Price level convergence, purchasing power parity and multiple structural breaks: An application to US cities[*]

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Abstract

This article provides a fresh methodological and empirical approach for assessing price level convergence and its relation to purchasing power parity (PPP) using annual price data for seventeen US cities. We suggest a new procedure that can handle a wide range of PPP concepts in the presence of multiple structural breaks using all possible pairs of real exchange rates. To deal with cross-sectional dependence, we use both cross-sectional demeaned data and a parametric bootstrap approach. In general, we find more evidence for stationarity when the parity restriction is not imposed, while imposing parity restriction provides leads toward the rejection of the panel stationarity. Our results can be embedded on the view of the Balassa-Samuelson approach, but where the slope of the time trend is allowed to change in the long-run. The median half-life point estimate are found to be lower than the consensus view regardless of the parity restriction.

Keywords: Price level convergence; Half-life; Pairwise analysis; Panel data; Multiple structural breaks; Cross-section dependence.

JEL Classification: C32, C33, E31, F41.

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1   Introduction

This paper takes a fresh look at price level convergence and Purchasing Power Parity
(PPP) hypothesis using annual price data from 1918 to 2005 for seventeen major US
cities. Understanding the dynamics of price differentials within a single currency area,
such as the US, is relevant not only for its own sake, it also offers insights for policymakers
in the Euro area which is a comparatively young currency union than the US. For instance,
in a recent paper Cecchetti, Mark and Sonora (2002) argue that the finding of a longer
half-life of price index convergence for the US cities could mean even a much slower price
adjustments for the Eurozone as it has a more rigid factor markets and decentralized fiscal
authority than the US.

Why should one be concerned about the existence and persistence of price differential
in a monetary union? As it stands out, there is some economic logic behind this concern.
Sustained price divergence not only dampens the law of one price or PPP, it may interferes
with the price stability goal of the monetary authority. In addition, significant price
differentials can give rise to real interest rate differences and may widen the gap between
market exchange rates and PPP exchange rates across markets within a region. For
many years (reduced) price dispersion has been used as a proxy for (increased) economic
integration – and there are potentials gains to be had from the integration of markets
and the compression of price divergence. For instance, if local price for an item in Boston
is lower than rest of the US, market integration and price convergence will benefit local
producers and workers more than they will harm local consumers. The opposite happens
when Boston's local price is higher than rest of the US.[1]

Recognizing this potential gain, over the last few years there has been a flurry of papers
analyzing the dynamics of price level convergence within a single monetary union. In this
regard, examining a panel of 51 prices from 48 US cities, Parsley and Wei (1996) found
that domestic tradable goods prices converge quickly with a estimated half-life of about 1
year. By contrast, Cecchetti et al. (2002) found quite long half-life when using aggregate
CPI data for 19 US cities. Chen and Devereux (2003) instead analyze absolute price level
dispersion and find that the dispersion of absolute price levels is lower for US cities than

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[1]See Warren, Hufbauer and Wada (2002) for an excellent account on the potential benefits of price
convergence.

internationally.[2]

For our purpose these results are interesting because they show evidence of mean reversion in city real exchange rates. However, the empirical support for PPP has been rather mixed.[3] PPP is a necessary if not a sufficient condition for markets to be fully integrated. Most existing studies rely on international data to unearth the rate at which real exchange rates converge to parity in the long run.[4] Rogoff (1996), while reviewing the empirical literature, reached to the consensus estimate of 3-5 year half-lives of PPP deviations.

A limitation of the considerable literature on price convergence (both internationally and within countries) is that the role of structural break is generally ignored. Accounting for parameter shifts is crucial[5] especially when using long spans of data that are more likely to be affected by structural breaks. The structural breaks can appear either because the data have been sampled across several different monetary arrangements or by the presence of shock such as the oil price ones. Additionally, aggregate price data are more susceptible to structural instability, because unlike individual price level aggregate price levels may not adjust so quickly due to differences in productivity among traded and non-traded goods sectors.[6] Left untreated, structural breaks may lead us to cast doubt on the reliability of the findings in previous work on price level convergence (e.g. Cecchetti et al. (2002)).

Structural instability is not the only issue that may characterize the data. Given the panel nature of the data, a closely related issue is that there are usually a high degree of dependence across different price levels. O'Connell (1998) shows the importance of cross-section dependence in PPP analyses. Recently, using simulation methods, Banerjee, Marcellino and Osbat (2004) show that panel data unit root statistics tend to conclude

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[2] For studies based on European price data, see among others, Rogers (2002) and Rogers et al. (2001).

[3] For instance, Cecchetti et al. (2002) are able to reject unit root null in real exchange rate, a finding that is consistent with the PPP hypothesis. By contrast, using an extended sample Chen and Devereux (2003) are unable to reject unit root null, which they claim is consistent with the broad version of PPP.

[4] Some recent and related contributions based on cross-country panel data include Canzoneri et al. (1999), Murray and Papell (2005), Imbs et al. (2005) and Choi et al. (2006).

[5] Perron (1989) demonstrated that erroneous omission of structural breaks in the series can lead to deceptive conclusion when performing the unit root tests using time series data.

[6] This is what the well-known Balassa-Samuelson effect indicates, an increase in the productivity of the traded goods sector in one region not only pushes the regional real wages up, it also drives up the regional relative prices of the non-traded goods. As argued by Canova and Pappa (2005), when productivity differences are persistent, differentials in output growth and inflation rates could also be observed.

in favor of stationarity, or convergence, when cross-section dependence is not considered. These authors warn about the cautions that should be taken when applying panel data statistics to tests the PPP hypothesis – see Banerjee (1999), Baltagi (2005), and Breitung and Pesaran (2005) for overviews of the field.

This paper incorporates these issues in the analysis and shows that the slow mean reversion in the real exchange rate improves significantly once both structural breaks and cross-sectional dependency are accounted for. This is to be expected if we think of studies that do not consider the presence of structural breaks are obtaining biased estimates of the autoregressive parameters, which constitute the half-life. Therefore, the potential pitfall in the computation of mean reversion is avoided through the joint consideration of both structural breaks and cross-sectional dependence.

The broad objective of the paper is to offer a framework that is useful to analyze the stochastic nature of spatial price variation and how it relates to the fulfillment of the PPP hypothesis. In so doing, we propose a new test statistic that is robust to multiple structural breaks while simultaneously entertaining the test for PPP hypothesis. More specifically, the proposed methodology can handle a wide range of PPP concepts depending on whether real exchange rate evolves around a constant or around a time trend – further details are given below.

