Dsge estimates with non stationary data*

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Abstract

A growing area of applied macroeconomic research is devoted to the employment of dynamic stochastic general equilibrium (dsge) models for policy analysis. In many cases, dsge models are intended to describe and capture the cyclical movements of the macroeconomic variables. This behavior barely matches the highly non-stationary pattern of the macroeconomic data. So, in order to take the data to the model, a researcher needs to extract the stationary component from the data using some filtering techniques, and it is well known that the improper use of filters can create spurious cyclical behavior and in some cases bias the structural parameters estimates. In this paper, I propose a method to estimate jointly the structural parameters of the dsge model and the filtering parameters using bayesian techniques; in particular, I consider linear detrending (ld), first difference (fd) and the Hodrick Prescott filter (hp). This method allows a researcher to validate different tend specifications, and above all to get estimates of the structural parameters robust to the filter used. Simulations results suggest that (a) the joint estimate of structural and filtering parameters are reliable under wrong specification of the trend, and (b) under correlation between trend and cycles this method improves in term of bias relative to the standard two steps approach.

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1 Introduction

Dynamic stochastic general equilibrium (dsge) models are now considered one of the most important framework for macroeconomic analysis. Massive progresses have been done in estimating the deep parameters of dsge models; from loosely calibrated parameters, applied researchers can now estimate densely parameterized models with classical and Bayesian methods. These improvements allowed a researcher to assess the degree of fitness of the model both in and out of sample, and in some cases to capture transmission mechanisms, to evaluate policy implications, to test counterfactuals hypothesis. In general, dsge model are taken more seriously as a tool for policy analysis because of rigorous econometric evaluations. In the literature of dsge estimation, it is common practice to linearize the equilibrium conditions around a pivotal point, the steady state, and interpret the movements of the variables around it as cyclical fluctuations. As it is well known, economic data are quite persistent and display clear trend. So, before taking the model to the data, a researcher *must* construct a data analog for the model concepts. Therefore, one needs a method to remove the trend component form the data and transform the highly stationary behavior of the data into stationary. There are mainly two ways to account for it: either a researcher could rewrite the model and transform the variables (i.e. rewrite the model in terms of ratios), or could filter the data. With the first method, there is no guarantee that the transformed variables are indeed stationary; for example, consumption-gdp ratio or investment-gdp ratio look stationary, but hour worked over gdp is hard to think as a stationary process (see Figure 1). Therefore, everybody opts for filtering the series. For the purpose of this paper, I will consider the linear trend (lt) filter, the Hodrick-Prescott (hp) filter and first difference (fd) filter, which are the most widely used filters[1]. In principle, all of them remove the low frequencies of the times series, as in Figures 2-4, but there are differences in the amplitude and the duration of the cycles, as in Canova (1998). Therefore, depending on the filter used, one could obtain different estimates and and possibly draw different policy implications[2]. In other words, a researcher could

[1]Table 1 reports a non exhaustive listing of notable papers that estimate dsge models and for which I report the assumption about the exogenous processes, the filter used to detrend the data and the estimation technique.

[2]Harvey and Jaeger (1993) showed that using the Hodrey Prescott filter inappropriately can result in creation of spurious cyclical behavior, and this point is illustrated with empirical examples.

use lt or hp or fd filter and obtain posterior estimates conditional on different data set, i.e. $g(\theta|y^{lt})$ or $g(\theta|y^{hp})$ or $g(\theta|y^{fd})$, where θ is the vector of parameters and y^j are the filtered data with $j = lt, hp, fd$. In this paper, I propose a method to estimate jointly the 'deep' parameter of the dsge model and the paraments governing the non-stationary component in the time series. For the sake of the argument, let θ^m be the set of 'deep' parameters of the model and θ^f the parameters of the filter. The MCMC algorithm will produce the joint posterior distribution of the 'deep' parameters and of the filter parameters conditional on the data and on the filter used, i.e. $g(\theta^m, \theta^{lt}|y, f^{lt})$ or $g(\theta^m, \theta^{hp}|y, f^{hp})$ or $g(\theta^m, \theta^{fd}|y, f^{fd})$. These three distributions are comparable because the likelihood is computed at the same data point. With these posterior distributions we can test trend specifications in a classical set up or in a bayesian framework take decisions build on some loss function. Moreover, given the uncertainty of the trend specification, we can construct estimates of the structural parameters that are robust to the trend specifications by bayesian averaging across posterior distributions.

The present analysis is tightly related to the literature on filtering data. On one hand, we learned that filtering affects the structural property of the time series: in fact, improper filtering can produce spurious cycle (e.g. Harvey and Jaeger (1993)), or can alter the persistence and volatility of the series (e.g. Cogley and Nason (1995)). On the other hand, structural parameters estimation can be corrupted by the wrong choice of the trend. Cogley (2001) shows that inappropriate choice of trend, either trend stationary or first difference stationary, can lead to strong bias in parameters estimates using Maximum Likelihood methods; in particular, when the wrong trend specification is confronted with the data, its deep parameters adjust to compensate for this distortion. Therefore, even when the reduced form is correctly specified, trend misspecification is likely to result in inconsistent estimate of deep parameters. On the same track, Gorodnichenko and Ng (2007) show that estimates can be severely biased when model trend specifications are inconsistent with the data or detrended data are inconsistent with model concept of stationarity. They also propose a robust approach to address these two problems simultaneously using simulated methods of moments.

They both point at the bias arising from the 'mismatch' between the concept of non stationarity in the model and the one in the data; they need to transform the series into stationary in a way that is coherent with the model and viceversa. In some sense my method is similar to theirs since neither takes a stand on the property of the trend dynamics before estimation, but there are differences. Mainly, I assume that the dsge model capture only the cyclical variations and not the long run dynamics. The latter assumption could seem a big restriction, but on the other hand gives great flexibility in terms of long run restriction, especially when data display very idiosyncratic trend. In the model of Ireland (2001) with money in the utility function, he argues that if the source of long run dynamics were given by a deterministic labor augmenting technology process, then output and real balances would grow at the same rate in the model. But distinct upwards trends appear in the series for output and real balances in the US data[3]. Chang, Doh and Schorfheide (2007) estimate a RBC model with bayesian techniques using non stationary data, they test the hypothesis of a permanent labor supply shocks that can generate a unit root in hours worked, and they found that the data support this view. Their point is on the specification of the exogenous processes from the model standpoint, they ask which is the specification of the exogenous processes that better confronts with the data. In the techniques used, my method resembles much their approach, by which they compute the likelihood of the entire system of equations, the ones governing the long run dynamics and the ones governing the cycles. My direction is partially different: given a stationary model, I am asking how to extract low frequencies altering as less as possible the structural parameters estimates. Moreover, as mentioned before, confining the explanatory power of the dsge model only to cycles allows to consider a more flexible structure for the long run dynamics; in particular, I can consider linear trend, unit root or smooth non linear trend. There is a clear trade off between the flexibility in the specification of the long run dynamics and the explanatory power of the dsge model; on one hand, a coherent model that explain all range of frequencies in the spectrum is clearly theoretically appealing. On the other hand, this approach is likely to produce specification and measurement errors in in both the cyclical component and the structural parameters.

