Housing Market Heterogeneity in a Monetary Union

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Abstract

This paper studies the implications of cross-country housing market heterogeneity for a monetary union. I develop a two-country new Keynesian general equilibrium model with housing and collateral constraints to explore this issue. Results show that consumption reacts more strongly to common shocks in countries with high loan-to-value ratios (LTVs), a high proportion of borrowers or variable-rate mortgages. As for asymmetric technology shocks, output and house prices increase by more in the country receiving the shock if it can conduct monetary policy independently. I also find that after country-specific housing price shocks consumption does not only increase in the country where the shock takes place, there is an international transmission. From a normative perspective, I conclude that housing-market homogenization in a monetary union is not beneficial per se, only when it is towards low LTVs or predominantly fixed-rate mortgages. Furthermore, when there are asymmetric shocks, benefits from forming a monetary union with respect to having a flexible exchange rate regime decrease significantly if there is housing market heterogeneity.

Keywords: Housing market, collateral constraint, monetary policy, monetary union
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"Several of the benefits of the euro are already clearly visible, such as the deepening of trade and financial links between euro area countries and the greater resilience of the euro area to external shocks. Today I will discuss both of these accomplishments, and I will also touch on some of the challenges that we continue to face. For instance, there is presently a degree of diversity among euro area countries".
Jean-Claude Trichet, October 8, 2007.

1 Introduction

Costs and benefits of monetary unions are a much discussed topic, especially in relation to the Europe’s Economic and Monetary Union (EMU). There are clear arguments in favor. A single currency eliminates exchange rate risk, allows rapid price comparison and lowers transaction costs across countries. However, costs can arise if countries are not sufficiently similar. Different national characteristics such as heterogeneous institutions, consumption patterns or financial structures can be a source of different transmission of common shocks in a monetary union. Also, country-specific shocks derived from member heterogeneity can enhance the possible divergence.

In this paper, I focus on housing markets. I consider how heterogeneous housing markets across members affect the transmission of shocks (both symmetric and asymmetric) in a currency area. I also use welfare analysis to evaluate whether housing market homogenization is beneficial in a monetary union and whether countries with asymmetric shocks should join in a monetary union, especially when they have different housing characteristics.

Countries in Europe clearly differ in their housing markets. There is evidence of different loan-to-value ratios (LTVs), different proportion of residential debt relative to GDP across countries and heterogeneous mortgage contracts. Also, house price movements do not show the same pattern in every country. Maclennan et al (1998) point out the importance of such heterogeneity in a monetary union. They conclude that there should be an effort toward institutional homogenization among European countries to alleviate possible tensions.

LTVs in Europe range from 16% in France\(^1\) to 73% in Germany or 95% in Sweden. European countries also differ in their proportion of borrowers. The residential debt to GDP ratio ranges from values such as 18.7% in Italy to 98.4% in the Netherlands or 100.8% in Denmark. In those countries with a high LTV or a high proportion of indebted consumers, housing collateral effects are stronger. Therefore, shocks

\(^1\)According to the European Mortgage Federation, the average loan-to-value ratio for first-time buyers reached a low 16% in 2004 due to house price inflation and low interest rates.
that affect the value of the collateral constraint could potentially have amplified effects on aggregate variables. This is a clear example of the financial accelerator mechanism, first modeled by Bernanke et al (1999).

Differences in mortgage contracts across countries are another important source of heterogeneity in Europe. In countries such as Germany or France, the majority of mortgages are fixed rate. On the contrary, the predominant type of mortgages in countries such as the United Kingdom, Spain or Greece is variable rate. Calza et al (2009) and Rubio (2009) show that the mortgage structure of an economy matters for the transmission of shocks, especially for those shocks that display more persistence.

Asymmetric shocks can also pose a problem for monetary unions. For example, different housing markets can also lead to an asymmetric evolution of house prices. Data on house price movements for European countries in the last years show such asymmetry. There are countries such as Spain, the United Kingdom or France that have experienced large house price increases. However, house prices have been pretty stable and even slightly decreased in Germany. Country-specific house price shocks can create extra divergence across monetary union members. It is important to assess to what extent asymmetric house price movements in a specific region can be transmitted to other areas. House prices increasing in one area increase consumer’s wealth and therefore consumption. Since countries are trading also production in other areas can increase. Furthermore, interest rates respond to inflation creating house price movements in the whole union. Asymmetric technology shocks can also be considered to study costs and benefits of forming a monetary union. If the shock occurs in one of the countries, the interest rate response would be different if the economy can conduct its independent monetary policy or if it is in a monetary union regime.

There is an extensive literature discussing differences in the transmission mechanisms between European countries but little focus on the consequences of housing market heterogeneity from a theoretical standpoint. A microfounded general equilibrium model is needed to understand the implications of housing market differences, explore all the interrelations that take place in the economy and do some normative analysis. Calza et al (2009) and Rubio (2009) use a closed economy framework to study some of these issues and thus cannot address housing market heterogeneity in a monetary union. Gilchrist et al (2002) build a two-country model with a financial accelerator and cross-country financial heterogeneity to explore differences between monetary regimes (monetary union vs. flexible exchange rates). Nevertheless, their model is silent about differences in housing markets. Iacoviello and Smets (2006) develop a monetary union model with housing market heterogeneity. However, they do not compare it with a
non monetary union framework. Also, they do not consider the role of mortgage contract heterogeneity. Aspachs and Rabanal (2008) have a two-country model with housing and collateral constraints but just focus on the case of Spain and the EMU. None of these papers does welfare analysis. Campbell and Hercowitz (2009) study the welfare implications of moving to high LTVs. Rubio (2009) analyzes welfare when mortgages can be fixed or variable rate. However, these two papers do the analysis in a closed economy framework. Carré and Collard (2003) study the implications asymmetric technology shocks both from a positive and a normative perspective. However, their model does not consider a housing market and collateral constraints.

This paper presents a two-country dynamic stochastic general equilibrium (DSGE) model that features a housing market. There is a group of individuals in each country that are credit constrained and need housing collateral to obtain loans. Countries trade goods and savers in each country have access to foreign assets. Across countries, I allow for differences in LTVs, in the proportion of borrowers and in the structure of mortgage contracts (fixed vs. variable rate). I also consider idiosyncratic house price and technology shocks. Under this general setting, I compare the case in which the two countries have independent monetary policy and different currencies with the case of a monetary union.