There are a number of other issues that we consider but have not received much attention in the literature. First and following Pesaran, Smith, Yamagata and Hvozdyk (2006), we conduct pairwise tests for PPP that are not sensitive to the base country effects. The pairwise tests focus on all possible $N(N-1)/2$ real exchange rate pairs between the individuals in the panel and can consistently estimate the proportion of pairs that do not satisfy the PPP. Second, we deviate from the restricted specification proposed in Papell (2002) and Harris, Leybourne and McCabe (2005) to a more general framework for PPP which permits multiple breaks, a possibility that the data do not reject.[7] In addition, we also consider a version of the PPP model that includes a time trend with both level and

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[7]As mentioned in Papell and Prodan (2006), most existing panel data based studies that test PPP hypothesis have considered either one or two changes in the mean. With one-time change in the mean long-run PPP never holds. By contrast, with two changes in the mean PPP may hold if the breaks are offsetting, but if the changes are not offsetting long-run PPP does not hold. In this regard, our multiple breaks specification offers more flexibility to incorporate events such as oil embargo or productivity shocks that may have affected the level as well as the slope of real exchange rate.

slope shifts. Third, to get a feeling of the nature of cross-sectional dependency among individuals, we have applied the tests developed by Pesaran (2004) and Ng (2006). In particular, the test by Ng (2006) allows to gain more insight in terms of how pervasive and strong is the cross-section correlation. Finally, to ensure consistency of our approach, the half-life calculations are obtained using the number and position of the structural breaks that have been obtained in the primary empirical model.[8]

The application of these procedures offers us a clear picture about the PPP hypothesis for the US cities. Thus, more evidence for stationarity around a changing level is found when the parity restriction is not imposed, while imposing parity restriction provides favorable evidence for the specification that accounts for changes in the slope of the trend. When choosing between these specifications, more favorable evidence is found in favor of the Balassa-Samuelson version of PPP. The median half-life point estimate are found to be lower than the consensus view regardless of the parity restriction.

The rest of the paper is structured as follows. Section 2 presents a brief account of the various PPP concepts used in this paper. An overview of the econometric methodologies is outlined in Section 3. Section 4 presents the data and main empirical results, while section 5 discusses estimates of half lives. Concluding remarks appear in Section 6. All proofs are relegated to the Appendix.

2   Price convergence, PPP and structural breaks

This section summarizes the different definitions and concepts that can arise when dealing with price convergence in the presence of structural breaks and how this relates to the PPP hypothesis. This is quite important provided that most of the papers that focus on the PPP hypothesis do not account for the presence of structural breaks and, more interestingly, those that consider this issue do not test for the real definition of PPP. Therefore, we believe that the discussion presented in this section can help to disentangle whether price

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[8]In this paper we do not address the source (e.g. distance) of relative price variability. In an influential work, Engel and Rogers (1996) show that both distance and border matter for relative price variability while examining disaggregated CPI data for 23 Canadian and US cities. Parsley and Wei (1996) concluded that distance alone cannot explain why convergence is faster within the US than across countries. Recently, Engel and Rogers (2001) found that sticky nominal prices play a more important role than distance in explaining the variation in prices between pairs of United States cities. Overall, the literature based on price data supports the idea that border barriers are significant. Obstfeld and Rogoff (2000) label the border effect on trade flows one of the 'six major puzzles in international macroeconomics.'

convergence occurs and whether it implies the fulfilment of the PPP hypothesis when multiple structural breaks are considered.

There is a flurry of papers in the economics literature that have investigated whether price convergence has taken part among individuals such as countries, regions or cities focusing on the time series $y_{i,j,t} = (\ln p_{i,t} - \ln p_{j,t})$, that is, the difference between the logarithm of the price of one individual $(p_{i,t})$ and the logarithm of the price of the benchmark individual $(p_{j,t})$, $i, j = 1, \ldots, N$, and $t = 1, \ldots, T$. Since our framework restricts to cities inside a country, $y_{i,j,t}$ can be seen as the real exchange rate provided that US cities share the same currency. The investigation of price convergence is mainly addressed through the assessment of the stochastic properties of the real exchange rates using unit root and stationarity statistics. When real exchange rates are characterized as stationary in variance processes – henceforth, I(0) stochastic processes – it is said that there is evidence in favor of the PPP hypothesis. However, the literature has defined two different concepts of PPP depending on whether real exchange rate evolves around a constant mean or around a time trend. Thus, when the deterministic component that is used in the computation of the unit root and stationarity tests is given by a constant term we are dealing with the Cassel (1918) definition of the PPP. Balassa (1964) and Samuelson (1964) devise a second concept of PPP when noticing that divergent international productivity lead to permanent deviations from the Cassel's PPP concept. This feature is captured through the specification of a long-run trend around which the real exchange rates would show stationary fluctuations, which defines the so-called "Trend PPP" (TPPP). Therefore, in this case unit root and stationarity test statistics have to use a linear time trend as the deterministic component when testing for TPPP.

Note that evidence in favor of either PPP or TPPP requires real exchange rate to be I(0). However, misspecification of the deterministic component of the models in which the unit root and stationarity statistics are based can lead to misleading conclusions. In this regard, Perron (1989) and Lee, Huang and Shin (1997) showed that the lack of accounting for the presence of structural breaks can bias the inference towards the non-stationarity in variance – see Perron (2006) for an overview. This feature has provoked, firstly, the introduction of structural breaks in the studies that analyze the order of integration of the real exchange rates and, secondly, the definition of new concepts of PPP that are

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compatible with the presence of structural breaks. To this end, Dornbusch and Vogelsang (1991) consider the presence of one structural break affecting the level of the real exchange rate and coin the term "Qualified PPP" (QPPP) to cover those situations in which real exchange rate is stationary around a changing deterministic component. One relevant feature is that Dornbusch and Vogelsang (1991) interpret this situation as evidence in favor of the Balassa-Samuelson model so that the inclusion of structural breaks can be nested in one of the accepted concepts of PPP in the literature. Other analyses have considered the presence of level shifts when testing the order of integration of real exchange rates with the deterministic specification given either by a constant – see, for instance, Perron and Vogelsang (1992), Hegwood and Papell (1998), and Gadea, Montañés and Reyes (2004) – or by a time trend – see Im, Lee and Tieslau (2005), and Papell and Prodan (2006). Following the convention established in Papell and Prodan (2006), we denote by QPPP those situations in which the real exchange rate evolves around a deterministic component given by a constant term with level shifts. Similarly, we denote by "Trend Qualified PPP" (TQPPP) the situation in which the real exchange rate evolves around a deterministic component given by a linear time trend with level shifts.