With this paper, I propose a methodology to estimate the structural parameters of a stationary dsge model using all the information contained in the non stationary observed data; in particular, I estimate jointly structural and filtering parameters. This method allows to test different trend specifications and by bayesian averaging it permits to construct estimates of the structural parameters that are robust to the trend uncertainty. A

[3]In Merha (1997), it has been shown that real M2 grows at a much slower rate than output since 1990.

parsimonious set of assumptions is considered; mainly, I assume that the dsge model is a reliable approximation of the cyclical movements of the macroeconomic variables. I provide simulation results and I found that (a) the joint estimate of structural and filtering parameters is able to back up the true trend data generating process, (b) the estimates are reliable under misspecification of the trend, and (c) under correlation between trend and cycles this method improves in term of bias relative to the standard two step approach by which one first filters the data and then estimates the structural parameters. The paper is organized as follows: Section 2 presents the dsge model followed by Section 3 in which I report the econometric methodology. In section 4, I provide simulation results to test the methodology proposed. Section 5 concludes.

2 Model

I consider a simple version of New Keynesian model with nominal rigidities in the spirit of Ireland (2001), but the method can be easily extended to more densely parameterized models. The representative household has a preference for variety: the consumption index is

$$c_t = \left(\int_0^1 c_t(j) dj \right)^{\frac{\theta_c}{\theta_c - 1}} \tag{1}$$

where $c_t(j)$ is the consumption of the good produced by firm j. The maximization of c_t w.r.t. $c_t(j)$ for a given total expenditure leads to a set of demand function of the type

$$c_t(j) = \left(\frac{p_t(j)}{P_t} \right)^{-\theta_c} c_t \tag{2}$$

where $p_t(j)$ is the price of the good produced by firm j. Moreover, the appropriate price deflator is given by

$$P_t = \left(\int_0^1 p_t(j)^{1-\theta_c} dj \right)^{\frac{1}{1-\theta_c}}$$

Conditional on such optimal behavior, it will be true that $P_t c_t = [\int_0^1 p_t(j) c_t(j) dj]$. The representative household faces standard intertemporal decisions by choosing a stream of consumption and leisure.

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\sigma_c} c_t^{1-\sigma_c} - \chi_t \frac{1}{1+\sigma_n} n_t^{1+\sigma_n} \right] \tag{3}$$

A demand shifter is assumed: χ_t affects the consumption-leisure intertemporal trade-off. We assume that the process is exogenous and (in logs) follows AR(1), i.e.

$$\ln \chi_t = \rho_\chi \ln \chi_{t-1} + \epsilon_t^\chi$$

where $\epsilon_t^\chi \sim N(0, \sigma_\chi^2)$. The nominal budget constraint is the following

$$P_t c_t + B_t = W_t n_t + R_t B_{t-1} \tag{4}$$

where $W_t n_t$ is labor income. The bond maturity gives a gross interest rate of R_t.

In the private sector there is a continuum of firms j each producing one differentiated final good with the following linear technology

$$y_t(j) = n_t(j) z_t$$

where z_t is a stationary labor-augmenting productivity shock. Each firms chooses its own price to maximize intertemporal profits defined as the difference between total revenues and total cost (inclusive of the price adjustment cost, which is scaled in terms of wholesale total output)

$$\max_{p_t(j)} E_0 \sum_{t=0}^{\infty} Q_{0,t} \{ p_t(j) y_t(j) - W_t n_t(j) - \frac{k}{2} \left(\frac{p_t(j)}{p_{t-1}(j)} - \pi \right)^2 P_t y_t \} \tag{5}$$

subject to the fact that output is demand-determined. The total demand for good j is equal to $y_t(j) = c_t(j)$; thus, firm will face an isoelastic demand function with price elasticity θ_c for its total demanded output. $Q_{0,t}$ is the stochastic discount factor.

The behavior of the interested rate is controlled by a monetary authority which follows a simple interest rate feedback rule a la Taylor characterized by a response of the nominal rate R_t to deviations from the steady state values of lagged inflation, contemporaneous output, i.e.

$$\widehat{r}_t = \rho_R \widehat{r}_{t-1} + (1-\rho_R)\rho_\pi \beta/\pi \widehat{\pi}_{t-1} + (1-\rho_R)\rho_y \beta/\pi \widehat{y}_t + \epsilon_t^m \tag{6}$$

where $\widehat{r}_t$ is the log deviation of interest rate from its steady state level, π is the long run target inflation, and $\epsilon_t^m \sim N(0, \sigma_m^2)$. The aggregate level of any quantity variable is $x_t(j)$ is given by

$$x_t = \int_0^1 x_t(j) dj$$

Equilibrium in market goods requires:

$$y_t = n_t z_t = c_t + ADJ_t \tag{7}$$

where ADJ_t stand for adjustment cost which, in real terms, are given by

$$ADJ_t = \frac{k}{2}(\pi_t - \pi)^2 y_t$$

Market clearing conditions in labor markets are obtained by setting firms' demand equal to the household supply.

3 Econometric Methodology

In this part, I develop the statistic set up to estimate the structural parameters of the model. The main idea is compute the likelihood of a system the embodies the low and medium frequencies of the data. More precisely, I assume that the linearized solution of the model provides a good approximation of the cyclical movements of the variables. These cyclical movements are combined with a non stationary behavior which is going to be estimated jointly with the cyclical part. I allow for different representations of the low frequencies process, mainly deterministic and stochastic representations.