Results show that in a monetary union, common shocks (monetary policy and technology) have a different impact across countries when there exists housing market heterogeneity. In particular, consumption reacts more strongly after a shock when the LTV is high, the proportion of borrowers is high, or when mortgages are predominantly variable rate. Concerning asymmetric house price shocks, I find that consumption increases in the country where a positive house price shock takes place but also in the other country. House price shocks are transmitted internationally. Asymmetric technology shocks have different effects on both economies depending on the monetary regime considered because the interest rate response is different in each case.

From a normative perspective I find that homogeneity per se is not necessarily beneficial. For instance, total welfare is higher in a situation where LTVs are asymmetric than in a situation where they are equal but very high, in line with the findings in Campbell and Hercowitz (2009). Also, for mortgage contracts, homogenization is welfare improving only if it is towards fixed-rate mortgages. As for benefits and costs of forming a monetary union when there is an asymmetric shock, benefits are reduced if there is housing market heterogeneity.

The paper is organized as follows. Section 2 presents both the baseline model (two countries with different currencies and independent monetary policies) and the monetary union version. Section 3
presents the dynamics of the model. Section 4 analyzes welfare. Section 5 concludes.

2 A Two-Country Model with Housing

I develop a two-country general equilibrium model with a housing market. As a starting point I consider the case in which each of the countries implements its own monetary policy, under a flexible exchange rate regime. In each country, the central bank sets the interest rate to respond to domestic output and inflation. I allow for mortgage and housing heterogeneity across countries.

2.1 The Model

I consider an infinite-horizon, two-country economy with a flexible exchange rate regime. Households consume, work and demand real estate. There is a financial intermediary in each country that provides mortgages and accepts deposits from consumers. Each country produces one differentiated good but households consume goods from both countries. Housing is a non-traded good. I assume that labor is immobile across countries. Firms follow a standard Calvo problem. In this economy, both final and intermediate goods are produced. Prices are sticky in the intermediate goods sector.

2.1.1 The Consumer’s Problem

There are three types of consumers in each country: unconstrained consumers, constrained consumers who borrow at a variable rate and constrained consumers who borrow at a fixed rate. The proportion of each type of borrower is fixed and exogenous. Consumers can be constrained or unconstrained, in the sense that constrained individuals need to collateralize their debt repayments in order to borrow from the financial intermediary. Interest payments for both mortgages and loans next period cannot exceed a proportion of the future value of the current house stock. In this way, the financial intermediary ensures that borrowers are going to be able to fulfill their debt obligations next period. As in Iacoviello (2005), I assume that constrained consumers are more impatient than unconstrained ones. This assumption ensures that the borrowing constraint is always binding, so that constrained individuals do not save and wait until they have the funds to self-finance their consumption.\(^2\) This generates an economy in which households are separated into the ones that mostly borrow and the ones that mostly save.\(^2\) Iacoviello (2005) shows this result holds in the steady state. However, in a situation with small shocks it can be generalized.
COUNTRY A

Unconstrained Consumers (Savers) Unconstrained consumers maximize:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left( \ln C_t^u + j_t \ln H_t^u - \frac{(L_t^u)^\eta}{\eta} \right),$$  (1)

Here, $E_0$ is the expectation operator, $\beta \in (0, 1)$ is the discount factor, and $C_t^u$, $H_t^u$ and $L_t^u$ are consumption at $t$, the stock of housing and hours worked respectively. $j_t$ represents the weight of housing in the utility function. I assume that $\log(j_t) = \log(j) + u_{jt}$, where $u_{jt}$ follows an autorregressive process. Shocks to $j_t$ can be interpreted as shocks to the house price. $1/\eta - 1$ is the aggregate labor-supply elasticity.

Consumption is a bundle of domestically and foreign produced goods. The consumption index is defined as: $C_t^u = (C_{At}^u)^n (C_{Bt}^u)^{1-n}$ where $n$ is the size of Country A.

The budget constraint, in units of Country A’s currency, is:

$$P_{At} C_{At}^u + P_{Bt} C_{Bt}^u + Q_t H_t^u + R_{At-1} B_{t-1}^u + e_t R_{Bt-1} D_{t-1} + \frac{\psi}{2} e_t D_t^2 \leq Q_t H_t^u +
$$

$$W_t L_t^u + B_t^u + e_t D_t + P_{At} F_t + P_{At} S_t,$$

where $P_{At}$ and $P_{Bt}$ are the prices of the goods produced in Countries A and B, respectively, $Q_t$ is the housing price in Country A, and $W_t^u$ is the wage for unconstrained consumers. Unconstrained consumers can hold bonds. $B_t^u$ represents domestic bonds denominated in home currency. $R_{At}$ is the nominal interest rate in Country A. Positive bond holdings mean borrowing and negative mean savings. However, as we will see, this group will choose not to borrow at all, they are the savers in this economy. $D_t$ are foreign bond holdings by savers in Country A. $R_{Bt}$ is the nominal rate of foreign bonds, which are denominated in foreign currency. $e_t$ is the exchange rate between currency in Country A and Country B. To ensure stationarity of net foreign assets, I introduce a small quadratic cost of deviating from zero foreign borrowing $\frac{\psi}{2} e_t D_t^2$. They obtain interests for their savings. $S_t$ and $F_t$ are lump-sum profits received from the firms and the financial intermediary in Country A, respectively.

Dividing by $P_{At}$, we can rewrite the budget constraint in terms of good A:

\footnote{It is assumed that housing services are proportional to the housing stock.}
\footnote{A shock to $j_t$ represents a shock to the marginal utility of housing. These shocks directly affect housing demand and therefore can be interpreted as a proxy for exogenous disturbances to house prices.}
\[
C^u_{At} + \frac{P_{At} C^u_{At}}{P_{At} C^u_{At}} + \frac{R_{At-1} b^u_{t-1}}{\pi_{At}} + \frac{e_t R_{At-1} D_{t-1}}{P_{At}} + \frac{\psi}{2 P_{At}} e_t D_t^2 \leq \frac{q_t H^u_{t-1} + w^u_t L^u_t + b^u_t + e_t D_t}{P_{At}} + F_t + S_t, \tag{3}
\]

where \(\pi_{At}\) denotes the inflation rate for the good produced in Country A, defined as \(P_{At}/P_{At-1}\).