In fact, the TQPPP definition can accommodate other specifications than the ones defined above. Thus, it is possible that events such as the oil embargo or shocks affecting the technological process may change the productivity of individuals in different ways, so that divergences in productivity can be reduced or increased after the shocks, which may imply a change in the slope of the long-run trend around which the real exchange rates would show stationary fluctuations. This feature can be accounted for including structural breaks that affect both the level and the slope of the time trend. Economically, the presence of structural breaks can be argued from the fact that productivity shocks may have affected traded and non-traded goods sectors differently.[9] In this regard and focusing on the US economy, Bernard and Jones (1996, Table 1) show that labor productivity gains in the traded-good sectors (e.g. mining and manufacturing) have been greater than the productivity gains in the non-traded-good sectors (e.g. construction and service) between 1963 and 1989. They further find that the variation in productivity levels across sectors

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[9]The economic literature on productivity suggests numerous explanations for the sources of productivity differences that could vary systematically across geographic areas. See Kalemli-Ozcan et al. (2005) for an excellent overview.

is consistent with a large amount of variation in productivity across states. Vohra (1998) points to a significant gain in productivity levels in the mining states until the end of the second oil price shock and a drastic fall thereafter. Although these results are based on US states, one can arguably conjecture that such changes may have had affected the major cities of these states. In this regard, our view of the TQPPP as a weaker version of the TPPP Balassa-Samuelson definition can be justified.[10] It is worth mentioning that this broader definition of the TQPPP concept nests the Balassa-Samuelson and Dornbusch and Vogelsang (1991) concepts of PPP.

Evidence in favor of QPPP or TQPPP does not imply that PPP as defined in Cassel (1918), Balassa (1964) and Samuelson (1964) is fulfilled, since in these cases PPP requires reversion towards a constant mean or a constant trend in the long-run. Therefore, in the presence of structural breaks, QPPP and/or TQPPP is necessary but not sufficient condition for the PPP to hold. In this case, when we have found evidence in favor of QPPP and/or TQPPP further investigations should be conducted to conclude that the PPP hypothesis is satisfied according to Cassel (1918), Balassa (1964) and Samuelson (1964). To be specific, we require to impose parity restrictions on the coefficients of the first and last breaks so that these coefficients are of the same magnitude but opposite sign. Note that after imposing the parity restrictions the deterministic component does not change in the long-run. This implies that after the last break has occurred, the deterministic component of the time series equals the one previous to the first structural break – see Papell (2002) and Papell and Prodan (2006). In this paper we consider all these cases, and propose a new way to accommodate for the presence of multiple structural breaks when testing for the different definitions of PPP that have been described.

Of special relevance in all these analyses is the selection of the benchmark individual against which the real exchange rate is computed. There are mainly two different possibilities. First, most empirical investigations have tested the PPP hypothesis using either the

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[10]It is appropriate to cite Engel (1999), who demonstrate that nearly all variability in real exchange rates against the United States can be attributed due to changes in the countries' relative consumer price of traded goods. Engel's (1999) result is a striking contradiction of the Balassa-Samuelson model which necessities that all variability in real exchange rates is due to changes in international differences in two countries' relative price of traded to non-traded goods. In contrast, our results show support for the weak version of Balassa-Samuelson model within the US. This is to be expected since findings by both Parsley and Wei (1996) and Chen and Devereux (2003) indicate a much faster convergence in the relative consumer price of traded goods within the US.

average of the price levels or the price level of the leading individual in the sample – for instance, when dealing with international data sets the US price level is usually taken as the reference. The main drawback that can be stated to this approach is that the results can be dependent on the choice of the benchmark.[11] Second and in order to avoid this drawback, Pesaran et al. (2006) have suggested the use of pairwise tests of PPP, which focus on the PPP hypothesis using all possible $N(N-1)/2$ real exchange rate pairs between the countries considered in the analysis. As noted in Pesaran et al. (2006), this analysis is invariant to benchmark effects and the proportion of pairs that do not satisfy the PPP hypothesis can be consistently estimated. In this paper we consider the proposal in Pesaran et al. (2006), which allows to obtain more robust results than the ones based on the first approximation.

3 Methodology

This section briefly discusses the panel stationarity tests proposed in Hadri (2000) and Carrion-i-Silvestre, del Barrio-Castro and López-Bazo (2005). These statistics are the ones applied in the paper to investigate the different definitions of PPP described in the previous section. This has led us to design a new procedure that allows to test the PPP hypothesis with the inclusion of multiple structural breaks. Then, we briefly discuss about the effects of cross-section dependence when assessing the stochastic properties of panel data sets. Finally, we present two statistics to formally test the hypothesis of cross-section independence. All these statistics are used throughout the paper.

3.1 Panel stationarity tests with structural breaks

Hadri (2000) proposes an LM panel data stationarity test without structural breaks, while Carrion-i-Silvestre et al. (2005) extend the analysis to account for the presence of multiple structural breaks. Since the latter proposal encompasses the former one, we proceed to present the approach in Carrion-i-Silvestre et al. (2005). As above, let

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[11]This issue is even more relevant at the international level where researchers often face the choice of numeraire currencies–with US dollar being the most preferred benchmark. In this regard, Canzoneri et al. (1999) obtain more favorable evidence for the Balassa-Samuelson model with German mark relative to US dollar. See Papell and Theodoridis (2001) for further discussion on the influence of the choice of reference currency in empirical work.

$y_{i,j,t} = (\ln p_{i,t} - \ln p_{j,t})$ be the difference between the logarithm of the price of two individuals, for which we assume that its behavior can be modeled through

$$y_{i,j,t} = \alpha_{i,j} + \sum_{k=1}^{m_{i,j}} \theta_{i,j,k} DU_{i,j,k,t} + \beta_{i,j}t + \sum_{k=1}^{m_{i,j}} \gamma_{i,j,k} DT^*_{i,j,k,t} + \varepsilon_{i,j,t} \qquad (1)$$

where $t = 1,...,T$ and $i,j = 1,...,N$, $i \neq j$, indexes the time series and cross-sectional units, respectively. The dummy variables $DU_{i,j,k,t}$ and $DT^*_{i,j,k,t}$ are defined as $DU_{i,j,k,t} = 1$ for $t > T_{b,k}^{i,j}$ and 0 elsewhere, and $DT^*_{i,k,t} = t - T_{b,k}^{i,j}$ for $t > T_{b,k}^{i,j}$ and 0 elsewhere, with $T_{b,k}^{i,j}$ denoting the k-th date of the break for the (i,j) pair of individuals, $k = 1,...,m_{i,j}, m_{i,j} \geq 1$, $\alpha_{i,j}$ and $\beta_{i,j}$ are the parameters of the constant and time trend, respectively, and $\varepsilon_{i,j,t}$ denotes the disturbance term. Note that the proposal in Hadri (2000) follows from setting $\theta_{i,j,k} = \gamma_{i,j,k} = 0 \ \forall i,j,k, \ i \neq j$, in (1). The model in (1) includes individual effects, individual structural break effects (i.e. shift in the mean caused by the structural breaks known as temporal effects where $\beta_{i,j} \neq 0$) and temporal structural break effects (i.e. shift in the individual time trend where $\gamma_{i,j} \neq 0$). In addition, the specification given by (1) considers multiple structural breaks, which are located at different unknown dates and where the number of breaks are allowed to vary across the members of the panel. Note that the different concepts of PPP that have been defined in the previous section appear as particular cases of the model in (1). Thus, for the (i,j)-th pair of individuals the Cassel's definition of PPP is achieved when $\alpha_{i,j} \neq 0$ and $\beta_{i,j} = \theta_{i,j,k} = \gamma_{i,j,k} = 0 \ \forall k$ in (1), TPPP is obtained when $\alpha_{i,j} \neq \beta_{i,j} \neq 0$ and $\theta_{i,j,k} = \gamma_{i,j,k} = 0 \ \forall k$ in (1), QPPP is found when $\alpha_{i,j} \neq \theta_{i,j,k} \neq 0$ and $\beta_{i,j} = \gamma_{i,j,k} = 0 \ \forall k$ in (1) and, finally, TQPPP is found when $\alpha_{i,j} \neq \beta_{i,j} \neq \theta_{i,j,k} \neq \gamma_{i,j,k} \neq 0 \ \forall k$ in (1).

