deviations band but here, we follow the suggestions of Uhlig (2005). Note that a one standard deviation band is also popular in this kind of empirical litterature. The impulse response functions presented hereafter look reasonable.

Figures (1) and (2) show the impulse responses for the monetary policy shock, the demand shock and the supply shock. Those are computed from the first model with current inflation rate, output gap and federal funds rate as endogenous variables. After a contractionary monetary shock, output gap decreases at short term then increases with time until it converges to zero at long term. Inflation rate decreases after a rise in federal funds rate and converges slowly to zero at long term. These two impulse response functions have a U shaped form. Following a supply shock, output gap increases and inflation rate decreases. When we identified a supply shock, we did not impose any sign restriction on federal funds rate impulse. The federal funds rate response decreases instantaneously then increases with time. For the period pre-1980, after one year federal funds rate becomes positive and stays positive across all horizons until it converges to zero at long term. While for the period post-1980, federal funds rate become quickly positive after a couple of months then decreases and becomes negative. The rapid sign reversion in this period, suggests that the Fed made a mistake and than tried to catch up. Before 1980, inflation is thought to be caused by cost-push pressure. Federal Reserve and the US government were more concerned by unemployment and economic expansion. Our results show that Federal Reserve responds cyclically to aggregate supply shock. If we suppose that during this period the Fed places more weight on output target than inflation target, the decrease in federal funds rate could be explained by the Fed's will to boost demand to meet aggregate supply. Easing monetary policy will increase prices and Federal Reserve will be later obliged to tighten monetary policy to reduce aggregate demand. After 1980, during Vocker's and Greenspan's chairmanship, the Fed's first objective was inflation rate and expected inflationary pressures.