Maximizing (1) subject to (3), we obtain the first-order conditions for the unconstrained group:

\[
\frac{C^u_{At}}{C^u_{Bt}} = \frac{n P_{At}}{(1 - n) P_{At}} \tag{4}
\]

\[
\frac{1}{C^u_{At}} = \beta E_t \left( \frac{R_{At}}{\pi_{At+1} C^u_{At+1}} \right), \tag{5}
\]

\[
\frac{1 - \psi D_t}{C^u_{At}} = \beta E_t \left( \frac{R_{At} e_t + 1}{\pi_{At+1} C^u_{At+1} e_t} \right), \tag{6}
\]

\[
w^u_t = (L^u_t)^{\nu-1} \frac{C^u_{At}}{n}, \tag{7}
\]

\[
\frac{j_t}{H^u_t} = \frac{n}{C^u_{At}} q_t - \beta E_t \frac{n}{C^u_{At+1}} q_{t+1}. \tag{8}
\]

Equation (4) equates the marginal rate of substitution between goods to the relative price. Equation (5) is the Euler equation for consumption. Equation (6) is the first-order condition for net foreign assets. Equation (7) is the labor-supply condition. These equations are standard. Equation (8) is the Euler equation for housing and states that at the margin the benefits from consuming housing have to be equal to the costs.

Combining (5) and (6) we obtain a non-arbitrage condition between home and foreign bonds:

\[
R_{At} = \frac{R_{At} E_t e_{t+1}}{(1 - \psi D_t) e_t}. \tag{9}
\]

Since all consumption goods are traded and there are no barriers to trade, I assume in this paper that the law of one price holds:

\[5\text{The log-linearized version of this equation could be interpreted as the uncovered interest rate parity.}\]
\[ P_{At} = e_t P^*_A \]  

\textbf{Constrained Consumers (Borrowers)}  

Constrained consumers are of two types: those who borrow at a variable rate and those who do it at a fixed rate. The difference between them is the interest rate they face. Whereas the fixed-rate borrower in country A faces \( \bar{R}_{At} \), set by the financial intermediary, the variable-rate constrained consumer faces \( R_{At} \), set by the central bank. The proportion of variable-rate consumers in country A is constant and exogenous and equal to \( \alpha_A \in [0, 1] \).

Constrained consumers are more impatient than unconstrained ones, so that \( \bar{\beta} < \beta \). Constrained consumers face a collateral constraint; the expected debt repayment next period cannot exceed a proportion of the expectation of tomorrow’s value of today’s stock of housing:

\[ E_t \frac{R_{At}}{\pi_{At+1}} b^{cv}_{At} \leq k_A E_t q_{t+1} H^{cv}_t, \]  

(11)

\[ E_t \frac{\bar{R}_{At}}{\pi_{At+1}} b^{cf}_{At} \leq k_A E_t q_{t+1} H^{cf}_t, \]  

(12)

where equations (11) and (12) represent the collateral constraint for the variable and the fixed-rate borrower, respectively.\(^6\) \( k_A \) is the loan-to-value ratio in Country A.

Without loss of generality, I present the problem for the variable-rate borrower, since the one for the fixed rate is symmetric. Variable-rate borrowers maximize their lifetime utility function:

\[ \max E_0 \sum_{t=0}^{\infty} \beta^t \left( \ln C^{cv}_t + j_t \ln H^{cv}_t - \frac{(L^{cv}_t)^{\eta}}{\eta} \right), \]  

(13)

where \( C^{cv}_t = (C^{cv}_A)^n (C^{cv}_{Bt})^{1-n} \), subject to the budget constraint (in terms of good A):

\[ C^{cv}_{At} + \frac{P_{Bt}}{P_{At}} C^{cv}_{Bt} + q_t H^{cv}_{At} + \frac{R_{At-1} b^{cv}_{At-1}}{\pi_{At}} \leq q_t H^{cv}_{t-1} + w^{cv}_t L^{cv}_t + b^{cv}_t, \]  

(14)

and the collateral constraint (11).

The first-order conditions for these consumers are:

\(^6\)The superscript \( cv \) stands for "constrained variable" while \( cf \) stands for "constrained fixed".
\begin{equation}
\frac{C^{cv}_{At}}{C^{cv}_{Bt}} = \frac{nP_{Bt}}{(1-n)P_{At}}
\end{equation}

\begin{equation}
\frac{n}{C^{cv}_{At}} = \beta E\left(\frac{nR_{At}}{\pi_{At+1}C^{cv}_{At+1}}\right) + \lambda^{cv}_{At}R_{At},
\end{equation}

\begin{equation}
\omega^{cv}_{t} = (L^{cv}_{t})^{n-1} \frac{C^{cv}_{At}}{n},
\end{equation}

\begin{equation}
\frac{\dot{H}^{cv}_{t}}{H^{cv}_{t}} = \frac{n}{C^{cv}_{At}}q_{t} - \beta E\left(\frac{nR_{At}}{C^{cv}_{At+1}}\right)q_{t+1} + \lambda^{cv}_{At}k_{A}E_{t}q_{t+1}\pi_{At+1}.
\end{equation}

These first-order conditions differ from those of the unconstrained individuals. In the case of constrained consumers, the Lagrange multiplier on the borrowing constraint \(\lambda^{cv}_{At}\) appears in equations (16) and (18). As in Iacoviello (2005), the borrowing constraint is always binding, so that constrained individuals borrow the maximum amount they are allowed to and their saving is zero.\(^7\)

### 2.1.2 Aggregate Variables and Market Clearing

Given \(\alpha_{A}\), the fraction of variable-rate borrowers in Country A, we can define aggregates across constrained consumers as the sum of variable-rate consumers aggregates and fixed-rate consumers aggregates, so that \(C^{c}_{t} = \alpha_{A}C^{cv}_{t} + (1 - \alpha_{A})C^{cf}_{t}\), \(L^{c}_{t} = \alpha_{A}L^{cv}_{t} + (1 - \alpha_{A})L^{cf}_{t}\), \(H^{c}_{t} = \alpha_{A}H^{cv}_{t} + (1 - \alpha_{A})H^{cf}_{t}\) and \(b^{c}_{t} = \alpha_{A}b^{cv}_{t} + (1 - \alpha_{A})b^{cf}_{t}\).