The test statistic is constructed by computing the stationarity test in Kwiatkowsky, Phillips, Schmidt and Shin (1992) – hereafter, KPSS test – for every member of the panel and then averaging the N individual statistics. The general expression for the test statistic is

$$LM(\lambda) = \frac{2}{N(N-1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \eta_{i,j}(\lambda_{i,j}), \qquad (2)$$

with $\eta_{i,j}(\lambda_{i,j}) = \hat{\omega}_{i,j}^{-2} T^{-2} \sum_{t=1}^{T} \hat{S}_{i,j,t}^2$, where $\hat{S}_{i,j,t} = \sum_{l=1}^{t} \hat{\varepsilon}_{i,j,l}$ is the partial sum process that is obtained using the estimated OLS residuals of (1). $\hat{\omega}_{i,j}^2$ denotes a consistent estimate

of the long-run variance of the error $\varepsilon_{i,j,t}$, which has been estimated following the procedure in Sul, Phillips and Choi (2005) – we use the Quadratic spectral kernel. In (2), $\lambda_{i,j}$ is defined as the vector $\lambda_{i,j} = \left(\lambda_{i,j,1}, ..., \lambda_{i,j,m_{i,j}}\right)' = \left(T_{b,1}^{i,j}/T, ..., T_{b,m_{i,j}}^{i,j}/T\right)'$, which indicates the relative position of the dates of the breaks on the entire time period, T, for each (i,j)-th pair of individuals – note that for the test in Hadri (2000) $\lambda_{i,j} = 0$ $\forall i, j$, $i \neq j$, since there is no structural breaks. Assuming that individuals are cross-section independent, Hadri (2000) and Carrion-i-Silvestre et al. (2005) show that $LM(\lambda)$ reaches the following sequential limit under the null of stationary panel with multiple shifts

$$Z(\lambda) = \frac{\sqrt{N}(LM(\lambda) - \bar{\xi})}{\bar{\varsigma}} \to N(0,1),$$

where $\bar{\xi}$ and $\bar{\varsigma}$ are the cross-sectional average of the individual mean and variance of $\eta_{i,j}\left(\lambda_{i,j}\right)$, which are defined in Hadri (2000) and Carrion-i-Silvestre et al. (2005).

In order to estimate the number of breaks and their locations, Carrion-i-Silvestre et al. (2005) follow the procedure developed by Bai and Perron (1998), which proceeds in two steps.[12] First, the breakpoints are estimated by globally minimizing the sum of squared residuals for all permissible values of $m_{i,j} \leq m^{\max}$, $i, j = 1, ..., N$, $i \neq j$. Second, we use the sequential testing procedure suggested in Bai and Perron (1998) to estimate the number of structural breaks. As a result, we obtain the estimation of both the number and position of the structural breaks. This procedure is then repeated N times to obtain the estimated number of breaks and their locations for each individual. It is worth mentioning that this approximation considers the situation of no structural breaks, so that the case in which some individuals might be not affected by the presence of structural breaks is taken into account.

3.2 PPP hypothesis with structural breaks

The use of the individual KPSS and the panel stationarity statistic that have been described so far allow to detect QPPP and/or TQPPP hypothesis when structural breaks

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[12]Note that the sequential approach in Bai and Perron (1998) can be used here since under the null hypothesis of the statistic we have that the units are stationary in variance. Therefore, the procedure in Bai and Perron (1998), which was proposed to estimate the number and position of the structural breaks in an I(0) framework, can be used. Under the alternative hypothesis of I(1) the test statistic is consistent even in the case where the break points are not properly estimated, as shown in Lee, Huang and Shin (1997), Kurozumi (2002) and Carrion-i-Silvestre (2003), among others.

are involved. Notwithstanding, evidence in favor of the QPPP and/or TQPPP does not imply that PPP holds. If we are interested in testing the PPP hypothesis we should include the parity restrictions in the model so that the parameters of the first regime equals the ones in the last regime. This has led us to extend the previous approach to consider these parity restriction when there are multiple structural breaks.

Let us consider the DGP given by (1) expressed using orthogonal regressors for the model that includes a time trend with both level and slope shifts as

$$y_{i,j} = x_{i,j}\delta_{i,j} + \varepsilon_{i,j} \tag{3}$$

where $x_{i,j} = diag\left(x_{i,j,1},\ldots,x_{i,j,m_{i,j}+1}\right)$, $x_{i,j,k,t} = (1,t)$, $\delta_{i,j} = \left(\delta_{i,j,1},\ldots,\delta_{i,j,m_{i,j}+1}\right)'$ and $\delta_{i,j,k} = \left(\mu_{i,j,k},\beta_{i,j,k}\right)'$ for $T_{b,k-1}^{i,j} < t \leq T_{b,k}^{i,j}$, $k = 1,\ldots,m_{i,j}+1$, with the convention that $T_{b,0}^{i,j} = 0$ and $T_{b,m_{i,j}+1}^{i,j} = T$, being $m_{i,j}$ the number of structural breaks for the (i,j)-th pair of individuals, $i,j = 1,\ldots,N$, $i \neq j$. The parity restrictions imply that the parameters of the first regime, $\delta_{i,j,1}$, and the ones for the last regime, $\delta_{i,j,m_{i,j}+1}$, have to be equal, while the parameters of the other regimes are left free. Note that this is the case because we has specified the model in a way that regressors are block orthogonal, so that the parameters on each orthogonal block are not affected by the values of the parameters in the previous blocks.[13] In this set-up these parity restrictions can be expressed as $R\delta_{i,j} = r$ with $R = \begin{bmatrix} I_l & 0_{l\times(m_{i,j}-1)l} & -I_l \end{bmatrix}$ and $r = 0_{(m_{i,j}+1)l\times 1}$, where I_l denotes the identity matrix, l is the number of regressors in $x_{i,j,k}$ – $l = 1$ in the constant only case and $l = 2$ in the case of the linear time trend – and $0_{a\times b}$ an $(a \times b)$-matrix of zeros. Using these elements, we can compute the restricted least squares estimator $\left(\hat{\delta}_{i,j}^*\right)$ of $\delta_{i,j}$ in (3) such that the estimator satisfies $R\delta_{i,j} = r$. It is worth mentioning that we require at least two structural breaks in order to impose the parity restrictions, since parity restrictions for the one break case will imply the absence of the structural break.