3.1 Time Series Specification

I assume that we observe a set of potential non-stationary times series; in particular, we observe the log of hour worked, gdp, real wages, i.e.

$$y_t = [n_t, y_t, \omega_t]$$

Harvey and Jaeger (1993) argued that detrending is best accomplished by fitting a structural times series model consisting of a trend and cycle. More precisely, as in Harvey, Trimbur and Dijk (2004) I assume that the data are made up of a non stationary trend component, y_t^t, a cyclical component, y_t^c, i.e. the identity follows

$$y_t = y_t^t + y_t^c \tag{8}$$

I assume that the cyclical component is given by the dsge linearized solution. The solution takes the from

$$y_t^c = RR(\theta^m)x_{t-1} + SS(\theta^m)z_t \tag{9}$$

$$x_t = PP(\theta^m)x_{t-1} + QQ(\theta^m)z_t \tag{10}$$

$$z_{t+1} = NN(\theta^m)z_t + \epsilon_{t+1} \tag{11}$$

The matrices $PP(\theta^m), QQ(\theta^m), RR(\theta^m), SS(\theta^m)$ are non linear function of the 'deep' parameters of the model. The behavior of the endogenous variables,

$$y_t^c = [\widehat{n}_t, \widehat{y}_t, \widehat{\omega}_t]$$

is driven by the state variables

$$x_t = [\widehat{mrs}_t, \widehat{r}_t, \widehat{\pi}_t]$$

and the set of exogenous processes,

$$z_t = [a_t, \chi_t, \epsilon_t^m]$$

The vector of structural parameters is

$$\theta^m = [\beta, \pi, \sigma_c, \sigma_n, \theta_c, \rho_R, \rho_\pi, \rho_y, \rho_a, \rho_\chi, \sigma_a, \sigma_\chi, \sigma_m]$$

Equations (8) - (11) plus a specification for the trend define a state space representation which can be estimated with likelihood based approach, i.e.

$$s_{t+1} = F(\theta)s_t + G(\theta)\omega_{t+1} \tag{12}$$

$$y_t = H(\theta)s_t + \eta_t \tag{13}$$

From an econometric perspective, s_t can be viewed as a (partially latent) state vector in a linear state space model where (12) is the transition equation; $F(\theta)$, $G(\theta)$, $H(\theta)$ are matrices which are function of θ, the vector of parameters. η_t and ω_{t+1} represent the measurement and the process noise, respectively, which are uncorrelated and normally distributed with zero mean an constant variance covariance matrix. Equation (13) is the measurement equation, where y_t is a set of observable variables. The following subsections present different trend specifications.

3.1.1 Hodrick Prescott filter (hp-dsge estimate).

Lets assume that the trend, y_t^t, is given by

$$y_{t+1}^t = y_t^t + \mu_t \tag{14}$$

$$\mu_{t+1} = \mu_t + \zeta_{t+1} \tag{15}$$

The trend generated by a Hodrick Prescott (hp) filter is approximated by a smooth trend in a cloud of points. In Harvey and Jaeger (1993) hp trend can be shown to be the optimal signal extractor filter in the identity, equation (8), when the trend, y_t^t, is specified as in (14) and (15). Equations (8)-(11) and (14)-(15) can be cast into the linear state space representation (12) and (13), by setting

$$
\begin{aligned}
s_t &= \begin{pmatrix} y_t^t & \mu_t & x_{t-1} & z_t \end{pmatrix} \\
\omega_{t+1} &= \begin{pmatrix} \zeta_{t+1} & \epsilon_{t+1} \end{pmatrix} \\
F &= \begin{pmatrix} I & I & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & PP & QQ \\ 0 & 0 & 0 & NN \end{pmatrix} \\
G &= \begin{pmatrix} 0 & 0 \\ I & 0 \\ 0 & 0 \\ 0 & I \end{pmatrix} \\
H &= \begin{pmatrix} I & 0 & RR & SS \end{pmatrix}
\end{aligned}
$$

The set of shocks, ω_t, of the state space model is the joint distribution of the structural shocks of the model, ϵ_{t+1}, and the stochastic part in the trend, ζ_{t+1}. I assume that $\zeta_{t+1} \sim N(0, \Sigma_\zeta)$, where Σ_ζ has on its main diagonal $\sigma_n^\zeta, \sigma_y^\zeta, \sigma_\omega^\zeta$, and zeros elsewhere. To make a link with the hp filter, the ratio between the trend standard deviation and the cycle standard deviation is the smoothing parameter of the hp filter. Usually, the smoothing parameters is set to 1'600 for quarterly values. Whereas everybody agrees on the smoothing parameter value for quarterly quarterly observations, there is no such a consensus for annual data. Since in this set up the ratio of the standard deviations is estimated from the data, the statistical framework is quite flexible to for the time unit. Therefore, the filter parameters to be estimated is given by $\theta^{hp} = \Sigma_\zeta$. I will refer to this specification as hp-dsge estimates.

3.1.2 First difference filter (fd-dsge estimate).

In this specification we assume that the data display a unit root pattern, i.e.

$$y_t^t = \gamma + \Gamma y_{t-1} + \eta_t$$

where γ is the drift and Γ is a diagonal matrix; for the moment I assume that Γ is equal to the identity matrix; η_t is a white noise normally distributed with zero mean and variance covariance matrix, Σ_η, on its main diagonal $\sigma_n^\eta, \sigma_y^\eta, \sigma_\omega^\eta$, and zeros elsewhere. The latter equation and equations (8)-(11) can be cast into the state space representation (12)-(13) by setting

$$\begin{aligned}
y_t &= y_t - \gamma - \Gamma y_{t-1} \\
s_t &= \begin{pmatrix} x_{t-1} & z_t \end{pmatrix} \\
F &= \begin{pmatrix} PP & QQ \\ 0 & NN \end{pmatrix} \\
G &= \begin{pmatrix} 0 & I \end{pmatrix}' \\
H &= \begin{pmatrix} RR & SS \end{pmatrix} \\
\omega_{t+1} &= \epsilon_{t+1}
\end{aligned}$$

Therefore, the filter parameters to be estimated is given by $\theta^{fd} = [\Sigma_\eta, \Gamma, \gamma]$. I will refer to this specification as fd-dsge estimates.

3.1.3 Linear trend filter (lt-dsge estimate).

In this specification, I assume that the data are made up of a linear trend, i.e.

$$y_t^t = A + B * t + \eta_t$$

where A and B are vectors of the same dimensionality of the observed times series. The latter with (8)-(11) can be cast into the linear state space representation (12)-(13), by setting

$$\begin{aligned}
y_t &= y_t - A - B * t \\
s_t &= \begin{pmatrix} x_{t-1} & z_t \end{pmatrix} \\
F &= \begin{pmatrix} PP & QQ \\ 0 & NN \end{pmatrix} \\
G &= \begin{pmatrix} 0 & I \end{pmatrix}' \\
H &= \begin{pmatrix} RR & SS \end{pmatrix} \\
\omega_{t+1} &= \epsilon_{t+1}
\end{aligned}$$

Therefore, the filter parameters to be estimated is given by $\theta^{lt} = [A, B, \Sigma_\eta]$. I will refer to this specification as lt-dsge estimates.