Therefore, economy-wide aggregates in Country A are \(C_{t} = C^{u}_{t} + C^{c}_{t}\), \(L_{t} = L^{u}_{t} + L^{c}_{t}\) and aggregate supply of housing is fixed, so that market clearing requires \(H_{t} = H^{u}_{t} + H^{c}_{t} = H\).

The market clearing condition for the final good in country A is \(nY_{At} = nC_{At} + (1-n)C^{*}_{At} + n\frac{\psi}{2}d^{2}_{t}\) and the world bond market clearing condition is \(nd_{t} + (1-n)\frac{P_{At}}{P_{At}}d^{*}_{t} = 0\), where \(d_{t}\) denotes the foreign bond in real terms. Variables in Country B are denoted with a star. Everything is similar in Country B.

\(^7\)From the Euler equations for consumption of the unconstrained consumers, we know that \(R_{A} = 1/\beta\) in steady state. If we combine this result with the Euler equation for consumption for the constrained individual we have that \(\lambda^{cv}_{At} = n\left(\beta - \bar{\beta}\right)\frac{C^{cv}_{At}}{C^{cv}_{At}} > 0\) in steady state. This means that the borrowing constraint holds with equality in steady state. Since we log-linearize around the steady state assuming that uncertainty is low, we can generalize this result to off-steady-state dynamics.
2.1.3 The Financial Intermediary

There is a financial intermediary in each country. The financial intermediary accepts deposits from savers, and extends both fixed and variable-rate loans to borrowers.

COUNTRY A The financial intermediary in Country A accepts deposits form savers, and extends both fixed and variable-rate loans to borrowers. The profits from the financial intermediary are:

\[
F_t = \alpha_A R_{At-1} b^c_{At-1} + (1 - \alpha_A) \overline{R}_{At-1} b^f_{At-1} - R_{At-1} b^b_{At-1}.
\]  

(19)

For simplicity, and given that typically the time horizon of a mortgage is large, I consider the maturity of mortgages to be infinite. This assumption is not crucial for the dynamics of the problem since we are interested in short-term business cycle fluctuations.

In equilibrium, borrowing and savings have to be equal, that is \( b^c_{At} = b^b_{At} \). Using this fact, (19) becomes:

\[
F_t = (1 - \alpha_A) b^f_{At-1} \overline{R}_{At-1} - R_{At-1} .
\]  

(20)

I assume a competitive framework for the financial intermediary. Therefore, optimality implies that the intermediary is indifferent between lending at a variable or fixed rate at each point in time \( \tau \). Hence, the expected discounted profits that the intermediary obtains by lending at a fixed interest rate have to be equal to the expected discounted profits the intermediary would obtain by lending at the variable rate:

\[
E_\tau \sum_{i=\tau+1}^{\infty} \beta^{i-\tau} \Lambda_{\tau,i} R_{At}^{OPT} (b^c_{At} - b^c_{At-1}) = E_\tau \sum_{i=\tau+1}^{\infty} \beta^{i-\tau} \Lambda_{\tau,i} R_{Ai-1} (b^c_{At} - b^c_{At-1}) ,
\]  

(21)

where \( \Lambda_{\tau,i} = \frac{C^u_{At}}{C^u_{At+i}} \) is the unconstrained consumer’s discount factor. Since the financial intermediary is owned by the savers, their stochastic discount factor is applied to the financial intermediary’s problem. Notice that this is not a condition on the stock of debt, but on the new amount obtained on a given period. New debt at a given point in time is associated with a different fixed interest rate. Both the fixed interest rate in period \( \tau \) and the new amount of debt in period \( \tau \) are going to be fixed for all periods. However, the fixed interest rate varies with the date the debt was issued, so that there is a new fixed interest rate associated with new debt in every period.
We can obtain the optimal value of the fixed rate in period $\tau$ from expression (21):

$$R_{A\tau}^{OPT} = \frac{E_\tau \sum_{i=\tau+1}^{\infty} \beta^{i-\tau} \Lambda_{\tau,i} R_{Ai-1}}{E_\tau \sum_{i=\tau+1}^{\infty} \beta^{i-\tau} \Lambda_{\tau,i}}.$$  

(22)

Equation (22) states that, for every new debt issued at date $\tau$, there is a different fixed interest rate that has to be equal to a discounted average of future interest rates. Notice that this is not a condition on the stock of debt, but on the new amount obtained in a given period. New debt at a given point in time is associated with a different fixed interest rate. Both the fixed interest rate in period $\tau$ and the new amount of debt in period $\tau$ are fixed for all future periods. However, the fixed interest rate varies with the date the debt was issued, so that in every period there is a new fixed interest rate associated with new debt in this period. If we consider fixed-rate loans to be long-term, the financial intermediary obtains interest payments every period from the whole stock of debt, not only from the new ones. Hence, we can define an aggregate fixed interest rate that is the one the financial intermediary effectively charges every period. This aggregate fixed interest rate is composed of all past fixed interest rates and past debt, together with the current period optimal fixed interest rate and new amount of debt. Therefore, the effective fixed interest rate that the financial intermediary charges for the stock of fixed-rate debt every period is:

$$R_{At} = \frac{R_{At-1}b_{t-1}^c + R_{A\tau}^{OPT}(b_{t}^c - b_{t-1}^c)}{b_{t}^c}.$$  

(23)

Equation (23) states that the fixed interest rate that the financial intermediary is actually charging today is an average between what it charged last period for the previous stock of mortgages and what it charges this period for the new amount. Importantly, this assumption is not crucial for results. Both $R_{A\tau}^{OPT}$ and $R_{At}$ are practically unaffected by interest rate shocks. This assumption is a way to reconcile the model with the fact that fixed-rate loans are not one-period assets but longer term ones.

As noted above, if any, profits from financial intermediation are rebated to the unconstrained consumers every period. Even if the financial intermediary is competitive and it does not make profits in absence of shocks, if there is a shock at a given point in time, the fact that only the variable interest rate is affected can generate non-zero profits.