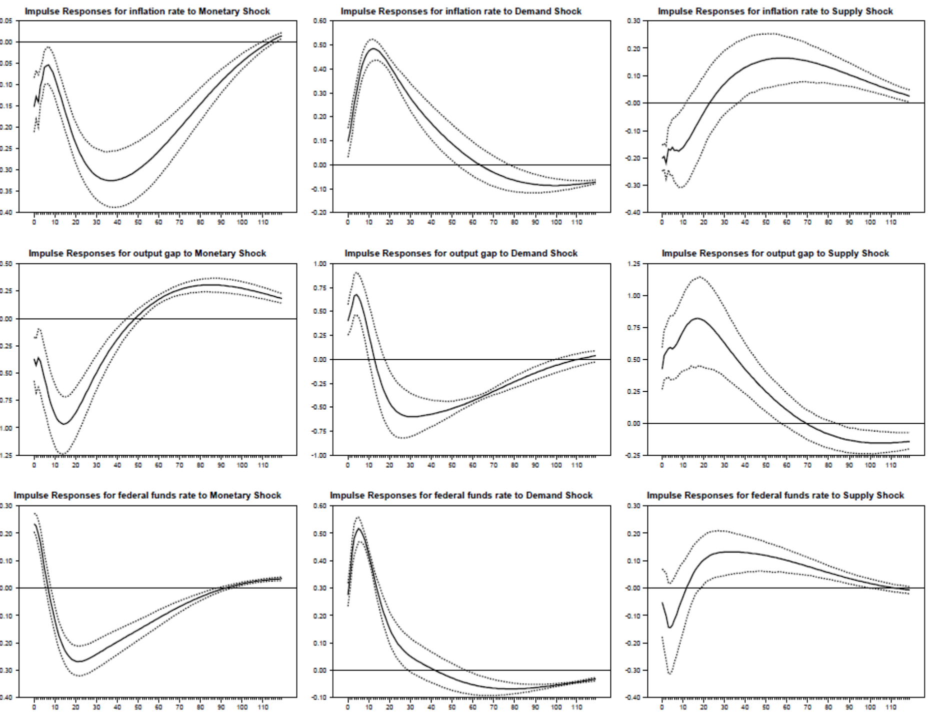


Figure 1: **Impulse Responses from model 1: 1960M1 to 1980M1**

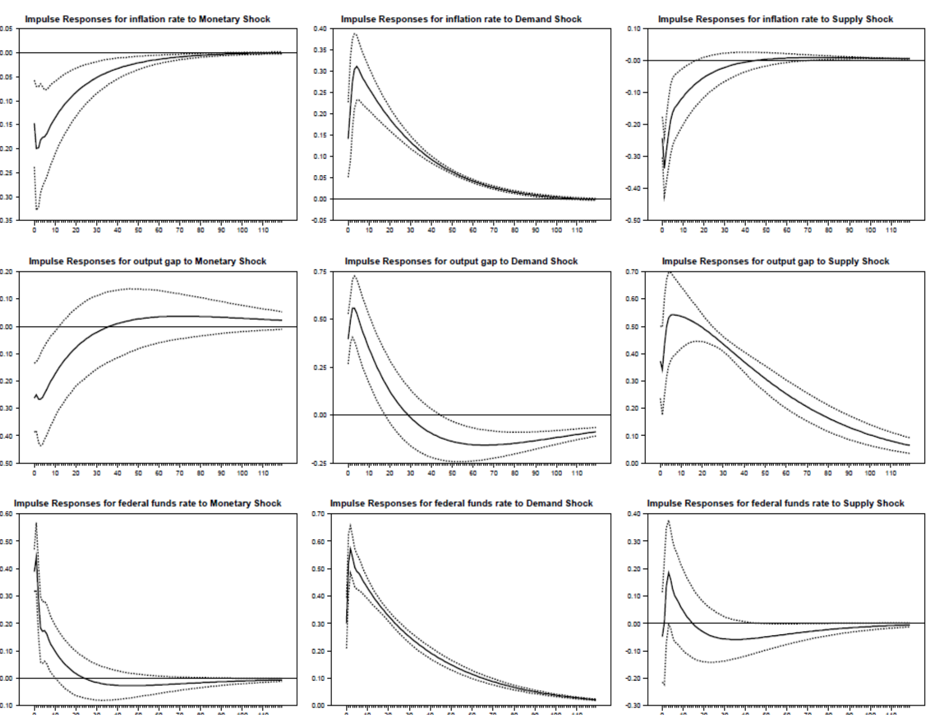


Figure 2: **Impulse Responses from model 1: 1980M2 to 2008M12**

Aggregate supply shock reduces inflation rate, which induces the Fed to stimulate the economy by increasing the Fed's funds rate. As for the supply shock, we do not impose any sign restriction on the impulse of the federal funds rate following a demand shock. We see that after a demand shock output gap increases at short term. It becomes slightly negative at long term and then converges to zero. Inflation rate increases at short term and becomes positive then decreases. It converges to zero at long term. Following the increases in prices, Federal Reserve has two choices. Either it accepts the higher inflation rate, but in this case national revenue is weaker than the one that could be attended if prices don't change or it responds by increasing interest rate to reduce demand. The latter is the only one that is tenable for a long run objective. It seems here, that the Fed leans against the wind in response to a positive aggregate demand shock. That is, the model predicts that the Fed responds countercyclically to demand shock. Thus, any increase in aggregate demand can only have a temporary effect since this shock creates inflation which will be controlled by a fall in the demand. The responses following a demand and a supply shocks fit well the scheme described in section (4). The Fed responds more vigourosly to demand shock than to supply shock.

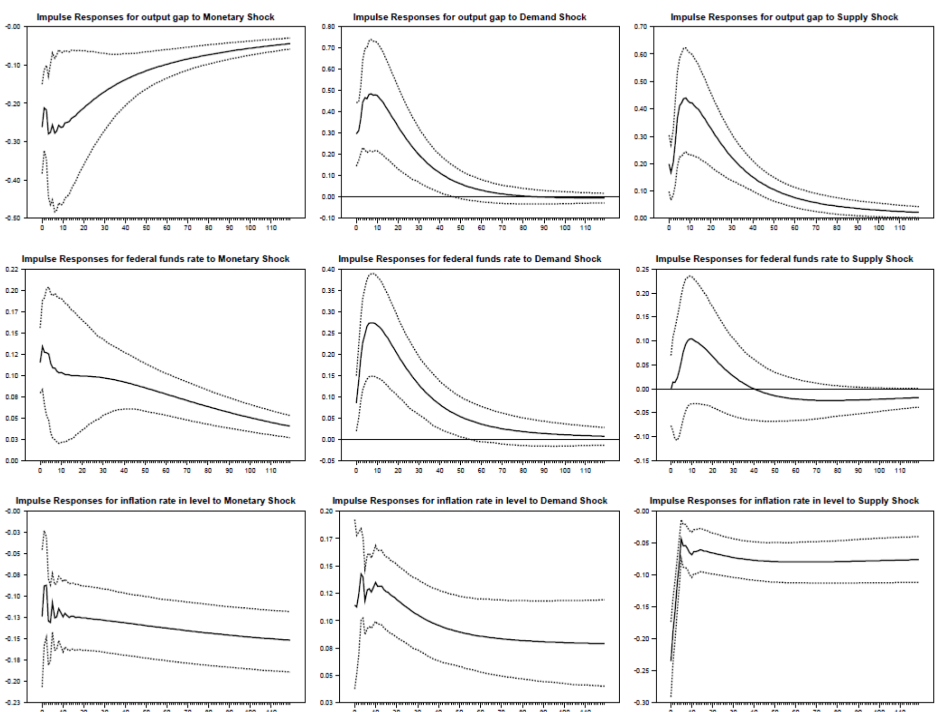


Figure 3: **Impulse Responses from model 2: 1982M10 to 2008M12**

Figure (3) shows the impulse responses for the monetary policy shock, the demand shock and the supply shock. Those are computed from model 2 with expected inflation rate, output gap and federal funds rate as endogenous variables. Note that expected inflation rate is taken in first difference in the VAR but we plot the accumulated effect on the level of inflation rate. The effect of a shock at time t on the level of inflation rate at time $(t+s)$, is the cumulative sum of the shocks over the period on the first difference of inflation rate. Since inflation rate is not stationary, the impact on inflation level is persistent and does not vanish with time, while the impact on the first difference of inflation rate is mean reverting and tend to zero at long term. When we estimate the second VAR model, replacing current inflation rate by expected inflation rate, we obtain the same shape for the impulse responses regarding contractionary monetary shock, demand shock and supply shock as the one described above. No new comments need to be added.

5.2 The explanatory power of monetary, demand and supply disturbances

Hereafter, we try to evaluate the contributions of monetary shock, demand shock and supply shock to output gap and inflation rates (current and expected) fluctuations. For this reason we calculate for each sub-period the forecast error variance decomposition of the variables. The results from the model 1, estimated over 1960M1-1980M1 and 1980M2-2008M12, are reported in figures (4) and (5). These figures displays three important features.

First, it seems that a small portion of output gap variability is due to monetary disturbances in the second sub-period. Only 15% to 25% of the fluctuations in output gap is explained by a monetary shock. While for the first sub-period, monetary disturbance accounts for a larger portion of output gap variability. It explains between 20% and 35% of the variance of output gap. This result is not as odd. The empirical studies investigating the contribution of monetary policy shocks to output fluctuations, do not lead to the same conclusions. Some studies like Leeper et al. (1996), Kim (1999) and Uhlig (2005) found that the contribution of monetary policy shock to output fluctuations is negligible. While others like Canova and De Nicolo (2003) concludes for a larger role of monetary shocks. When we split the full sample into two stable sub-samples, our results suggest that federal funds rate may not be the principal cause of fluctuations in output gap between February 1980 and Decembre 2008. Thus, it seems that monetary shock may not be the principal source of recessions and recoveries during this period. But monetary shock accounts for the third of the variation in output gap between January 1960 and January 1980. During this sub-period, federal funds rate played a much more important role in explaining fluctuations in output gap. It seems that most of the output gap variations in the fist sub-period is due to demand shock, while in the second sub-period it is caused by demand and supply shock.