3.2 Parameters Estimation

The set of parameters that we are interested in is the union of the model structural parameters and of the filtering parameters. At the end of the day, we obtain the joint posterior distribution of the set of parameters conditional on the data, on the model and on the trend-filter specification, i.e. $g(\theta^m, \theta^f|y, M, f)$ with $f = hp, fd, lt$. Previous dsge estimates were intended to get the posterior distribution of the structural parameters of the model conditioning on the model once the data were already filtered, i.e. $g(\theta^m|y^f, M)$. The advantage of having the joint posterior distribution of $\theta = [\theta^m, \theta^f]$ is twofold. On one hand, we can evaluate trend specifications by calculating the relative posterior support, i.e. the Posterior Odd. The Posterior Odd ratio is constructed by comparing the bayes factor, which is the ratio of the predictive density of the data conditional on the model. In standard dsge estimates, data are already filtered in some way and one could not compare posterior density of different data. In other words, the ratio between the posterior density of hp filtered data with the posterior density of linear detrended data would not be meaningful, because the likelihood is computed at different data point, i.e. $L(y^{hp}|M)$ with $L(y^{lt}|M)$. With the joint distributions of $[\theta^m, \theta^f]$, we can calculate the posterior density of the data conditioning on the model and on the filter used. More precisely, we can take Posterior Odds by taking the ratio of the posterior density of hp filtered data with the posterior density of linear detrended data conditional on a certain model, i.e. $f(y|M, hp)$ and $f(y|M, lt)$. With the Posterior Odd and a loss function, we can compare different trend specifications in a bayesian context, or test hypothesis in a classical statistics set up. The second main advantage of this formulation is that we can construct estimates of the structural parameters that are robust to trend uncertainty. Given that we do not know the 'true' data generating process and given that the wrong use of the filter biases also the structural parameters estimates (Cogley (2001)), by averaging across trend specifications we can consistently account for trend uncertainty. In particular, we can calculate

$$g(\theta^m|y, M) = \sum_{j=hp,fd,lt} \frac{f(y|M, j)}{\sum_{k=hp,fd,lt} f(y|M, k)} \int g(\theta^m, \theta^j|y, M, j)d\theta^j$$

For the sake of the argument, suppose that a researcher knows that the cyclical properties are correctly specified, but he does not know the correct specification of the trend, and suppose that the true trend data generating process (dgp) is linear. If one employs a standard 2 steps estimate, he would first filter the data, with hp or linear filters, and then estimate the structural parameters of the model; according to the filter used he would get estimates of the structural parameters, but he can not retrieve the trend specification. Estimating jointly the structural and filtering parameters, a researcher can compare different Posterior Odd and choose the correct trend specification. He can do more: since in most of the cases the true trend dgp is not know, by bayesian averaging one could construct posterior estimates of the structural parameters that account to the trend uncertainty.

To obtain the non normalized posterior distribution of the parameters, $\theta = [\theta^m, \theta^f]$, I used Monte Carlo Markov Chain simulators, that generate the sample from the posterior target distribution. In particular, following Schorfheide (2000) I used the Random Walk Metropolis algorithm (RWM), which proceeds in two steps. First, the linear expectations is solved to obtain equations (9)-(11). If the parameter value θ^m implies indeterminacy (or non-existence of a stable rational expectations solution), then $L(\theta|y)$ is set to 0. If a unique solution exists, then the Kalman filter is used to evaluate the likelihood function associated with the linear state space system (12) and (13). The second step is to generate a number of draws from the posterior distribution with the RWM algorithm. This class of algorithm generates Markov Chains with stationary distributions that correspond to the posterior distributions of interest. The algorithm is as follow, starting from an initial value θ_0 for $\ell = 1, ..., L$

1. draw a candidate $\theta^* = \theta_{\ell-1} + N(0, \Sigma)$
2. solve the linear expectations system given θ^*; if indeterminacy or no-existence set $L(\theta|y) = 0$
3. evaluate the likelihood of the data given θ^* with the Kalman filter, $L(y|\theta^*)$[4]
4. calculate $\breve{g}(\theta^*|y) = g(\theta^*)L(y|\theta^*)$
5. calculate the ratio $\chi^* = \frac{\breve{g}(\theta^*|y)}{\breve{g}(\theta_{\ell-1}|y)}$
6. draw u from $U[0, 1]$; if $\chi^* > u$ then we accept the draw and we set $\theta^* = \theta_\ell$, otherwise set $\theta_{\ell-1} = \theta_\ell$

[4]Since the hp-dsge state spaces is not stationary, we can not use the unconditional moments to start KF algorithm and we need to start from an arbitrary point. I picked $s_{1|0} = [y_1, \mathbf{0}, \mathbf{0}, \mathbf{0}]$. Same reasoning applies to the initial variance of the system. I chose $\Omega_{1|0} = 10 * I$.

Iterated a large number of times, the RWM algorithm ensure that we get to the limiting distribution which is the target distribution that we need to sample from (for further details see also Canova (2006), Ch. 9).

4 Simulations

4.1 Set up and Bias Computation

In this part, I generate data using wide range of population values; in Table 2, I report the values for the structural parameters of the dsge model. To generate a data set, I had also to specify a set of values for the non stationary component of times series. For each structural parameter vector (i.e. each column of Table 2), I generate two data set, one with a deterministic trend, i.e. a linear trend, and the other with a stochastic trend, i.e. an integrated random walk. Therefore, I constructed 10 data set, five of which have a deterministic trend and the remaining five have a stochastic trend. Each data set is composed by three times series vectors of 300 observations length; I discarded the first 140 observations, in order to remove the dependence on the initial condition.
Table 3 to 10 report bias in the estimates using the method proposed in the previous section, hp-dsge and lt-dsge, and the bias using current bayesian dsge estimates, by which the data are first filtered and then the structural parameters are estimated; I will refer to the latter as 2 steps estimate. The estimation bias is calculated using the following algorithm

1. for each simulated dataset, d^s for $s = 1, ..., 10$, run a RWM algorithm as specified in section 3.2 until the convergence is achieved[5].
2. discard the first 300,000 draws and keep one every 1,000 draws, θ_j^s, and compute
$$bias_\ell^s = \frac{1}{L}\sum_{j=1}^{L}(\frac{\theta_j^s - \theta_{true}^s}{\theta_{true}^s})^2$$
with $L = (N - 300,000)/1,000$ and N is the number of iterations of the RWM.
3. do 2. 100 times and take the average bias, i.e. $BIAS^s = \frac{1}{100}\sum_{\ell=1}^{100} bias_\ell^s$
4. do 2) and 3) for $s = 1, ..., 10$.