The financial intermediary problem for Country B is symmetric.
2.1.4 Firms

Final Goods Producers  In Country A, there is a continuum of final goods producers that aggregate intermediate goods according to the production function

\[ Y_{1t}^k = \left[ \int_0^1 Y_{1t}^k (z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^\frac{\varepsilon}{\varepsilon-1}, \tag{24} \]

where \( \varepsilon > 1 \) is the elasticity of substitution between intermediate goods.

The total demand of intermediate good \( z \) is given by \( Y_{At} (z) = \left( \frac{P_{At}(z)}{P_{At}} \right)^{-\varepsilon} Y_{At} \), and the price index is \( P_{At} = \left[ \int_0^1 P_{At} (z)^{1-\varepsilon} dz \right]^\frac{1}{1-\varepsilon} \).

Intermediate Goods Producers  The intermediate goods market is monopolistically competitive. Following Iacoviello (2005), intermediate goods are produced according to the following production function:

\[ Y_{At} (z) = Z_t L_t^u (z)^{\gamma_A} (L_t^c (z))^{(1-\gamma_A)}, \tag{25} \]

where \( Z_t \) represents technology. I assume that \( \log Z_t = \rho_Z \log Z_{t-1} + u_{Zt} \) where \( \rho_Z \) is the autorregressive coefficient and \( u_{Zt} \) is a normally distributed shock to technology. \( \gamma_A \in [0, 1] \) measures the relative size of each group in terms of labor. This Cobb-Douglas production function implies that labor efforts of constrained and unconstrained consumers are not perfect substitutes. This specification is analytically tractable and allows for closed form solutions for the steady state of the model. This assumption can be economically justified by the fact that savers are the managers of the firms and their wage is higher.\(^8\)

Experimenting with a production function in which hours are substitutes leads to very similar results in terms of model dynamics because these two assumptions are comparable. Under the Cobb-Douglas specification each household has mass one and \( \gamma_A \) represents the economic size of the patient household. In the alternative specification, the absolute size of savers in the population would be specified.

The first-order conditions for labor demand are the following:\(^9\)

\[ w_t^u = \frac{Z_t}{X_t} \gamma_A \frac{Y_{At}}{L_t^u}; \tag{26} \]

\(^8\)It could also be interpreted as the savers being older than the borrowers, and therefore more experienced.

\(^9\) Symmetry across firms allows to avoid the index \( z \).
\[ w_t^* = \frac{Z_t}{X_t} (1 - \gamma_A) \frac{Y_{At}^*}{L_t^*}, \tag{27} \]

where \(X_t\) is the markup, or the inverse of marginal cost.

The price-setting problem for the intermediate goods producers is a standard Calvo-Yun setting. An intermediate good producer sells good at price \(P_{At}(z)\), and \(1 - \theta\) is the probability of being able to change the sale price in every period. The optimal reset price \(P_{At}^{OPT}(z)\) solves:

\[
\sum_{k=0}^{\infty} (\theta \beta)^k E_t \left\{ \Lambda_{t,k} \left[ \frac{P_{At}^{OPT}(z)}{P_{At+k}} - \frac{\varepsilon}{(\varepsilon - 1)} \right] Y_{At+k}^{OPT}(z) \right\} = 0. \tag{28} \]

The aggregate price level is given by:

\[
P_{At} = \left[ \theta P_{At-1}^{\varepsilon} + (1 - \theta) \left( P_{At}^{OPT} \right)^{1 - \varepsilon} \right]^{1/(1 - \varepsilon)}. \tag{29} \]

Using (28) and (29) and log-linearizing, we can obtain the standard forward-looking Phillips Curve.\(^\text{10}\)

The firm problem is analogous in Country B.

### 2.1.5 Monetary Policy

The model is closed with a Taylor Rule with interest-rate smoothing for interest-rate setting by each country’s central bank.\(^\text{11}\) In Country A,

\[
R_{At} = (R_{At-1})^{\rho A} \left( \pi_{At}^{(1 + \phi_{\pi,A})} R_A \right)^{1 - \rho A} \varepsilon_{AR,t}, \tag{30} \]

\(0 \leq \rho A \leq 1\) is the parameter associated with interest-rate inertia. \((1 + \phi_{\pi,A})\) measures the sensitivity of interest rates to current inflation. \(\varepsilon_{AR,t}\) is a white noise shock process with zero mean and variance \(\sigma^2_\varepsilon\).

In Country B, \(R_{Bt}\) is set similarly.

### 2.2 The Monetary Union Case

Now we can consider the case in which Country A and Country B form a monetary union. The problem for consumers in this case differs from the previous one in that prices are denominated in a common

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\(^{10}\)This Phillips curve is consistent with other two-country models with financial accelerator. See for instance Gilchrist et al (2002) or Iacoviello and Smets (2006).

\(^{11}\)This rule is consistent with the primary objective of the ECB being price stability. This type of rule is also used in other monetary union models. See Iacoviello and Smets (2006) or Aspachs and Rabanal (2008)
currency and therefore there is no need for the use of the exchange rate. Monetary policy is now conducted by a single central bank that reacts to inflation and output in both countries weighted by its relative size.

3 Dynamics

3.1 Parameter Values

We can use the model to explore how shocks are transmitted across different experiments. I linearize the equilibrium equations around the steady state. The discount factor for savers, $\beta$, is set to 0.99 so that the annual interest rate is 4% in steady state. The discount factor for borrowers, $\tilde{\beta}$, is set to 0.98. The steady-state weight of housing in the utility function, $j$, is set to 0.1 in order for the ratio of housing wealth to GDP in steady state to be approximately 1.40. I set $\eta = 2$, implying a value of the labor supply elasticity of 1. For the loan-to-value ratio, I pick $k_A = k_B = 0.8$ for the baseline calibration, consistent with a weighted average of LTVs in 2004 calculated by the European Mortgage Federation (EMF) on European countries. However, one of the experiments I perform consists of testing the sensitivity of results to this parameter. The labor income share of unconstrained consumers, $\gamma_A = \gamma_B$, is set to 0.7 as a reference point. Nonetheless, as for the LTV ratio, experiments with different values of this parameter will be performed. I pick a value of 6 for $\varepsilon$, the elasticity of substitution between intermediate goods. This value implies a steady-state markup of 1.2. The probability of not changing prices, $\theta$, is set to 0.75, implying that prices change on average every four quarters. For the Taylor Rule parameters I use $\rho_A = \rho_B = 0.8$, $\phi_{\pi A} = \phi_{\pi B} = 0.5$. The first value reflects a realistic degree of interest-rate smoothing. $\phi_{\pi A}$ and $\phi_{\pi B}$ are consistent with the original parameters proposed by Taylor in 1993. For the baseline model, I consider $\alpha_A = \alpha_B = 1$, that is, all mortgages are variable rate. Results for the case of fixed-rate mortgages are also checked.