Second, at all horizons, most movements in federal funds rate for the first sub-period are explained by a demand shock. Supply shock has a less important effect on the variability of the interest rate. At short term, supply shock has a negligible effect on federal funds rate variations. It increases with time and explaines around 30% of the variations in Fed's funds rate at long term. Thus, we can say that movements in federal funds rate are responses to the state of the economy. This may suggest that some exogenous factors generate inflation and Federal Reserve only allows interest rate to adjust. In the first sub-period, around 55% and 40% of the variations in federal funds rate are due to demand shock. In the second sub-period, between 30% and 40% of the variability in the variance of the federal funds rate is due to monetary shock. Nevertheless, supply shock has a non negligible effect on interest rate fluctuations at long term. 25% of the fluctuations in federal funds rate is explained by a supply shock at long term for the two sub-periods.

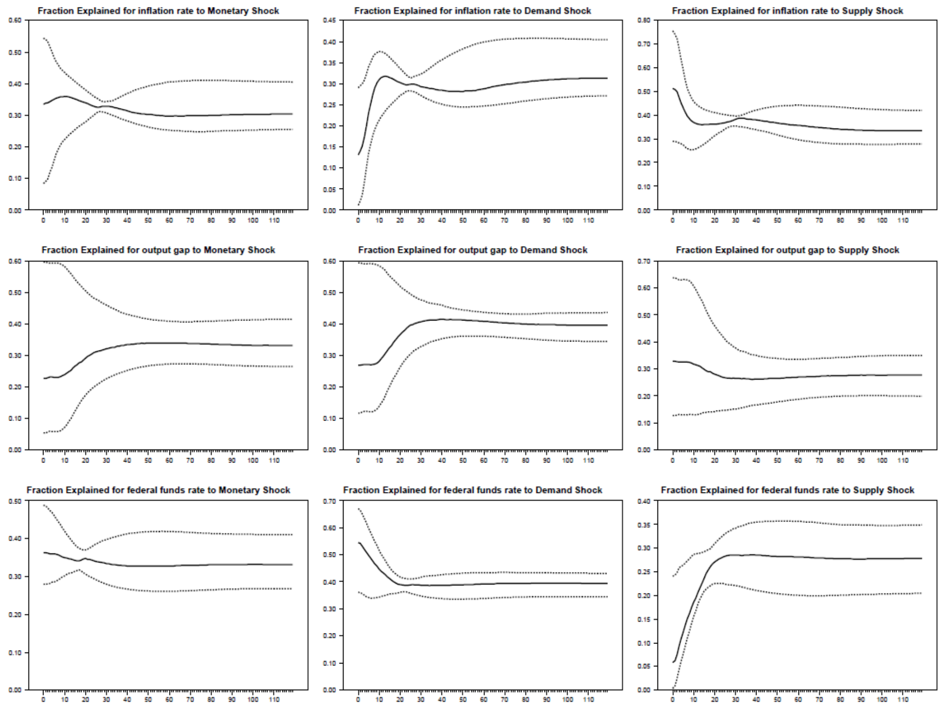


Figure 4: **FEVD from model 1: 1960M1 to 1980M1**

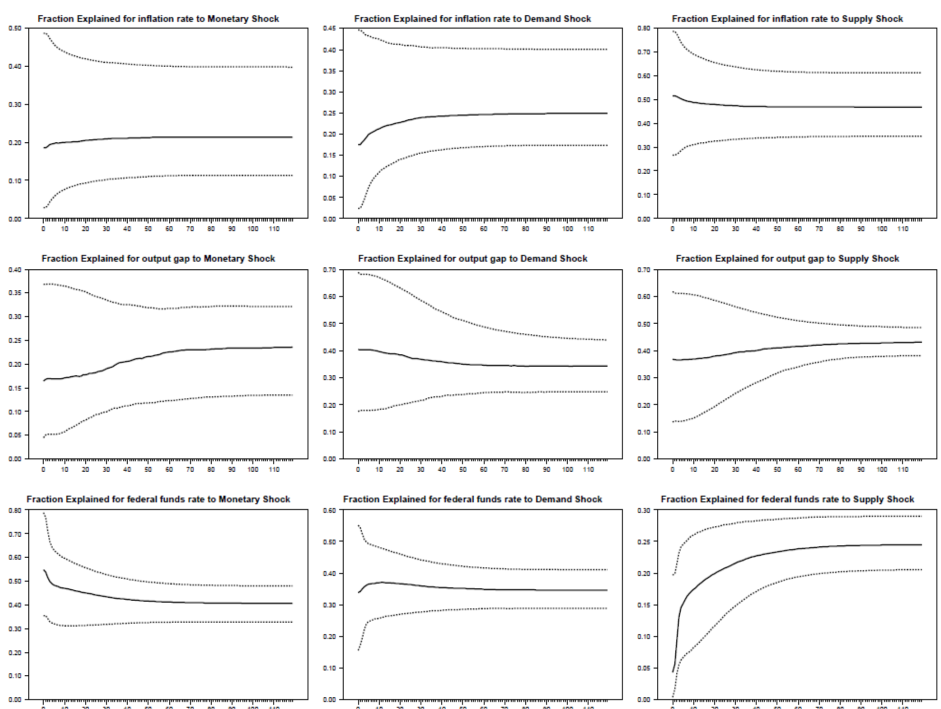


Figure 5: **FEVD from model 1: 1980M2 to 2008M12**

Third, in the tow sub-periods fluctuations in the variance of current inflation rate, are rather caused by a supply shock than a demand shock. The contribution of supply disturbances to inflation variability is between 60% and 35% for the first sub-period and between 50% and 40% for the second sub-period. At long horizon, monetary shock has also a non negligible effect on current inflation rate fluctuations. It explains for the first sub-period around 30% of the variability in current inflation rate. Its magnitude is less important in the second sub-period, 20% of the variations in current inflation rate is due to federal funds rate disturbances. This suggest that monetary policy in USA was not so bad conducted.