The estimation of the restricted least squares estimator of $\delta_{i,j}$ is carried out using the dynamic programming algorithm recently proposed in Perron and Qu (2006) which permits the consideration of multiple structural breaks with restrictions among the parameters of

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[13]This would not be the case if the model is not specified in terms of orthogonal regressors. Then, the restrictions on the parameters should be imposed so that the sum of all the coefficients for the level shifts (and the slope shifts if we deal with a trending variable) should be equal to zero.

the different regimes. The proposal that we suggest in this paper proceeds in two stages: (i) we estimate the number of structural breaks using the unrestricted dynamic algorithm in Bai and Perron (1998) and, conditional to the number of structural breaks, (ii) minimize the restricted sum of squared residuals to estimate the position of the structural breaks with the vector of parameters $\delta^*_{i,j}$ satisfying $R\delta_{i,j} = r$, with R and r defined above.

The restricted estimated disturbance term $\hat{\varepsilon}^*_{i,j}$ can be used to compute the stationarity tests proposed in the previous section, i.e. the individual-by-individual restricted KPSS statistic given by

$$\eta^*_{i,j}(\lambda_{i,j}) = \hat{\omega}^{-2}_{i,j} T^{-2} \sum_{t=1}^{T} \hat{S}^{*2}_{i,j,t} \tag{4}$$

and the corresponding restricted panel data $(Z^*(\lambda))$ statistic. The following Theorem provides the limiting distribution of the individual KPSS with multiple structural breaks that considers the parity restrictions.

Theorem 1 *Let $\{y_{i,j,t}\}_{t=1}^{T}$ be the stochastic process generated according to (3), with $\{\varepsilon_{i,j,t}\}_{t=1}^{T}$ a stochastic process satisfying the strong mixing regularity conditions defined in Phillips (1987) and Phillips and Perron (1988). Furthermore, let $\delta^*_{i,j}$ the vector of parameters that satisfies $R\delta_{i,j} = r$, with $R = \begin{bmatrix} I_l & 0_{l \times (m_{i,j}-1)l} & -I_l \end{bmatrix}$, where $l = 1$ for the model that includes a constant and level shifts, and $l = 2$ for the model that includes a linear time trend with both level and slope shifts. Thus, as $T \to \infty$ and $T^{i,j}_{b,k} \to \infty$ in a way that $\lambda_{i,j,k} = T^{i,j}_{b,k}/T \; \forall k, \; k = 1, \ldots, m_{i,j}$, remains constant, the $\eta^*_{i,j}(\lambda_{i,j})$ statistic given by (4) converges to*

$$\eta^*_{i,j}(\lambda_{i,j}) \Rightarrow \int_0^{\lambda_{i,j,1}} M_{i,j,1}(\lambda_{i,j})^2 \, dr + \cdots + \int_{\lambda_{i,j,m_{i,j}}}^{1} M_{i,j,m_{i,j}+1}(\lambda_{i,j})^2 \, dr,$$

where $\Rightarrow$ indicates weak convergence to the associated measure of probability and $M_{i,j,k}(\lambda_{i,j})$ denotes the orthogonal projection of a standard Brownian motion onto the space spanned by the regressors and the terms involved in the definition of the restrictions.

The proof of the Theorem is given in the Appendix. The limit distribution of the statistic depends on the number of structural breaks as well as on their relative position in the sample. We have computed asymptotic critical values for $m = 2, \ldots, 9$ structural breaks, for all possible combinations of $\lambda_k = \{0.1, 0.2, \ldots, 0.9\}$, $k = 1, \ldots, m$. In order

to summarize the information, we have estimated response surfaces to approximate the asymptotic critical values, for which we have essayed the following functional form

$$cv\left(\lambda\right) = \beta_{0,0} + \beta_{0,1}m + \sum_{k=1}^{9} \beta_{1,k}\lambda_k + \sum_{l=1}^{3} \sum_{k=1}^{7} \sum_{j=k+1}^{8} \delta_{l,k,j}\left|\lambda_k - \lambda_j\right|^l + u,$$

where λ is a (9×1)-vector of the sorted (in ascending order) values of the break fraction parameters.[14] In addition, we have use the same functional form to approximate the mean and the variance of the statistics for each combination of break fractions, which is required in the computation of panel data statistics similar to those defined above for the non-restricted case. The estimated coefficients of the response surfaces for the percentiles of the 90, 95, 97.5 and 99%, along with those for the mean and the variance, are reported in Table 1 for the model that includes a constant term and level shifts, and in Table 2 for the model that includes a time trend with both level and slope shifts.[15]

We have conducted a small set of Monte Carlo simulations to assess the finite sample performance of the restricted panel data statistic. The DGP is given by (1) with $\alpha_{i,j} = \beta_{i,j} = 0 \ \forall i,j$, $\theta_{i,j,k} \sim U\left(4,5\right)$, $\gamma_{i,j,k} \sim U\left(0.4,0.5\right)$, with two structural breaks $(m_{i,j} = 2 \ \forall i,j)$, where U denotes the uniform distribution. The disturbance term $\varepsilon_{i,j,t}$ is generated according to $\varepsilon_{i,j,t} = \rho_{i,j}\varepsilon_{i,j,t-1} + v_{i,j,t}$ where $\rho_{i,j} \sim U\left(0.5,0.6\right)$ for the empirical size and $\rho_{i,j} = 1 \ \forall i,j$, for the empirical power analyses with $v_{i,j,t} \sim iid \ N\left(0,1\right)$. The simulations are carried out for all combinations of $T = \{100, 200, 300\}$ observations and $N = \{20, 40\}$ individuals considering both the QPPP and TQPPP specifications. The long-run variance has been estimated as described above.

Table 3 shows that the empirical size of the statistic is around the nominal 5% level as T increases for the QPPP specification, but the test is mild undersized for the TQPPP specification. As for the empirical power, we see that all values equal one for the QPPP specification, while the one for the TQPPP model it is bit lower but, as expected, increases with T and N. Note that the lower power shown by the TQPPP compared to the QPPP

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[14]When there are less than nine structural breaks $(m_{i,j} < 9)$, the first $m_{i,j}$ positions of the λ vector collect the break fractions and the other $(9 - m_{i,j})$ positions are zero.