Monetary policy shocks are represented by a one percent increase of the interest rate. A technology

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12 Lawrance (1991) estimates discount factors for poor consumers between 0.95 and 0.98 at quarterly frequency.
13 This value corresponds to the US. I assume here that the ratio is similar across most industrialized countries, given the lack of housing wealth data for European countries. See Aspachs and Rabanal (2008).
14 Microeconomic estimates usually suggest values in the range of 0 and 0.5 (for males). Domeij and Flodén (2006) show that in the presence of borrowing constraints this estimates could have a downward bias of 50%.
15 The countries that are included in the sample are Belgium, Germany, Greece, Spain, France, Italy, Hungary, Poland, Sweden and the United Kingdom.
16 This value is in the range of the estimates of Iacoviello (2005), Iacoviello and Neri (2008) and Campbell and Mankiw (1991) for the US, Canada, France and Sweden.
18 This value makes the model comparable with the standard models where fixed-rate mortgages are not considered.
shock will be a one percent positive technology with 0.9 persistence\textsuperscript{19}. House price shocks have a 0.8 persistence.\textsuperscript{20} I set the size of the shock to the housing demand parameter to 20% so that house prices increase roughly by 1%.

3.2 Common Shock in a Monetary Union with Housing Market Heterogeneity

3.2.1 LTV Asymmetry

When countries in a monetary union are asymmetric in their housing markets, a common shock can affect them differently. The first source of asymmetry that I consider is differences in LTVs. The loan-to-value ratio is a crucial parameter because it implies the degree of credit accessibility for borrowers and therefore the strength of the financial accelerator. When LTVs are high, shocks that affect the value of the collateral are amplified due to the financial accelerator effect.

\textsuperscript{19}This high persistence value for technology shocks is consistent with what is commonly used in the literature. Smets and Wouters (2002) estimate a value of 0.822 for this parameter in Europe, Iacoviello and Neri (2008) estimate is 0.93 for the US.

\textsuperscript{20}The persistence of the house price shock is consistent with the estimates in Iacoviello (2005) and Iacoviello and Neri (2008).
Figure 1 shows the effects of a monetary policy shock in a monetary union when countries differ in their LTVs. We consider Country B to be a country with a low LTV of 0.2, as opposed to the rest of the union which has an LTV of 0.8. The size of Country B is set to 0.1. This theoretical experiment could illustrate the case of France in 2004, when LTVs for first-time buyers reached a low 16% due to house price inflation and low interest rates.

An increase in the interest rate contracts the economy. Savers substitute intertemporally and prefer to save today to consume tomorrow. For borrowers, there is both a direct and an indirect effect that make their consumption decrease. First, their mortgage payments increase and therefore they consume less. The second effect comes through the collateral constraint. Since housing prices decrease following the interest rate increase, the value of their collateral decreases. Impatient agents are able to borrow less and hence consume less. This collateral effect, however, is stronger the higher the LTV parameter. We can see that the effects of this shock are amplified if the country has a high LTV, meaning that the financial accelerator is stronger there. Notice however, that savers, who have access to international financial markets, are able to compensate the differences between the two scenarios.

The experiment for a common technology shock is not shown here because it would be analogous. Also in this case total consumption would react more in the country that has a high LTV ratio. The interest rate would decrease and housing prices in both countries increase. The collateral effect is greater in that country with the higher LTV and therefore its consumption would increase by more.

3.2.2 Borrowers Proportion Asymmetry

The proportion of borrowers is also a source of cross-country asymmetry that matters for the transmission of shocks. The economic size of this group in Country A is captured by $1 - \gamma_A$ in the model. We consider Country B having a higher proportion of borrowers ($\gamma_B = 0.2$) than the rest of the union (Country A) where $\gamma_A = 0.7$. Similarly to the LTV heterogeneity case, we expect that when borrowers are very numerous, collateral constraints are a more pervasive feature of the economy. Figure 2 confirms this intuition. For borrowers, consumption decreases by more when they have a more important economic size. In this case, savers are able to offset only part of these differences through international financial markets. Housing also reacts more strongly in the country with more borrowers. In the aggregate we see that after a monetary policy shock consumption reacts more strongly where the proportion of borrowers is higher.
3.2.3 Mortgage Contract Asymmetry

Another source of heterogeneity in housing markets is the cross-country mortgage structure heterogeneity. Let us analyze now the case in which the structure of mortgage contracts in Country A is fixed rate and variable rate in Country B. This could be seen as Country B being for instance Spain and Country B the rest of EMU. Consider first an interest rate shock in a monetary union. For those consumers with variable-rate mortgages, after a positive interest-rate shock, interest rate payments increase by more than for the fixed-rate case. Also, the value of their collateral decreases by more. Then, the monetary policy shock hits strongly those individuals that are constrained. We can observe in Figure 3 that consumption and housing demand for borrowers decrease slightly more persistently in the country in which consumers borrow at a variable rate. For housing demand the mortgage contract makes a difference, for both borrowers and savers housing demand reacts by more in the variable-rate scenario. For aggregate consumption differences between the two countries are quantitatively small. General equilibrium effects partially offset aggregate differences: On the one hand, there is a redistribution between borrowers and savers. On the other hand, there are important wealth effects in the labor-
Figure 3: Impulse Responses to a Monetary Policy Shock in a Monetary Union.

supply decision, that is, variable-rate borrowers can simply decide to work more to compensate their consumption loss.21

In Figure 4 we see that a more persistent shock, such as a technology shock delivers larger aggregate differences between the two countries when the structure of mortgage contracts differs among them. In particular, we see strong differences in the behavior of housing demand and house prices across countries. Total consumption reacts by more in the fixed-rate case due to the procyclicality of the real interest rate. Variable-rate borrowers consume less because increase in real rate affects them negatively. However, fixed-rate consumers are better off in comparison and they can consume more. In Rubio (2009) it is also the case that aggregate differences increase with the persistence of the shock.
3.3 Asymmetric Shocks

3.3.1 Technology Shock

Even if countries are symmetric, they can suffer asymmetric shocks. Figure 5 shows impulse responses of a technology shock that occurs only in Country B. I compare the monetary union versus the flexible exchange rate regime. When technology improves in Country B, interest rates react little in a monetary union because the interest rate setting rule takes into account an inflation average. However, if Country B conducts monetary policy independently, the interest rate in Country B decreases significantly as compared to the monetary union regime. Now Country B can set monetary policy according to its own inflation. As a consequence, under the flexible exchange rate regime, housing prices in Country B increase by more in response to the change in the interest rate. This enhances the wealth effect and consumption also increases by more.