The results from the second VAR model, with $\Delta\pi_t^e$, $\widetilde{y}_t$ and i_t as endogenous variables, estimated over 1982M10-2008M12 are reported in figure (6). From the forecast error variance decomposition we can report three essential points.

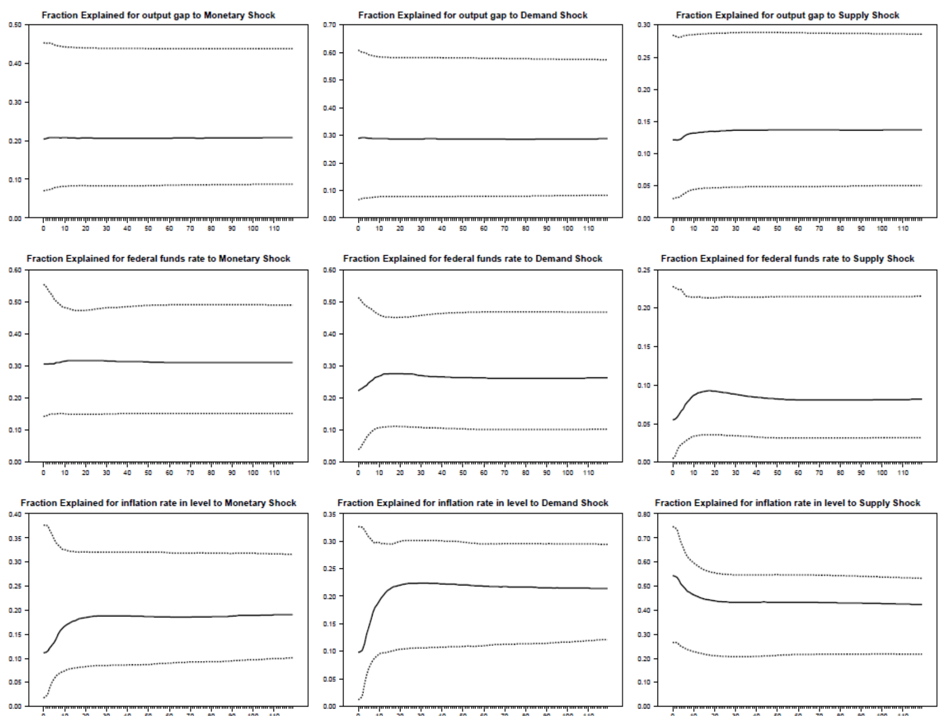


Figure 6: **FEVD from model 2: 1982M10 to 2008M12**

First, we clearly see that most of the fluctuations in expected inflation rate are caused by a supply shock. The latter explains between 55% and 40% of the variability in the variance of expected inflation rate.

Second, third of the federal funds rate variations is explained by a monetary shock. As we found earlier, demand shock has a more important effect on federal funds rate variability than supply shock. The earlier explains around 30% of the interest rate variations, while the later explains 10%.

Third, once again monetary policy has a small real effect on output gap fluctuations. It explains 20% of output gap variations. In this sub-period, around 30% of the fluctuations in output gap are due to demand shock. 15% are due to supply shock

The overall effect of a shock could be small in terms of volatility, while it might be a dominant source in some sub-periods in terms of levels. As pointed out by Kim (1999), if it is the case, the impulse responses or forecast error variance decompostion may be misleading. With the historical decomposition, we can analyze the effect of the shock on a specified variable in a specific period. This methodology has been developed by Sims (1980) and applied, among others, by Choueiri and Kaminsky (1999), Kim (1999), Andrade and Divino (2001), Andrade and Divino (2005), Barnett and Straub (2008) and Dungey and Fry (2009). Thus, in order to evaluate the relative importance of each shock on output gap, federal funds rate, current inflation rate and expected inflation rate over time, we move away from impulse response functions and variance decomposition to look into their historical decomposition.

6 Historical decomposition

6.1 Methodology

Hereafter we highlight the historical decomposition techinique.
The idea is best understood by considering the moving average representation of a structural model. Consider the general model represented in equation (17) in appendix (7):

$$Y_t = C_\mu(L)\mu_t + C(L)\widetilde{A}Qv_t \tag{5}$$

where the vector Y_t represents the endogenous variables π_t^c, $\widetilde{y}_t$ and i_t for the first model and $\Delta\pi_t^e$, $\widetilde{y}_t$ and i_t for the second model. The vector μ contains the deterministic part of the model, a constant, with the term $C_\mu(L)$ representing a polynomial matrix giving the effects of μ on the variables in Y. The vector v_t contains the structural shocks. Finally the matrix $C(L)\widetilde{A}Q$ contains the estimated impulse response functions. Equation (5) states that the dynamics of the endogenous variables, can be expressed as the sum of the deterministic and the stochastic component of the model. For a particular period $(t + j)$, equation (5) can be written as:

$$Y_{t+j} = (\mu + \sum_{s=0}^{j-1} C_s\widetilde{A}Qv_{t+j-s}) + \sum_{s=j}^{\infty} C_s\widetilde{A}Qv_{t+j-s} \tag{6}$$

with $C\widetilde{A}Q$ denoting the impulse responses to a structural innovation. Equation (6) represents the historical decomposition of the variables in the vector Y. It is apparent from (6) that the variable Y_{t+j} is the summation of two terms. The term on the far right contains the information that is available at time t. Based on this information the expectation of Y_{t+j} can be computed. This is the so-called "base projection" of Y_{t+j}. However the base projection is unlikely to coincide with Y_{t+j}, because in the time period from $(t+1)$ to $(t+j)$ new structural innovations hit the system. By their very nature, these shocks are unexpected; hence the first term on the right hand side can be interpreted as the forecast error of Y_{t+j}. It represents the part of Y_{t+j} that is due to innovations in periods $(t + 1)$ to $(t + j)$. The historical decomposition is based on this part of the system, thereby allowing one to attribute the unexpected variation of Y_{t+j} to individual innovations hitting the economy, which is useful for exploring the sources of fluctuations. Thus, the main idea is to decompose the forecast error in terms of the structural shocks arising from each of the three variables underlying the VAR.