[5]Convergence checks are available upon request

We are interested only in the bias of the structural parameters estimates. I fix β and π in order to have a net interest rate of 3% roughly. Throughout these simulations, the acceptance rate played a crucial role. Mainly, I observed that the bigger was the acceptance rate the bigger was the bias; this is quite intuitive if we think that the acceptance rate is inversely related with the variance of the RWM algorithm. With a small RWM variance, the step of the algorithm is small; that makes life hard for the algorithm to explore the entire parameters space and to get close to the true values. I tried to keep the acceptance rate between 15% and 35% as the literature suggests. In the following subsections, I report the simulation results for the method; in particular, I confront first the bias and the marginal likelihood across hp-dsge and lt-dsge estimate when the trend is deterministic and stochastic. Then, I confront the bias relative to traditional 2 steps estimate.

4.2 Baseline results

Tables 3-6 report the bias across different specifications. Simulations results suggest that the method is able to retrieve the true data generating process (dgp) throughout the marginal likelihood when the trend is an integrated random walk. Confronting the marginal likelihood of Tables 4 and 6, we can notice that hp-dsge marginal likelihood is always higher relative to lt-dsge one. This is not alwas the case when the trend is linear. This can be due to the fact that the unobserved component set up, equations (14) and (15), encompasses the linear trend specification by setting $\zeta_t = 0$.
Overall, the bias arise in both methods, suggesting that dsge parameters are hard to identify, Canova and Sala (2006). Nevertheless, the bias does not increase under the wrong trend specification. In particular, in Table 4 I report the bias of lt-dsge estimate when the true trend is stochastic. Compared with the bias using the correct specification, Table 6, we do not observe big changes in term of bias, meaning that under misspecification we are not increasing the bias in the structural parameters estimate. Same applies when the true trend is stochastic.

4.3 Comparison with traditional 2 steps estimate

In this part, I repeated the same exercise with also 2 steps estimates; I gave the best chance to perform to the traditional methods by filtering the series with the proper filter: if the true trend dgp was deterministic, I filtered the data with a linear filter, viceversa for a stochastic dgp. I found that there are almost no differences between the method I propose and 2 steps method[6]. This was somehow predictable given that by construction trend and cycle are independent; filtering first and then estimating or doing it at once would not matter as long as these two components are uncorrelated. Things change if we allow for some correlations between trend and cycles in the simulated data. If we allow for correlation, then differences in bias arise. The correlation is induced only in the simulation part. All the estimation methods (2 steps, lg-dsge or hp-dsge) are misspecified in the sense that they all assume that trend and cycles are uncorrelated. To impose the correlation structure in the simulated data, I assume that $\eta_t = A_1 z_t + v_t$ or $\zeta_t = A_1 z_t + v_t$, if the true data generating process is deterministic or stochastic, respectively; v_t is white noise. This assumption induce that trend and cycles are correlated, i.e. $corr(y_t^t, y_t^c) \neq 0$. Tables 7 - 10 report the bias in the parameters estimates allowing the trend and the cycles to be correlated in the simulated data. Looking at Tables 7 and 8, we can confront the bias between the 2 steps and the lt-dsge methods when the true dgp is deterministic. Overall, both specifications present bias is the parameters estimates. Nevertheless, important differences come out: the bias of ρ_R and ρ_y is systematically bigger. The 2 steps estimate produces an average bias of 75% for ρ_R whereas for lt-dsge estimate the bias is around 31%; similarly, on average the bias for ρ_y is around 60% for the 2 steps estimate, whereas is below 35% for the lt-dsge method. There are also big differences in the bias of the standard deviation of the monetary policy innovation, σ_{mp}. For the other parameters, the differences in bias are undistinguishable. When the trend is stochastic, Tables 9 and 10, big gains in terms of bias are relative to coefficients of the exogenous processes, ρ_z and $\rho\chi$, and to the standard deviation of the monetary policy innovation, σ_{mp}. On average, the 2 steps method produces a bias of about 65% and 50% for ρ_z and $\rho\chi$ respectively, whereas the hp-dsge method gives a bias around 20% for both parameters. The intuition for this outcome stands on the fact that in the lt-dsge or hp-dsge estimates we are computing the likelihood of the entire system, whereas in the 2 steps estimate we are computing the likelihood of just the cyclical component. In the 2 steps estimate, we are confining all the misspecification to the cycles, producing more bias. To make the statement clearer, suppose that the true trend dgp were linear and we observe the combination of a linear trend, cycles and their corre-

[6]Simulation results assuming independence between trend and cycles are available upon request.

lation; we do not observe the single components, just their combination. If we employ a 2 steps method, we would first estimate A and B in a linear specification with standard least square regression and then estimate the structural parameters using the as cycles residual of the regression. A and B turn out to be unbiased; hence all the distortion given by the correlation between trend and cycles would be absorbed by the residuals, therefore affecting the structural parameters estimate. In the lt-dsge set up, the misspecification of the correlation is going to be absorbed by the entire system, producing bias in the filtering parameters (A and B are biased as well) and reducing the bias of the structural parameters.

5 Conclusions

I build up a flexible set up to estimate structural parameters of a stationary dsge model using non stationary data. The method takes explicitly into account the uncertainty embodied in the trend specification and in the fact that we do not observe trend and cycles separately. To do this I compute the likelihood function that includes the long and medium run dynamics of the times series. I assume that the dsge linearized solution is a good approximation of the cycles, and I allow for different trend specifications, linear, unit root and stochastic smooth trend. simulation results suggest that the method is robust to trend misspecification and it improves the performance in terms of bias relative to traditional two step estimates (first filter and then estimate), when trend and cycles are correlated.

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Paper	Exog Process	Filter	Est
Smets and Wouters (2003)	stat	hp	B
Ireland (2001)	stat	lt	MLE
Bouakez, Cardia and Ruge-Murcia (2005)	stat	lt	MLE
Rabanal and Rubio-Ramirez (2003)	stat	lt*	B
McGrattan (1994)	stat	lt	MLE
Rabanal (2007)	stat	qt*	B
Burnside, Eichenbaum and Rebelo (1993)	stat	hp	GMM
Christensen and Dib (2008)	stat	hp	MLE
Smets and Wouters (2005)	stat†	fd	B
McGrattan, Rogerson and Wright (1997)	stat†	lt	MLE
DelNegro, Schoerfheide, Smets and Wouters (2007)	unit root	fd	B
Justiniano and Primiceri (2006)	unit root	fd	B

Table 1: Summary of Selected papers.* the nominal variable are demeaned. † the technology process is stationary, and there is a deterministic trend in labor.