In Country A, when it conducts its own monetary policy, interest rates do not move because the shock happened in Country B and inflation is not changing. However, under the monetary union regime,

\[21\] These results are in line with Rubio (2009) that shows that in a closed economy, GHH preferences, which eliminate wealth effects in the labor supply, are able to generate larger aggregate differences.
the common interest rates goes down and that expands Country A’s economy. Furthermore, the decrease in interest rates makes house prices in Country A. This increases consumption and output further due to the positive wealth effects for borrowers.

### 3.3.2 House Price Shock

An important source of asymmetry within Europe is the house price evolution. In this framework I can study how asymmetric house price shocks are transmitted across countries. In a closed economy, a positive house price shock increases the value of the collateral, and total consumption increases, mainly due to the increase in consumption of borrowers. However, in an open economy, a country-specific house price shock can be transmitted internationally to other countries. If that were the case, the divergence caused by an asymmetric shock would be alleviated. Figure 6 shows the effects of a house price shock in Country A. Consumption in this country increases initially because of wealth effects. Housing demand by borrowers also increases. However, this asymmetric shock is slightly transmitted to Country B where consumption also increases because the countries are trading. Interest rates, especially in the
union, decrease and this makes house prices in Country B increase as well, giving an extra impulse to consumption in Country B.

4 Welfare Analysis

The remaining questions are whether countries in a monetary union should homogenize their structures even if shocks are common and whether heterogeneity reduces the benefits of forming a monetary union when shocks are asymmetric. Welfare analysis is needed to address these issues.

In this section, I numerically evaluate how cross-country asymmetries affect welfare for a given policy rule. Notice that in this economy there are two types of distortions, price rigidities and credit frictions. The individual welfare for savers and borrowers in Country A respectively is defined as follows:22

\[
V_{u,t} = E_t \sum_{m=0}^{\infty} \beta^m \left( \ln C_{t+m}^u + j \ln H_1^u - \frac{(L_{t+m}^u)^{\eta}}{\eta} \right),
\]

\[(31)\]

22 I numerically compute the second order approximation of the utility function as a measure of welfare.
\[ V_{ci,t} \equiv E_t \sum_{m=0}^{\infty} \beta^m \left( \ln C_{t+m}^{ci} + j \ln H_{t+m}^{ci} - \frac{(L_{t+m}^{ci})^\eta}{\eta} \right), \] (32)

Following Mendicino and Pescatori (2007), I define social welfare as a weighted sum of individual welfare for the different type of households:

\[ V_t = (1 - \beta) V_{a,t} + \left( 1 - \beta \right) \left[ \alpha V_{cv,t} + (1 - \alpha) V_{cf,t} \right]. \] (33)

Borrowers and savers’ welfare are weighted by \( \left( 1 - \beta \right) \) and \( (1 - \beta) \) respectively, so that the two groups receive the same level of utility from a constant consumption stream. As in Mendicino and Pescatori (2007), I take this approach to be able to evaluate the welfare of the three types of agents separately.23

Everything is symmetric for Country B.

Total welfare is defined as a weighted sum of the welfare in the two countries:

\[ W_t = nV_t + (1 - n)V_t^*. \] (34)

Table 1 presents welfare comparisons in a monetary union when countries are both symmetric and asymmetric and are hit by a common technology shock. Country B is a small country.

<table>
<thead>
<tr>
<th>Welfare</th>
<th>Symmetric</th>
<th>Asymmetric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>( k = .2 ) ( \gamma = .2 ) ( \alpha = 0 )</td>
<td>( k_A = .8 ) ( \gamma_A = .7 ) ( \alpha_A = 0 )</td>
</tr>
<tr>
<td></td>
<td>( k_B = .2 ) ( \gamma_B = .2 ) ( \alpha_B = 1 )</td>
<td></td>
</tr>
<tr>
<td>Social B</td>
<td>-3.486</td>
<td>-3.087</td>
</tr>
</tbody>
</table>

Table 1: Welfare values. Monetary Union. Common Technology Shock.

The first column displays welfare values for the baseline case, a symmetric case in which the loan-to-value ratio is 0.8, the saver’s labor income share is 0.7 and mortgage rates are variable in the whole union. For the sake of comparison, I also consider other symmetric cases in which LTVs are low, the proportion of borrowers is high and mortgage rates are fixed. Finally, I consider asymmetric cases in which the small country is different from the rest of the union.

23 See Monacelli (2006) for an example of the Ramsey approach in a model with heterogeneous consumers.
The most clear and intuitive results are with respect to the variability of mortgage rates. Homogenization towards fixed-rate mortgages is welfare improving. We are in an economy where a group of individuals are constrained. In Monacelli (2006), Mendicino and Pescatori (2007) and Rubio (2009) it is shown that there is an inflation channel through which the distortion introduced by the collateral constraint is reduced. Inflation lowers debt repayments in real terms. However, this inflation channel is stronger when nominal rates for repayments are fixed. This comes at the expense of savers, who have to bear all the risk associated with interest rate variability but the welfare improvement of the borrowers compensates it. Then, countries with variable-rate mortgages are worse off than countries with fixed-rate mortgages. For mortgage contracts, the heterogeneity by itself is not welfare worsening, in fact moving to a scenario in which both countries have variable-rate mortgages is worse than being heterogeneous. This result has important policy implications. It suggests that countries such as Spain or the United Kingdom (if it entered the EMU) should increase their proportion of fixed-rate contracts.