We can see, for instance, whether movements in certain variable of the vector Y at date t were the results of innvovations in the other variables of the vector Y a year earlier. Therefore the actual level of Y is given by the sum of the baseline projections made at the beginning of the period and the effect of shock that hit Y thereafter.

The historical decomposition is computed by replacing the unknown parameters by their estimated values. The "structural shocks" are computed from equation (15) in appendix (7) by taking the inverse of $\widetilde{A}Q$ to get

$$v_t = (\widetilde{A}Q)^{-1}u_t \tag{7}$$

where u_t are the residuals obtained from the estimated VAR. $Y = (\pi_t, \widetilde{y}_t, i_t)'$ with π_t either current inflation rate (π_t^c) or expected inflation rate in first difference ($\Delta\pi_t^e$). Thus, the above decomposition will compute the historical decomposition of $\Delta\pi_t^e$. To express the historical decomposition in terms of the levels of expected inflation rate (π_t^e), we just cumulate (6).

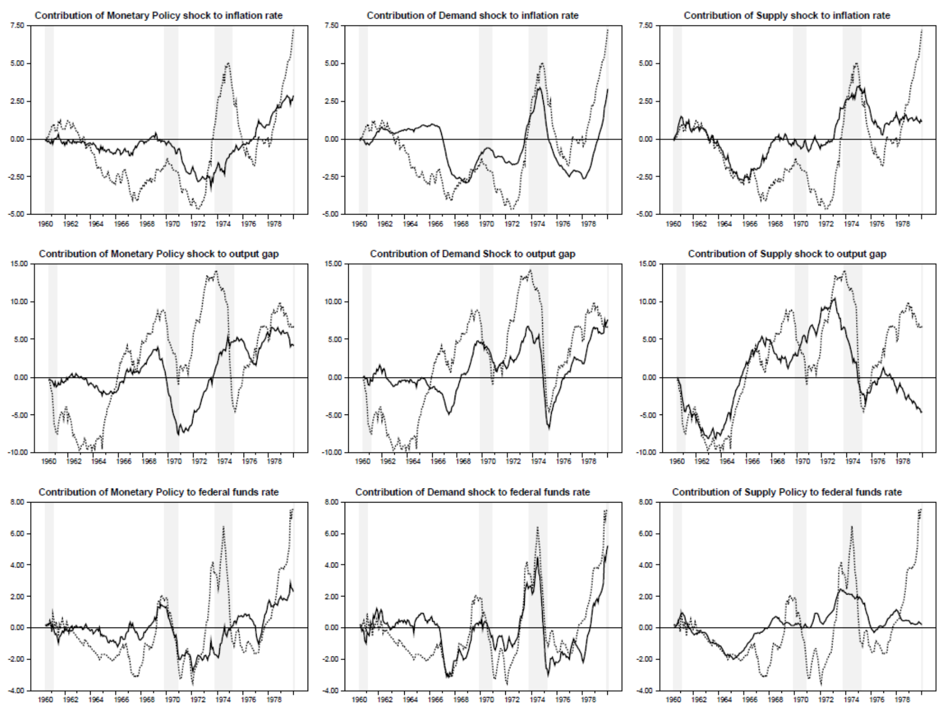
The contribution of the shocks is defined as the difference between the base projection and the projection that includes the associated shocks. To have an idea to what extent and during which periods each shock contributes the most to the fluctuations of federal funds rate, output gap and inflation rates, we simply compare the contribution of each shock to the total effect of all the shocks.

6.2 Results

Figures (7) to (9) show the historical decomposition of the two VARs model variables, for the three sub-periods. The shaded areas correspond to the recession periods as defined by the NBER. Each recession period should begin with a peak and end with a trough. Historical decomposition shows the accumulated effect of current and past shocks. The dotted line shows the total effects of all the shocks, this is given by the forecast error of the dynamic forecast from the start of the sample period. The solid line shows the contributions of each shock. The importance of the contribution of one shock to the total effects of all the shocks is determined by the extend to which it contributes to close the gap between the dotted line and the solid line. If the contribution of each shock follows the trend and the shape of the total effects; we say that the former plays a role in explaining the variable fluctuations. These figures confirm some of the variance decomposition analysis results. In addition to this, they show us in which specific period each shock contributes the most to the fluctuations in current inflation rate, expected inflation rate, output gap and federal funds rate.

Figure (7) shows the historical decomposition from January 1960 to January 1980. We find that demand shock is the most important factor in explaining output gap fluctuations between 1970-1980. It is significant but less important between 1966-1970. We can see that after 1970 the peak and the trough of the demand shock match closely the recession and expansion periods. Thus, the latter are obviously due to demand shock. The results are quite loose for the first expansion period. If demand or supply shock were causing business cycles, then they should be upward sloping in this expansion period. Or, this is not the case. Between 1960-1967 most of output gap fluctuations are caused by supply shock. Demand shock plays also an important role in the explanation of the federal funds rate variations especially after

1967. Before this date, federal funds rate fluctuations are mostly explained by supply shock. Supply shock proved to play relatively very important role in explaining current inflation rate fluctuations between 1960-1967. After 1967 till 1980, demand shock becomes more relevant in explaining current inflation rate fluctuation.