[15]All possible combinations of break fraction values has given 502 observations that are used to estimate the response surfaces. We have computed robust standard errors using the Newey and West (1994) estimator. All reported parameters in Tables 1 and 2 are statistical significant at the 10% level. The adjusted coefficient of determination for all estimated functions is at least of 0.96.

is due to the larger number of parameters involved in the former specification. In all, we have seen that the statistic that we have proposed shows good performance, especially for large T, since the empirical size is close to the nominal one and the presence of structural breaks does not cause power loses.

3.3 Testing for cross-section independence

Recent developments in the literature offer the possibility of testing for the presence of cross-section dependence among individuals. Pesaran (2004) designs a test statistic based on the average of pair-wise Pearson's correlation coefficients $\hat{p}_j$, $j = 1, 2, \ldots, n$, $n = N(N-1)/2$, of the residuals obtained from ADF-type regression equations. The CD statistic in Pesaran (2004) is given by

$$CD = \sqrt{\frac{2T}{n}} \sum_{j=1}^{n} \hat{p}_j \rightarrow N(0, 1).$$

This statistic tests the null hypothesis of cross-section independence against the alternative of dependence.

Besides, Ng (2006) relies on the computation of spacings to test the null hypothesis of independence. In brief, the procedure in Ng (2006) works as follows. First, we get rid of autocorrelation pattern in individual time series through the estimation of an AR model. As for the test in Pesaran (2004), this allows us isolating cross-section regression from serial correlation. Taking the estimated residuals from the ADF-type regression equations as individual series, we compute the absolute value of Pearson's correlation coefficients ($\bar{p}_j = |\hat{p}_j|$) for all possible pairs of individuals, $j = 1, 2, \ldots, n$, where as above $n = N(N-1)/2$, and sort them in ascending order. As a result, we obtain the sequence of ordered statistics given by $\{\bar{p}_{[1:n]}, \bar{p}_{[2:n]}, \ldots, \bar{p}_{[n:n]}\}$. Under the null hypothesis that $p_j = 0$ and assuming that individual time series are Normal distributed, $\bar{p}_j$ is half-normally distributed. Furthermore, let us define $\bar{\phi}_j$ as $\Phi\left(\sqrt{T}\bar{p}_{[j:n]}\right)$, where Φ denotes the cdf of the standard Normal distribution, so that $\bar{\phi} = \left(\bar{\phi}_1, \ldots, \bar{\phi}_n\right)$. Finally, let us define the spacings as $\Delta\bar{\phi}_j = \bar{\phi}_j - \bar{\phi}_{j-1}$, $j = 1, \ldots, n$.

Second, Ng (2006) proposes splitting the sample of (ordered) spacings at arbitrary $\vartheta \in (0, 1)$, so that we can define the group of small (S) correlation coefficients and the

group of large (L) correlation coefficients. The definition of the partition is carried out through minimization of the sum of squared residuals

$$Q_n\left(\vartheta\right) = \sum_{j=1}^{[\vartheta n]} \left(\Delta\bar{\phi}_j - \bar{\Delta}_S\left(\vartheta\right)\right)^2 + \sum_{j=[\vartheta n]+1}^{n} \left(\Delta\bar{\phi}_j - \bar{\Delta}_L\left(\vartheta\right)\right)^2,$$

where $\bar{\Delta}_S\left(\vartheta\right)$ and $\bar{\Delta}_L\left(\vartheta\right)$ denotes the mean of the spacings for each group respectively. Consistent estimate of the break point is obtained as $\hat{\vartheta} = \arg\min_{\vartheta\in(0,1)} Q_n\left(\vartheta\right)$, where definition of some trimming is required – we follow Ng (2006) and set trimming at 0.10.

Once the sample has been splitted, we can proceed to test the null hypothesis of non-correlation in both sub samples. Obviously, rejection of the null hypothesis for the small correlations sample will imply rejection for the large correlations sample provided that the statistics are sorted in ascending order. Therefore, the null hypothesis can be tested for the small, large and the whole sample using the Spacing Variance Ratio $SVR\left(\eta\right)$ in Ng (2006), with $\hat{\eta} = \left[\hat{\vartheta} n\right]$ being the number of statistics in the small correlations group. Ng (2006) shows that under the null hypothesis that a subset of correlations is jointly zero, the standardized statistic $svr\left(\eta\right) \rightarrow N\left(0,1\right)$.

One advantage of the approach in Ng (2006) is that it allows us gaining some insight about the kind of cross-section dependence in terms of how pervasive and strong is the cross-section correlation. The use of these statistics will help us to decide in which panel stationarity statistic we should most base the statistical inference.

3.4   Cross-section dependence

The presentation of the panel statistics so far has assumed that individuals are cross-section independent. However, this assumption might be restrictive in practice since the analysis of macroeconomic time series for different countries are affected by similar major events that might introduce dependence among individuals in the panel data set. There are different approximations in the literature to deal with cross-section dependence. In this paper we account for cross-section dependence in two ways. First, we follow the suggestion in Levin, Lin and Chu (2002) and proceed to remove the cross-section mean, which is equivalent to include temporal effects in the panel data set. Second, we follow Maddala and Wu (1999) and compute the empirical distribution by means of parametric

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bootstrap. These two approaches are applied for all test statistics described above.

Other proposals in the literature that deal with cross-section dependence are O'Connell (1998), who estimates a SUR specification, and Bai and Ng (2004a, b), Moon and Perron (2004), Harris, Leybourne and McCabe (2005) and Pesaran (2006), who use common factor models. However, in our case the statistic in Ng (2006) that is computed below indicates that the presence of cross-section dependence is not pervasive, so that a common factor structure does not appear to be a suitable characterization of the cross-section dependence in the sample that is used.

4 Empirical Results

4.1 Data

We extend the data set used by Cecchetti et al. (2002) and Chen and Devereux (2003) to include more recent observations consisting annual Consumer Price Index (CPI) covering the period 1918 to 2005 ($T = 88$) for $N = 17$ US cities: Atlanta, Boston, Chicago, Cincinnati, Cleveland, Detroit, Houston, Kansas City, Los Angeles, Minneapolis, New York, Philadelphia, Pittsburgh, Portland, San Francisco, Seattle, and St. Louis.[16] All data were extracted from the Bureau of Labor Statistics's (BLS) webpage (www.bls.gov).

The limitation of the BLS price indices is that they do not measure absolute city price levels and, hence, they do not provide information on the relative cost of living across cities at a point in time. In order to overcome this drawback we follow Chen and Devereux (2003) to compute price level series. In brief, Chen and Devereux (2003) use the absolute cost of living prices that Koo, Phillips and Sigalla (2000) obtained using disaggregated BLS data for 1989. Thus, we take the absolute price level for 1989 as the starting point and apply (backward and forward in time) the inflation rates calculated from individual city price indices to obtain absolute price indices for other years. The 1989 benchmark estimates are based on Koo et al. (2000). Chen and Devereux (2003) also check the reliability of their estimates on the basis of two other benchmark years (1935 and 1975) and concluded that their proxy for price dispersion is close to alternative benchmarks.