Results also show that heterogeneity in LTVs is not necessarily welfare worsening. Homogenization improves welfare if countries move towards low LTVs. This may seem counterintuitive because a high LTV relaxes the borrowing constraint. However, this is a result which has been already found in similar models. Campbell and Hercowitz (2009), perform a welfare analysis in a DSGE model with borrowers and savers and obtain that although high LTV ratios have a direct positive effect on welfare through the constraint relaxation, there may be other indirect effects that dominate. The mechanism is the following. Higher LTVs lead to higher consumption levels. We are in a second best economy where borrowing constraints are always binding, the more you offer the borrowers the more they will take. But this, in turn, changes relative prices. In particular, higher consumption levels imply higher interest rates. This could lead to a situation of excessive borrowing in the sense that high repayments could offset the positive effects on the constraint relaxation. Over a long horizon, a symmetric economy that has high LTVs does worse than an asymmetric economy in which one of the countries has a low LTV ratio or a symmetric economy with low LTVs. Smith (2009) shows that these results do not rely on Campbell and Hercowitz (2009) specific assumptions, even in the simplest model with borrowers, savers and collateral constraints this effect takes place. Hugget (1997) finds also a similar result but in this case is the reduction of the precautionary motive for saving, driven by the looser borrowing constraints, what leads to the increase in the interest rate.

Regarding $\gamma$, the economy as a whole achieves a higher welfare when the labor income share of

\(^{24}\)Rubio (2009) has a detailed discussion on this issue.
savers is small and countries are symmetric. When $\gamma$ is lower, that is, the economic size of constrained individuals is larger, the economy is more distorted by the collateral constraint. However, there is a trade-off between the two groups welfare and savers are better off. Overall, since $\gamma$ does not directly affect the collateral constraint as $k$, the welfare improvement of savers outweighs that of the borrowers. The economy has higher welfare with low values of $\gamma$. Country B especially, the small heterogeneous country improves welfare significantly by homogenization. $\gamma$ is however a parameter which is less institutional than $\alpha$ or $k$ and therefore its implications for policy are weaker.

Another important issue in the monetary union discussion is whether countries should form a currency area when there are asymmetric shocks. Carré and Collard (2003) take a step in this direction. They show that in a two-country model, when there is a positive asymmetric technology shock in just one of the countries, implementing a monetary union is beneficial to the households living in the country receiving the shock while detrimental to foreigners. Here I add the housing heterogeneity dimension to compare with their results.

Table 2 displays welfare comparisons when there is an asymmetric technology shock hitting only Country B, in the spirit of Carré and Collard (2003). I consider both housing market symmetry and asymmetry and both the monetary union and the flexible exchange rate regime.

<table>
<thead>
<tr>
<th>Symmetric</th>
<th>Asymmetric</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_A = .8/k_B = .2$</td>
<td>$\gamma_A = .7/\gamma_B = .2$</td>
</tr>
<tr>
<td>$\alpha_B = 0/\alpha_B = 1$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Welfare</th>
<th>MU</th>
<th>Flex. ERs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social A</td>
<td>-3.259</td>
<td>72.58</td>
</tr>
<tr>
<td>Social B</td>
<td>-2.960</td>
<td>-1373.23</td>
</tr>
<tr>
<td>Total</td>
<td>-3.229</td>
<td>-71.99</td>
</tr>
</tbody>
</table>

Table 2: Welfare values. Monetary Union vs. Flexible Exchange Rates. Technology Shock in Country B

The first two columns compare welfare under the two regimes when Country B is hit by an asymmetric technology shock. For the country that experiments the shock, it is beneficial to be in a monetary union. However, the other country is better off under a flexible exchange rate regime. These results are in line with Carré and Collard (2003). Overall, the economy is better off if countries form a monetary union. When there is a positive technology shock in Country B, production costs go down and output increases
under both regimes. However, the benefits obtained by exporting to the big country are enhanced in the monetary union case because there is not exchange rate volatility.

However, benefits from being in a monetary union when there are asymmetric shocks are reduced when there is also housing market heterogeneity. For LTV asymmetry I still find that this economy is better off in the monetary union. However, results are reversed if the asymmetry comes from the proportion of borrowers or differences in mortgage contracts. The latter is an important result when the variable-rate UK considers whether or not to enter the mainly fixed-rate EMU.

5 Concluding Remarks

This paper explores first how cross-country housing market heterogeneity affects the transmission of shocks in a monetary union. Since there is clear evidence of such heterogeneity across countries in Europe, it is relevant to evaluate to what extent this is important. Then, some normative conclusions are also presented.

For this purpose, I build a two-country DSGE model that features a housing market. A group of individuals in each country are credit constrained and need housing collateral to obtain loans. I consider two monetary regimes: the two countries conducting its own monetary policy under a flexible exchange rate system and a monetary union between the two countries.

I find that after a common monetary policy or technology shock, variables respond more strongly if the country has a high LTV, a high proportion of borrowers or mainly variable-rate mortgages. As for country-specific shocks, I find that the effects of a house price shock in one country are slightly transmitted to the other country. The effects of asymmetric technology shocks depend on the monetary regime.

The recommendation that European countries should move towards institutional homogenization is often heard. I perform welfare analysis to explore this issue. From a normative perspective, I find that housing market homogenization per se is not necessarily beneficial. In line with recent studies, homogenization towards high LTVs decreases welfare, indirect effects dominate the direct effect of relaxing the borrowing constraints. As for mortgage contracts, results suggest that countries with predominantly variable-rate contracts should move towards fixed-rate contracts because they reduce the distorting effects of the collateral constraint.

In terms of analyzing costs and benefits of forming a monetary union when there are asymmetric
shocks, I find that, as in Carré and Collard (2003), a country receiving an asymmetric technology shock is better off under a monetary union. However, benefits decrease with housing market heterogeneity. In fact, I find that if the small country has a high proportion of borrowers or variable-rate mortgages, as opposed to the rest of countries in the union, the monetary union regime is welfare worsening.

For future research, it would be interesting to make an attempt towards an analytical welfare result to be able to fully disentangle all the mechanisms which take place and study what is the optimal monetary policy under the different sources of asymmetry. Another extension could be to take a step towards estimation.

References