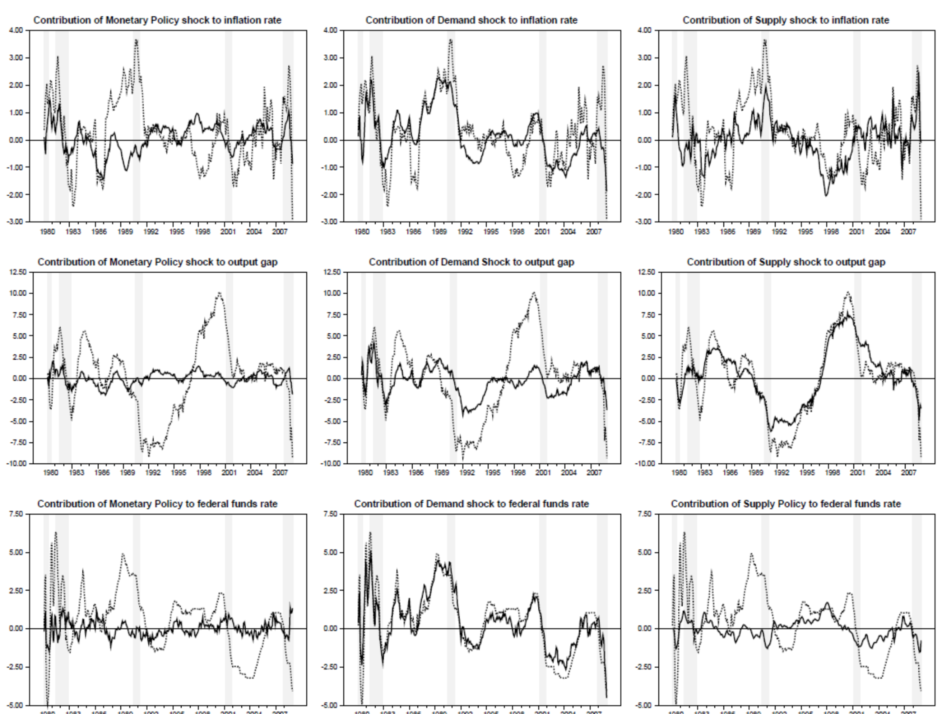


Note: The shaded areas correspond to the recession periods as defined by the NBER. The dotted line shows the total effects of all the shocks. The solid line shows the contributions of each shock.

Figure 7: **HD from model 1: 1960M1 to 1980M1**

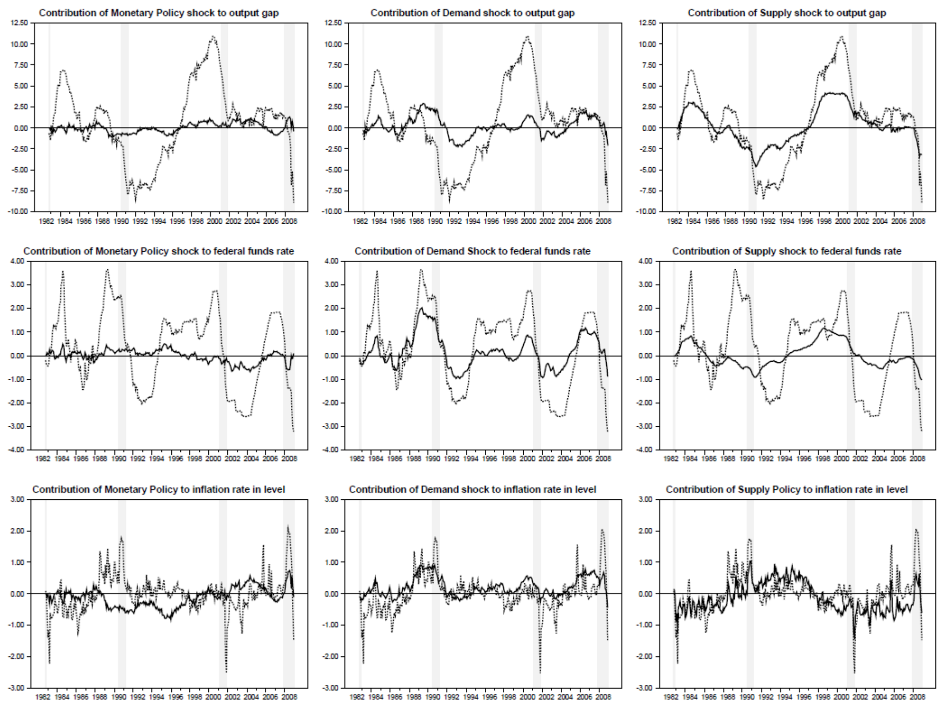
The historical decomposition of the second period covering February 1980 up to December 2008 is presented in figure (8). Between 1980-1988 and after 2006 most of output gap fluctuations are due to demand shock. This is reasonable for the following reason. In 1979, the fed's funds rate averaged 11.2% was raised by Volcker to a peak of 20% in June 1981. The prime rate rose also to 21.5% for the same year. This implied capital flow to the USA which induced the dollar appreciation. The results of Volcker's economic policy was a decrease in U.S. exportation. Thus, international demand for american goods turned down and balance trade becomes negative. These changes contributed to the significant recession the U.S. economy experienced in early 1980s, which included the highest unemployment levels ever encountered. Supply shock plays a less important role in explaining the recessions and expansion before 1988. The peaks and the troughs of demand component correspond well to the beginning and the end of these recessions and expansion. In order to reduce the U.S. current account deficit, and to help the U.S. economy to emerge from the previous described recession the government of France, West Germany, Japan, the United States, and the United Kingdom, signed the Plaza Agreement. The objective is to depreciate the U.S. dollar in relation to the Japanese yen and

German Deutsche Mark by intervening in currency markets. The dollar devaluation made U.S. exports cheaper and more competitive. This boosted demand for american goods and services. Thus the peak and trough of the demand component match the beginning and the end of the expansion period between 1983 and 1990. After 1988, fluctuations in output gap are mostly explained by supply shock. The peaks and the troughs of the supply component correspond well to the beginning and the end of the expansion period between 1990-2000. Demand shock during this period is also upward sloping but less steep. During the 1990s, the GDP rose by 69%, and the stock market as measured by the S&P 500 grew more than three-fold. From 1994 to 2000 real output increased, inflation was manageable and unemployment dropped to below 5%, resulting in a soaring stock market known as the *Dot-com* boom. The second half of the 1990s was characterized by the emergence of High-Tech and 'dot-com' companies. The recession period that begun in 2001 is often blamed on September 11 attack. During this recession, demand component decreases. It increases in the following recovery period. It then decreases in the last recession known as the subprime mortgage crisis. The peak and trough of demand component match closely the beginning and the end of the last two recessions. For the whole period, demand shock explains most of the variations in federal funds rate. Before 1992 and after 2001, most of current inflation fluctuations are explained by demand shock. Between 1990-2000, supply shock played the most significant role in explaining inflation fluctuations. The contribution of monetary policy to output gap and inflation fluctuations is less pronounced then the one in the first sub-period.