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[16]Note that, the original Cecchetti et al. (2002)'s sample consists of 19 cities including Baltimore and Washington DC. However, since 1996, the Bureau of Labor Statistics no longer maintains separate data for these two cities. As a result, these cities are excluded from the analysis.

We only dispose of information for general price level but not disaggregate price level since we do not have the baskets for different concepts. Therefore, we maintain the analysis at general level. This may explain why the presence of time trend is required since, unlike individual price levels, aggregate price levels may not adjust so quickly due to differences in productivity among traded and non-traded sectors. Thus, when the absolute city prices are converging, bilateral real exchange rates may contain a time trend with or without structural breaks depending on the pattern of convergence. In fact, Chen and Devereux (2003) graphically identified the evidence of structural breaks and time trends in the US city real exchange rate for the period 1918-2000. However, there might be more than one structural break affecting the real exchange rates so that the analysis should cover general situations in which both the number and location of the structural breaks are unknown.

4.2 Price convergence and structural breaks

Robust conclusions to the specification of the benchmark can be obtained if we base the study on all possible pair of exchange rates, i.e. $N(N-1)/2 = 136$ pairs in this case. As mentioned above, the number of break points is estimated following the procedure in Bai and Perron (1998) with the sequential testing or the LWZ information criterion in Liu, Wu and Zidek (1998) depending on the presence of broken linear trends. The initial maximum number of structural breaks is $m^{\max} = 5$, although in few cases the maximum was achieved for which $m^{\max}$ has been increased to $m^{\max} = 8$ – the new maximum is never reached.

Table 4 summarizes the proportion of rejections of the null hypothesis of I(0) in each case – detailed results are available from the authors in a companion Appendix. We can see that more evidence against the null hypothesis is found when the TQPPP specification is considered. Thus, using 5% critical values the null hypothesis is rejected for 13.2% of all possible pairs when using the QPPP specification, while the proportion increases up to the 35.3% for the TQPPP one. We have reported the proportion of rejections that is obtained when the BIC information criterion is used to select between the QPPP and the TQPPP hypothesis specifications for each time series (mixed case). In this situation, the proportion of rejections is 33.8% using the critical values at the 5% level of significance. Therefore, we cannot conclude that there is strong evidence against the null hypothesis of

either QPPP, TQPPP or the mixture QPPP/TQPPP versions of the PPP hypothesis.

This individual information can be combined to define panel data statistics. Before proceeding, we have computed the statistics in Pesaran (2004) and Ng (2006) to test the hypothesis of cross-section independence. The CD statistic for the constant with level shifts equals 27.113, while for the linear trend with level and slope shifts is 23.66. As can be seen, for both deterministic specifications the null hypothesis of independence is strongly rejected. The same conclusion is found when we compute the statistic in Ng (2006). Thus, for the constant with level shifts we obtain $svr^W(\hat{\eta}) = 26.610$ (p-value 0.000), $svr^L(\hat{\eta}) = 23.935$ (p-value 0.000) and $svr^S(\hat{\eta}) = 2.260$ (p-value 0.012), which indicate strong rejection of the null hypothesis of cross-section independence. In this case, the proportion of individuals in the small sample is 0.503. For the deterministic component given by the linear trend with level and slope shifts we have $svr^W(\hat{\eta}) = 23.657$ (p-value 0.000), $svr^L(\hat{\eta}) = 21.714$ (p-value 0.000) and $svr^S(\hat{\eta}) = 2.786$ (p-value 0.003), with the proportion of individuals in the small sample that equals 0.58. From these results we can conclude that cross-section is present among the individuals that define the panel data set, although this dependence is not strong, provided that half of the correlations are located in the small sample.

The results on the computation of the panel data statistics are reported in Table 5. When cross-section dependence is taken into account through cross-section demeaning, the null hypothesis of I(0) cannot be rejected either for the specification consistent with the QPPP hypothesis at the 5% level of significance, although the null is clearly rejected for the one given by the TQPPP hypothesis and the mixture of the QPPP/TQPPP hypotheses. Conclusive results are obtained if we base the inference on the bootstrapped critical values. In this case, the null hypothesis of I(0) is not rejected at the 5% level for either the QPPP, the TQPPP or the mixture of the QPPP/TQPPP hypotheses regardless of the statistic that is used.[17]

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[17]As above, the selection of the model that is used to compute the panel data statistics with the mixture of the QPPP and TQPPP specifications is based on the BIC information criterion. Only for eighteen out of the one hundred and thirty-six pairs of cities the QPPP specification is selected.

4.3 PPP and structural breaks

The consideration of the parity restrictions in the analysis provides the results reported in Table 4, where proportions of rejections of the null hypothesis of I(0) are presented. First of all, we can see that restricting the parameters of the QPPP model gives a proportion of rejections of 31.5% using the critical values at the 5% level of significance, while the proportion for the TQPPP hypothesis is of the 8.8%. These results show that imposing parity restrictions on the QPPP model specification may imply incredible restrictions that are not to be satisfied in practice. However, the converse is found for the TQPPP model specification, which shows that the more flexible specification that defines the TQPPP hypothesis is more likely to satisfy the parity restrictions. As expected, the use of the BIC information criterion to select between these two specifications for each individual gives a proportion that lies between both situations – the QPPP specification is selected in the 23.8% of the cases.

Table 4 also reports the proportions of rejection for the non-restricted model that are obtained for the same time series for which the parity restrictions are of application. We can see that the proportions for the QPPP hypothesis are smaller (13%) when we do not impose the parity restrictions, while it increases for the TQPPP hypothesis (39.8%) and the mixed QPPP/TQPPP hypothesis (40.3%). These results indicate that more evidence is found in favor of the TPPP hypothesis than for the PPP hypothesis in those cases for which parity restrictions can be imposed.

The picture based on the individual statistics can be completed with the results from the panel data statistics. Table 5 indicates that the null hypothesis of I(0) is rejected at the 5% level for both the restricted QPPP and TQPPP hypotheses when using the bootstrap critical values. These results show that there is no evidence in favor of the PPP hypothesis when we consider the whole panel data set. Therefore, we have to conclude that the PPP hypothesis does not hold for all the pairs of real exchange rates that have been considered in this paper.

5   Half-life estimates

In this section we estimate the half-life (HL) of shocks affecting the real exchange rates. We have followed the approach in Andrews and Chen (1994) in order to compute median unbiased (MU) estimates of the autoregressive parameters in which the computation of the HLs relies on.[18] To do so, we have estimated an AR model using the number and position of the structural breaks that have been obtained in the previous sections.