True Values	1	2	3	4	5
β	0.99	0.99	0.99	0.99	0.99
π	1.022	1.022	1.022	1.022	1.022
σ_c	0.80	0.70	0.60	0.50	1.80
σ_n	2.20	2.30	3.00	2.10	4.00
θ_c	6.00	6.20	5.70	6.50	6.00
k	5.30	6.00	5.30	5.00	5.00
ρ_R	0.80	0.90	0.70	0.75	0.60
ρ_y	0.50	0.60	0.65	0.60	0.40
ρ_π	2.50	2.00	1.70	1.90	2.40
ρ_z	0.80	0.88	0.70	0.50	0.40
ρ_χ	0.80	0.82	0.70	0.60	0.40
σ_z	0.35	0.10	0.21	0.03	0.21
σ_χ	0.28	0.09	0.11	0.02	0.11
σ_{mp}	0.21	0.08	0.05	0.01	0.05

Table 2: Population values for the structural parameters of the dsge model.

TV	1	2	3	4	5
Marg Like	727	747	731	561	565
σ_c	53(1.83)	2(0.62)	10(1.24)	36(1.50)	20(2.15)
σ_n	78(1.57)	57(0.84)	82(1.24)	69(1.17)	54(1.70)
θ_c	1(0.30)	1(0.12)	7(0.33)	5(0.27)	4(0.25)
k	4(0.60)	16(0.26)	3(0.67)	2(0.73)	1(0.61)
ρ_R	22(2.08)	75(0.91)	38(2.11)	81(2.69)	75(2.57)
ρ_π	14(3.34)	7(1.46)	11(2.36)	32(2.51)	97(3.92)
ρ_y	21(0.77)	2(0.34)	15(0.90)	5(0.97)	19(0.72)
ρ_z	34(15.53)	6(5.32)	30(22.36)	86(13.72)	134(17.38)
ρ_χ	18(7.25)	53(7.96)	33(8.26)	45(25.18)	22(30.23)
σ_z	76(10.79)	18(25.84)	61(19.20)	186(133.62)	59(19.43)
σ_χ	50(29.76)	52(54.91)	30(71.53)	615(384.06)	29(70.83)
σ_{mp}	14(44.70)	526(99.47)	343(258.89)	5567(1866.00)	1041(346.03)

Table 3: Bias with the lt-dsge filter. The DGP is generated with the values in Table 2 and with a deterministic trend. The bias values are expressed in % terms, with the standard deviations in parenthesis in % as well.

TV	1	2	3	4	5
Marg Like	624	661	423	609	621
σ_c	10(2.41)	20(2.70)	44(4.39)	33(1.09)	77(2.98)
σ_n	79(2.00)	71(3.18)	78(3.67)	25(0.94)	68(2.36)
θ_c	0(0.36)	3(0.55)	6(1.21)	6(0.20)	1(0.43)
k	6(1.00)	16(1.15)	6(2.11)	1(0.59)	4(1.05)
ρ_R	79(2.74)	10(1.98)	76(7.70)	31(1.91)	65(2.24)
ρ_π	77(3.76)	17(6.01)	30(8.87)	49(2.23)	59(6.44)
ρ_y	15(0.88)	5(1.72)	13(3.59)	10(0.64)	12(0.91)
ρ_z	5(14.15)	8(8.86)	22(35.43)	89(13.31)	120(34.32)
ρ_χ	19(8.37)	8(19.21)	18(40.96)	13(13.72)	146(6.14)
σ_z	55(34.67)	67(23.95)	54(39.51)	85(9.49)	85(10.12)
σ_χ	60(30.52)	49(46.50)	42(48.45)	50(28.41)	72(22.60)
σ_{mp}	25(41.53)	14(70.58)	1452(1139.75)	50(54.07)	14(61.25)

Table 4: Bias with the lt-dsge filter. The DGP is generated with the values in Table 2 and with a stochastic trend. The bias values are expressed in % terms, with the standard deviations in parenthesis in % as well.

TV	1	2	3	4	5
Marg Like	729	739	717	740	740
σ_c	29(6.31)	68(3.92)	60(4.52)	55(3.92)	28(6.44)
σ_n	60(4.76)	34(5.69)	67(3.74)	71(3.69)	57(5.49)
θ_c	3(1.82)	2(1.86)	6(1.93)	6(1.59)	2(1.73)
k	7(2.37)	18(1.54)	7(2.33)	2(2.22)	1(2.28)
ρ_R	87(14.62)	94(9.37)	88(14.00)	91(12.51)	88(15.86)
ρ_π	74(22.22)	51(14.37)	36(15.94)	52(14.52)	126(26.35)
ρ_y	21(5.16)	6(5.46)	11(6.84)	6(5.04)	22(4.10)
ρ_z	8(14.49)	54(11.69)	16(17.08)	62(24.39)	82(30.96)
ρ_χ	6(14.48)	18(12.53)	20(15.59)	41(19.80)	98(27.74)
σ_z	75(21.76)	17(60.56)	59(34.44)	180(222.63)	60(33.63)
σ_χ	54(45.26)	26(120.03)	23(114.73)	469(483.72)	11(55.26)
σ_{mp}	139(228.65)	543(659.70)	907(1116.72)	5686(4047.11)	975(1028.69)

Table 5: Bias with the hp-dsge filter. The DGP is generated with the values in Table 2 and with a deterministic trend. The bias values are expressed in % terms, with the standard deviations in parenthesis in % as well.

TV	1	2	3	4	5
Marg Like	764	760	758	739	761
σ_c	45(6.21)	45(5.10)	58(4.00)	61(5.00)	52(6.56)
σ_n	54(4.99)	58(6.22)	66(4.17)	57(4.37)	35(5.67)
θ_c	2(1.70)	2(1.97)	7(2.04)	7(1.82)	4(1.75)
k	6(2.24)	18(2.03)	7(1.95)	2(2.26)	2(1.95)
ρ_R	91(11.63)	90(10.83)	89(14.50)	89(13.45)	90(14.45)
ρ_π	81(17.64)	47(14.91)	35(17.27)	47(18.56)	126(22.83)
ρ_y	25(4.67)	6(6.41)	14(6.68)	3(6.76)	20(4.82)
ρ_z	8(13.08)	6(10.77)	18(15.78)	52(27.03)	25(26.38)
ρ_χ	7(11.34)	7(13.40)	18(18.31)	32(20.96)	79(27.08)
σ_z	61(29.14)	70(26.75)	60(30.27)	80(17.59)	84(12.48)
σ_χ	65(35.43)	55(43.17)	49(50.29)	61(38.05)	77(22.49)
σ_{mp}	25(98.52)	160(269.79)	1019(1005.11)	89(191.87)	139(233.32)

Table 6: Bias with the hp-dsge filter. The DGP is generated with the values in Table 2 and with a stochastic trend. The bias values are expressed in % terms, with the standard deviations in parenthesis in % as well.