Note: The shaded areas correspond to the recession periods as defined by the NBER. The dotted line shows the total effects of all the shocks. The solid line shows the contributions of each shock.

Figure 8: **HD from model 1: 1980M2 to 2008M12**



Note: The shaded areas correspond to the recession periods as defined by the NBER. The dotted line shows the total effects of all the shocks. The solid line shows the contributions of each shock.

Figure 9: **HD from model 2: 1982M10 to 2008M12**

In the next figures we plot the historical decomposition for the second VAR model replacing current inflation rate by expected inflation rate. Figure (9) shows the historical decomposition from October 1982 to December 2008. Supply shock has no effect on federal funds rate variations, but contributes to the explanation of output gap and expected inflation rate fluctuations. The contribution of supply shock to expected inflation rate fluctuations is more important before 1995. Output gap fluctuations are essentially explained by supply shock. We also find a close match between the beginning and the end of expansion period between 1990 and 2000, and the peak and trough of supply contribution. Once again, demand shock, is an important factor in explaining the variations in federal funds rate and expected inflation rate. It has a significant effect on output gap fluctuations only after 2005. The monetary policy contribution to expected inflation rate and output gap fluctuations is not very important comparing to the other two shocks. These results do not differ very much from those find when we estimate the first model.

The historical decomposition shows that monetary policy is not the main source of fluctuations in output gap, current inflation and expected inflation. Our results suggest that Federal Reserve accomodates monetary policy to the state of the economy. Monetary policy follows the trend of the total effect of output gap and inflation variability but do not contribute the most to their fluctuations. Fluctuations in these macroeconomic aggregate variables are caused by either a demand shock or a supply shock. Here, we defined in which period each shock was more relevant and we tried to propose explanation for this.

7 Conclusion

In this paper we apply Qu and Perron (2007) test to test the stability of our two VAR models. We find one structural break for the VAR with current inflation rate and one break date for the VAR with expected inflation rate. The break date is January 1980. Then, we estimate the two VARs for the different sub-periods using the pure sign restriction approach initiated by Uhlig (2005). We identify three shocks: monetary policy shock, demand shock and supply shock. We consider that the conventional theory concerning the three macroeconomic shocks is verified, and we evaluate the effect of each shock on the fluctuations of current and expected inflation rate, federal funds rate and output gap using forecast error decomposition and historical decomposition. We find that contractionary monetary policy is not the main fluctuations source neither in current and expected inflation rate nor in output gap. Those are mostly due to demand and supply shocks. Federal funds rate seems to be an exogenous monetary instrument used by Federal Reserve to accomodate output and inflation volatility. Our conclusion is not surprising because the litterature has not yet converged on a set of assumptions for identifying the effects of a monetary policy shock. Inflation and output gap variations seem to be caused by other shocks to the economy and Federal Reserve simply accommodates federal funds rate to the state of the economy. Forecast error variance decomposition gives more weight to supply shock in explaining variations in current and expected inflation. While, it explains most of output fluctuations by demand shock. Historical decomposition analysis defines in which sub-periods supply shock was more important than demand shock in explaining current or expected inflation and output gap fluctuations.

Appendix

A- The pure sign restriction approach

Consider a vector autoregressif in reduced form

$$Y_t = c + B_1 Y_{t-1} + \ldots + B_p Y_{t-p} + u_t \qquad \text{for} \qquad t = 1, \ldots, T \tag{8}$$

where Y_t is an n-dimensional vector, B_p for $p = (1, \ldots, n)$ is a squared $(n \times n)$ matrix and $E(u_t u_t') = \Sigma$.
The Vector Moving Average representation of (8) is

$$Y_t = \mu + C(L) u_t \tag{9}$$

where $C(L) = [I - B(L)]^{-1}$ and $\mu = c[I - B(L)]^{-1}$. $C(L)$ and $B(L)$ are polynomials in the lag operator L.
Written in a matrix form, (9) become

$$\begin{aligned}
\begin{pmatrix} y_{1t} \\ \vdots \\ y_{nt} \end{pmatrix} &= \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_n \end{pmatrix} + \begin{pmatrix} c_{11}(L) & \ldots & c_{1n}(L) \\ \vdots & \vdots & \vdots \\ c_{n1}(L) & \ldots & c_{nn}(L) \end{pmatrix} \begin{pmatrix} u_{1t} \\ \vdots \\ u_{nt} \end{pmatrix} \\
&= \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_n \end{pmatrix} + \sum_{k=0}^{\infty} \begin{pmatrix} c_{11}(k) & \ldots & c_{1n}(k) \\ \vdots & \vdots & \vdots \\ c_{n1}(k) & \ldots & c_{nn}(k) \end{pmatrix} \begin{pmatrix} u_{1t} \\ \vdots \\ u_{nt} \end{pmatrix}
\end{aligned} \tag{10}$$