Let us first focus on the results for the unrestricted specifications. Provided that the panel data statistics that have been applied in the previous section indicates that the null hypothesis of I(0) cannot be rejected for either the QPPP, TQPPP or the combination of the QPPP/TQPPP specification, we have computed MU HLs estimates for all pairs of real exchange rates. In order to save space, we only report detailed results for the combination of the QPPP/TQPPP specifications in Table 6 – results for the QPPP and TQPPP specifications are available upon request. As can be seen, most of the HL estimates are below the consensus range of 3–5 years mentioned in Rogoff (1996).[19] Overall, the median half-life estimate shows a faster adjustment to PPP than the consensus view. Similar results are also found when using the QPPP and TQPPP specifications. In order to get a complete picture we have summarized in Table 7 the percentage of HLs that are below, within and above the 3–5 years consensus. Note that the vast majority of HLs are below or within the consensus for the three different situations that we consider, which indicates that taking into account the presence of structural breaks that might be affecting the time series reduces the persistence of the difference in the price levels of the US cities.

We have shown that the consideration of the parity restrictions does not allow us to conclude in favor of any of the PPP, TPPP and PPP/TPPP hypothesis when using the whole panel data set of individuals. Therefore, we have only computed HL estimates for those individuals for which the null hypothesis of I(0) is not rejected (individual-by-individual analysis). As above, we present the summary of the HL estimates in Table 7

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[18]Formally, idea behind the concept of median-unbiasedness can be explained as follows. Let $m(\alpha_i)$ denote the median function of an arbitrary estimator, $\widehat{\alpha}_i$ say, of α_i. This function is defined by $P(\widehat{\alpha}_i < m(\alpha_i)) = 0.5$, which can be inverted to obtain another estimator $m^{-1}(\widehat{\alpha}_i)$ of α_i. By construction, this estimator satisfies $P(m^{-1}(\widehat{\alpha}_i) < \alpha_i) = 0.5$ so the probability of underestimation is equal to the probability of overestimation. An estimator that has this property is said to be median-unbiased.

[19]Our results are different from, but not in contradiction to, Cecchetti et al. (2002), who find a half-life of nearly nine years for aggregate CPI data.

according to whether they are below, within or above the 3–5 years consensus.

5.1 Related literature

Empirical modeling of the half-life is subject to several statistical biases that can pollute the estimation results. Some commonly known biases are: Nickell's small-sample attenuating bias, cross sectional aggregate bias and time aggregation bias.[20] Early panel studies[21] that report shorter half-lives relative to the consensus view were sharply criticized by Murray and Papell (2005) for improper estimation of the autoregressive coefficients that cause downward bias in the half-life. Murray and Papell (2005) address this problem by applying the approximate median-unbiased (MU) estimation method of Andrews and Chen (1994) to post-1973 quarterly data for 20 OECD countries. They arrive at a point estimate of 4.6 years with a 95% confidence interval of 2.7–5.3 years, which nicely embraces the consensus range. Unfortunately, Murray and Papell (2005) restrict the autoregressive coefficients to be identical across units, which is the main source for cross sectional aggregation bias.

Choi, Mark and Sul (2006) address the three biases directly. They first rule out aggregation by testing the homogeneity restriction in the data and unable to find strong evidence against homogeneity, indicating that pooling is appropriate. Since (pure) Nickell bias and the time aggregation bias work in opposite directions (i.e. downward versus upward bias), Choi et al. (2006) first estimate ρ by least squares dummy variable (LSDV) and then use the inverse function to simultaneously correct for the two biases.[22] They obtain an unbiased half-life of 3 years with a 95% confidence interval of 2.3–4.2 years.

The availability of micro data across countries have helped researchers to analyze the deviation from PPP more carefully. In a highly cited article, Imbs et al. (2005) demonstrate that in the presence of heterogeneity in the dynamics of relative price adjustment across disaggregated sectors, standard estimators that do not allow for such heterogeneity are inconsistent and a positive bias is induced equivalent to systematically assigning a larger weight to those components of the aggregate series that are most important. Using disaggregate monthly data on sectoral real exchange rates against the US for 19 sectors

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[20]See, for instance, Choi et al. (2006) for relevance of each bias in applied PPP work.

[21]See, for instance, Oh (1996), Wu (1996), Papell (1997, 2002), Fleissig and Strauss (2000), Papell and Theodoridis (2001), to mention a few).

[22]To control for cross section dependence Choi et al. (2006) follow the procedure in Phillips and Sul (2007) which is valid if a single common factor is present in the composite error term.

and 12 countries for the period 1981-1995 and, employing the preferred mean group esti-
mator that allows for cross section dependence in the residuals, Imbs et al. (2005) obtain
an average aggregate half life of just 11 months, much less than the consensus range of
three to five years. More recently, Crucini and Shintani (2006) study the dynamics of
(good-by-good) real exchange rates using a micro panel of 270 goods prices coming from
major cities in 63 countries and 258 goods prices drawn from 13 major US cities. For the
OECD samples, they find that non-traded good has a half-life of 1.9 years compared to
1.2 years for traded goods. More importantly, aggregating the micro-data increases the
persistence in the OECD by 6 months to 1.5 years, well below the consensus range.

6   Conclusions

The main objective of this paper is to bring new light on the question of price level
convergence and how it relates to PPP hypothesis. We suggest a new procedure for
testing PPP, which is robust to multiple structural breaks and cross-sectional dependence.
Our approach can handle different notions of PPP hypothesis that have evolved in the
literature over the last several decades. Close calls, such as whether price revert faster
to Boston or San Francisco, have an important effect in empirical work based on based
country/city effect, but none at all in our approach based on pairwise approach tests. The
pairwise approach focuses on all possible $N(N-1)/2$ pairs of real exchange rate between
two cities in the sample and estimate the proportion of the pairs that are stationary.

As an empirical application, we have utilized aggregate annual price data from 1918
to 2005 for seventeen US cities. We find evidence for stationarity around a changing
level when the parity restriction is not imposed, while imposing parity restriction provides
favorable evidence for the specification that accounts for changes in the slope of the trend.
When choosing between these specifications, more favorable evidence is found in favor
of the PPP hypothesis thus corroborating the Balassa-Samuelson version of PPP. The
median half-life point estimate are found to be lower than the consensus view regardless
of the parity restriction.

According to the Balassa-Samuelson model deviations from PPP are due to cross-
country (or cross-city in the present analysis) differentials in the productivity of technol-

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ogy to produce traded and non-traded goods. Our results provide empirical support for the notion of Balassa-Samuelson model in which real exchange rate evolves around a deterministic component given by a linear time trend with level and slope shifts. Finally, it is worth mentioning that our framework can be used in other fields of economic as well. For instance, the approach can be used in those applications that analyze the interest rate parity, convergence in wages, Fisher effect, among other. We expect that these and related applications will be exciting avenues for future research.

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