TV	1	2	3	4	5
σ_c	57(31.81)	46(43.60)	103(54.17)	93(55.83)	45(16.75)
σ_n	53(13.49)	48(12.93)	67(10.47)	44(14.01)	69(6.84)
θ_c	2(1.07)	2(1.08)	8(1.03)	5(1.04)	1(1.03)
k	1(1.19)	1(1.29)	2(1.25)	3(1.23)	1(1.27)
ρ_R	78(8.21)	73(7.31)	77(9.26)	86(6.86)	76(9.86)
ρ_y	67(13.07)	35(9.90)	29(8.80)	46(11.35)	119(15.15)
ρ_π	25(2.67)	8(3.08)	10(3.49)	3(3.45)	21(2.66)
ρ_z	38(8.15)	50(8.10)	24(10.53)	43(13.48)	32(18.44)
ρ_χ	46(8.04)	61(8.08)	44(10.06)	46(9.28)	14(15.22)
σ_z	73(23.85)	10(54.32)	55(38.03)	221(248.18)	54(39.14)
σ_χ	48(52.50)	79(174.98)	28(114.81)	596(713.18)	33(108.48)
σ_{mp}	257(239.44)	691(845.10)	1436(1292.04)	6801(6540.22)	1124(1041.38)

Table 7: Bias with the 2 step estimate. The DGP is generated with the values in Table 2 and with a deterministic trend allowing for correlation between trend and cycles. The bias values are expressed in % terms, with the standard deviations in parenthesis in % as well.

TV	1	2	3	4	5
Marg Like	688	697	666	692	550
σ_c	46(1.80)	75(1.21)	33(6.32)	100(13.44)	33(3.05)
σ_n	73(1.78)	50(1.20)	75(1.13)	79(2.70)	76(1.61)
θ_c	1(0.34)	1(0.24)	9(0.30)	7(0.45)	1(0.58)
k	1(0.88)	1(0.65)	1(0.72)	2(1.17)	1(1.09)
ρ_R	13(2.32)	35(1.56)	3(2.47)	31(2.48)	73(5.29)
ρ_y	38(4.01)	4(2.55)	11(3.01)	13(5.76)	101(7.11)
ρ_π	24(0.79)	8(0.71)	16(1.09)	5(1.48)	26(1.39)
ρ_z	59(18.32)	6(8.68)	44(20.62)	14(36.33)	133(27.65)
ρ_χ	34(20.84)	52(12.91)	32(21.02)	58(11.19)	18(48.31)
σ_z	73(17.89)	14(40.36)	53(35.59)	198(231.76)	54(42.81)
σ_χ	53(38.48)	55(98.72)	25(78.74)	570(573.69)	36(139.04)
σ_{mp}	20(62.96)	221(158.09)	807(308.97)	1952(1661.07)	1120(577.75)

Table 8: Bias with the lg-dsge estimate. The DGP is generated with the values in Table 2 and with a deterministic trend allowing for correlation between trend and cycles. The bias values are expressed in % terms, with the standard deviations in parenthesis in % as well.

TV	1	2	3	4	5
σ_c	19(29.64)	51(43.37)	29(26.45)	43(46.37)	59(14.67)
σ_n	43(13.37)	45(11.84)	51(7.53)	33(11.34)	62(6.40)
θ_c	3(1.11)	2(1.00)	7(1.00)	4(0.85)	3(0.98)
k	2(1.11)	1(1.05)	1(1.01)	2(1.12)	1(1.01)
ρ_R	86(6.36)	82(6.25)	89(7.39)	90(6.90)	82(8.63)
ρ_y	73(12.25)	40(10.53)	37(9.06)	47(7.63)	121(12.78)
ρ_π	25(2.52)	5(2.86)	10(3.10)	4(2.59)	16(2.26)
ρ_z	69(6.68)	56(6.49)	73(7.50)	65(10.47)	57(10.71)
ρ_χ	52(6.42)	48(7.37)	54(8.06)	57(9.51)	35(11.79)
σ_z	83(15.59)	80(17.42)	56(36.09)	79(18.12)	83(13.70)
σ_χ	58(43.49)	39(58.99)	45(51.94)	66(34.55)	69(31.32)
σ_{mp}	31(88.44)	74(165.58)	985(919.93)	126(185.69)	144(233.60)

Table 9: Bias with the 2 step estimate. The DGP is generated with the values in Table 2 and with a stochastic trend allowing for correlation between trend and cycles. The bias values are expressed in % terms, with the standard deviations in parenthesis in % as well.

TV	1	2	3	4	5
Marg Like	766	721	778	781	755
σ_c	60(15.54)	58(21.88)	12(10.62)	100(20.82)	67(7.75)
σ_n	56(5.09)	50(6.76)	68(3.99)	49(5.03)	59(2.64)
θ_c	2(1.79)	2(2.01)	8(1.67)	6(1.43)	2(2.30)
k	2(2.79)	1(2.60)	1(2.14)	2(2.22)	2(2.16)
ρ_R	88(13.39)	74(13.64)	90(12.63)	91(11.17)	90(12.61)
ρ_y	75(19.80)	37(17.85)	39(14.16)	52(13.88)	123(27.23)
ρ_π	23(4.48)	5(7.64)	8(7.49)	4(5.01)	20(5.59)
ρ_z	7(13.93)	5(12.70)	18(16.16)	33(24.85)	22(19.25)
ρ_χ	7(12.82)	5(14.37)	18(16.41)	41(21.01)	35(20.57)
σ_z	85(13.28)	81(17.03)	60(33.26)	81(16.67)	84(12.96)
σ_χ	62(36.15)	44(65.37)	52(43.28)	72(23.46)	71(27.74)
σ_{mp}	24(89.16)	35(152.96)	928(888.30)	97(217.20)	137(245.91)

Table 10: Bias with the hp-dsge estimate. The DGP is generated with the values in Table 2 and with a stochastic trend allowing for correlation between trend and cycles. The bias values are expressed in % terms, with the standard deviations in parenthesis in % as well.