The cumulated effects of unit impulses in u_t can be obtained by the appropriate summation of the impulse response function coefficients. For example note that after l periods, the effects of the j-th shock, u_{jt} on the value of the i-th variable, $y_{i(t+l)}$; is $c_{ij}(l)$. Thus after l periods the cumulated sum of the effects of u_{jt} on the y_{it} sequence is

$$\sum_{k=0}^{l} c_{ji}(k) \tag{11}$$

The key identification is to represent the one step ahead prediction errors, u_t, into economically meaningful shocks. We consider a linear combination between u_t and some orthogonalized "structural" shocks v_t. let

$$u_{t(n\times1)} = A_{(n\times n)} v_{t(n\times1)} \tag{12}$$

The n-fundamental shocks v_t are mutually orthogonal with variance-covariance equal to unity. Thus $E(v_t v_t') = I$. The Vector Moving Average representation (9) in terms of the structural shocks (12) become

$$Y_t = \mu + C(L) A v_t \tag{13}$$

The j-th column of A, a^j, represents the immediate impact, or *impulse vector*, of a one standard error innovation to the j-th fundamental innovation, v_{jt}.
Definition
the vector $a^j \in R^n$ is called an impulse vector, if there is a square matrix A of order n; such

as $AA' = \Sigma$ and so that a^j is a column vector of A.
Impulse response of the i-th variable to an impulse to the j-th shock is given by

$$C_i(L) \times a^j \tag{14}$$

where $C_i(L)$ is the i-th row of the polynomial matrix $C(L)$ and a^j is the j-th column of matrix A. The matrix A can be estimated using the information given by the covariance matrix of the reduced form Σ. In general there are a large number of full rank matrices A that reproduce Σ i.e Cholesky decomposition, matrix factorization. Hence there is no unique decomposition of Σ. Any two decompositions $\Sigma = AA'$ or $\Sigma = \widetilde{AA'}$ have to satisfy

$$A_{(n\times n)} = \widetilde{A}_{(n\times n)}Q_{(n\times n)} \tag{15}$$

where Q is a square orthonormal matrix of order n with $QQ' = I$. From (15) we deduce that any impulse vector a^j can be written as

$$a^j = \widetilde{A}q^j \tag{16}$$

where a^j is the j-th column of matrix A and q^j is the corresponding j column of matrix Q. q^j is an n-dimensional vector of unit length, $||q^j|| = 1$.
If we substitute (15) in (13), we obtain

$$\begin{aligned} Y_t &= \mu + C(L)\widetilde{A}Qv_t \\ \begin{pmatrix} y_{1t} \\ \vdots \\ y_{nt} \end{pmatrix} &= \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_n \end{pmatrix} + \begin{pmatrix} r_{11}(L) & \dots & r_{1j}(L) \\ \vdots & \vdots & \vdots \\ r_{i1}(L) & \dots & r_{ij}(L) \end{pmatrix} \begin{pmatrix} q_1^1 & \dots & q_1^j \\ \vdots & \vdots & \vdots \\ q_n^1 & \dots & q_n^j \end{pmatrix} \begin{pmatrix} v_{1t} \\ \vdots \\ v_{nt} \end{pmatrix} \end{aligned} \tag{17}$$

where $r_{ij}(L) = C_i(L) \times a^j$
We define $r_{ij}(k)$ as the impulse response of the i-th variable at horizon k to the j-th column of $\widetilde{A}$. Let $r_i(k)$ be the vector response at horizon k to the j-th shock. The n-dimensional column vector $r_i(k)$ is $r_i(k) = [r_{1i}(k), r_{2i}(k), \dots, r_{ni}(k)]'$. Then the n-dimensional impulse response $r_a(k)$ at horizon k to the impulse vector a^j for $j = (1, \dots, n)$ is given by

$$\begin{aligned} r_a(k)_{(n\times 1)} &= \begin{pmatrix} q_1^j r_{11}(L) + \dots + q_n^j r_{1j}(L) \\ q_1^j r_{21}(L) + \dots + q_n^j r_{2j}(L) \\ \vdots \\ q_1^j r_{i1}(L) + \dots + q_n^j r_{ij}(L) \end{pmatrix} \\ &= q_1^j r_1(k) + q_n^j r_2(L) + \dots + q_n^j r_n(L) \\ &= \sum_{i=1}^{l} q_i^j r_i(k) \end{aligned} \tag{18}$$

where q_i is the i-th element of vector q^j. $r_a(k)$ is a vector column of dimension n. The elements of this impulse vector are the impulses of the j-th shock to each variable in the system at horizon k.
A sign restriction on the impulse reponse of variable n at horizon k imply that the n-element of vector $r_a(k)$ is < or > 0.

B- Data figures

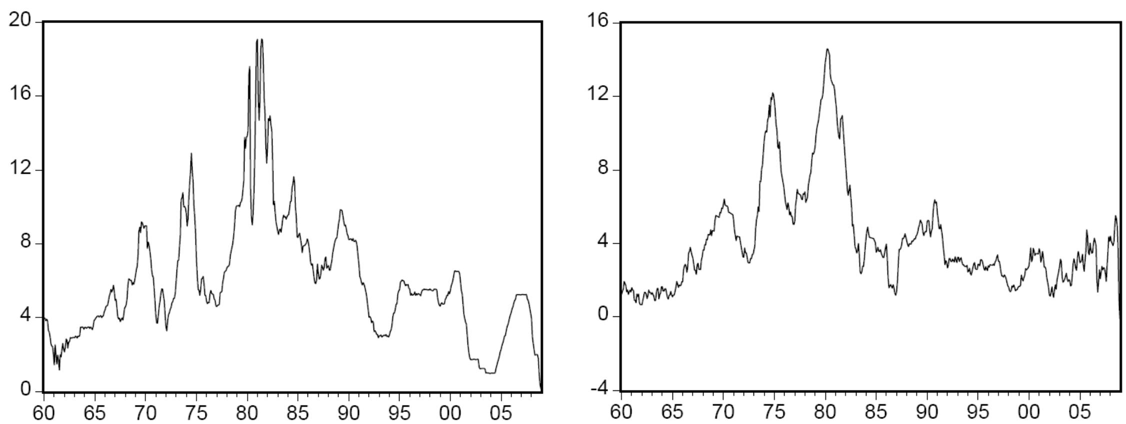


Figure 10: **Federal Funds Rate and Current inflation rate: 1960M1 to 2008M12**

Figure 11: **Output gap and Expected inflation rate: 1960M1 to 2008M12**

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