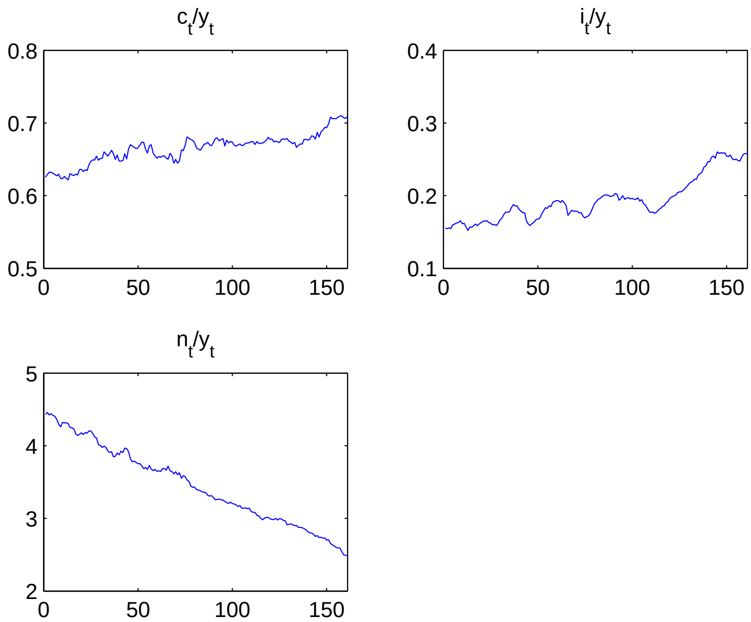

Figure 1: From top, investment, consumption, hour worked over gdp from 1964:1 to 2007:2.

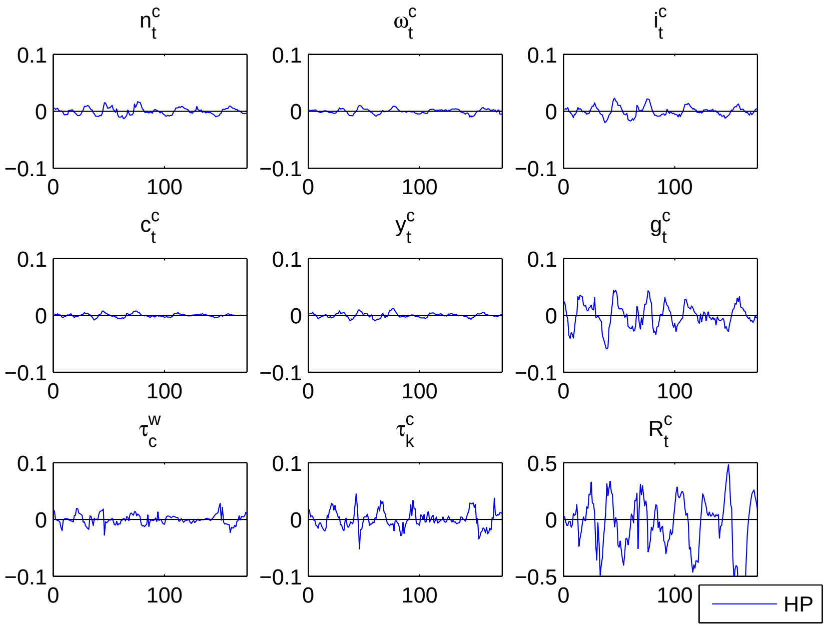

Figure 2: HP Filtered series. From top left, hour worked, wages, investment, consumption, gdp, government spending, labor tax, capital tax and interest rate from 1964:1 to 2007:2.

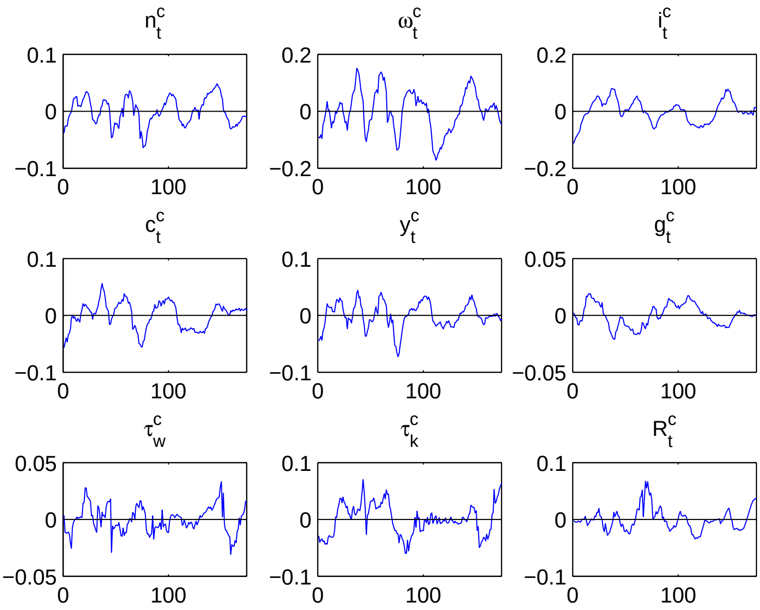

Figure 3: Quadratic Trend Filtered series. From top left, hour worked, wages, investment, consumption, gdp, government spending, labor tax, capital tax and interest rate from 1964:1 to 2007:2.

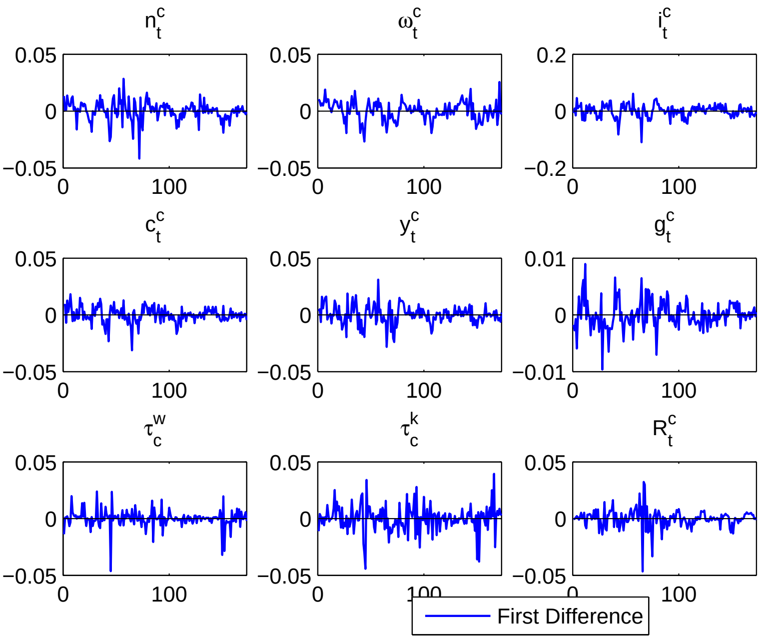

Figure 4: First Difference series. From top left, hour worked, wages, investment, consumption, gdp, government spending, labor tax, capital tax and interest rate from 1964:1 to 2007:2.