Endogenous Timing in a Mixed Duopoly with Capacity Choice

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An endogenous order of moves is analyzed in a mixed duopoly in which firms first choose their capacity levels strategically and then compete in prices. We show that, in equilibrium, firms set prices simultaneously while capacities are chosen sequentially. Therefore, there are two equilibria: in one of them the public firm is the leader in capacities and in the other, the follower. Social welfare is higher when the public firm is the follower and it is proved that the government can give the private firm a payment great enough to achieve that this firm become the leader in capacities. As a result, both the private firm and the government are better off in the second equilibrium.

Keywords: Mixed Duopoly; Endogenous Timing; Excess Capacity; Heterogeneous Products; Price Competition.

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1 Introduction

There is a conventional wisdom that holding excess capacity plays an essential role as a strategic device in the pure oligopoly market (see, for example, Dixit, 1980; Brander and Spencer, 1983; Horiba and Tsutsui, 2000). However, in many advanced countries private firms compete in the product market not only with other private firms but also with public firms (mixed markets). As public firms maximize social welfare while private firms maximize profits, the result obtained in a mixed oligopoly could differ from that obtained in a pure oligopoly. This last question has been analyzed by the literature on public firms.

Assuming a mixed market in which firms produce a homogeneous good and compete in quantities, Wen and Sasaki (2001) and Nishimori and Ogawa (2004) show that the public firm strategically chooses under-capacity while the private firm chooses excess capacity. Therefore, they show that the capacity decision of the public firm is different than that of the private firms. Ogawa (2006) extends the analysis assuming heterogeneous products. He shows that the public firm chooses over-capacity when products are complements and under-capacity when products are substitutes. Finally, Bárcena-Ruiz and Garzón (2007) show that the type of competition in the product market affects firms' capacity decisions. They show that when firms compete in prices it is obtained the opposite result the result than Ogawa (2006).

In the above cited papers it is assumed that firms take decisions simultaneously. However, as Pal (1998) points out, a sequential order of moves may give rise to significantly different results from those obtained in a simultaneous game.[1] In this regard, Lu and Poddar (2005) analyze the

[1] The literature on industrial organization has analyzed whether firms decide quantities (prices) sequential or simultaneously assuming private firms (see, for example, Gal-Or, 1985; Dowrick, 1986; Hamilton and Slutsky, 1990). This analysis has been extended to consider a mixed oligopoly. In this regard, Pal (1998) shows that when firms in a mixed oligopoly produce a homogeneous good they take production decisions sequentially, while in a private oligopoly firms decide quantities simultaneously. Bárcena-Ruiz (2007) shows that in a mixed duopoly for differentiated goods firms choose prices simultaneously while in a private duopoly firms decide prices sequentially.

strategic choice of capacity in a sequential game assuming a mixed duopoly. They consider that firms produce a homogeneous good and choose their capacity level either sequential or simultaneously and then decide quantities either sequential or simultaneously. They show that, the public firm never chooses excess capacity, while the private firm never chooses under-capacity.[2]

Lu and Poddar (2005) assume that the order of moves in which firms take decisions is exogenously given and, thus, firms do not decide whether decisions are taken sequential or simultaneously. We extend their analysis by assuming that the order of moves is endogenously determined by firms and that firms compete in prices. In order to carry out this analysis we consider a mixed duopoly in which firms produce a differentiated product with the same technology. We propose a four stage game with the following timing. In the first stage firms decide whether to choose their capacities simultaneously or sequentially. In the second stage, firms choose their capacities. In the third stage, firms decide whether to set prices simultaneously or sequentially. Finally, in the fourth stage firms decide prices.

We show that, for a given capacity level, both firms want to behave as leaders in prices since the greatest profit of the private firm and the greatest social welfare are obtained in this case.[3] Thus, both firms set prices simultaneously. This means that there are three relevant cases to be analyzed. In the first case, capacity decisions are taken simultaneously and it is obtained that the public (private) firm chooses over (under) capacity.[4] In the second case, the public firm decides its capacity before the private firm, obtaining that the public firm chooses over-capacity if and only if products are low substitutes and the private firm chooses under-capacity. Therefore, when the public firm is the leader in capacities the result depends on the degree in which goods are substitutes. Finally, in the third case, the private firm decides its capacity before the public firm and it is obtained that both firms choose over-capacity. These results are in contrast to those

[2] It can be shown that the same result is obtained if firms compete in quantities with substitute products. When products are complements both the public firm and the private firm never choose under-capacity.

[3] It has to take to be noted that the objective of a public firm is to maximize social welfare while that of a private firm is to maximize its own profit.

[4] This result is shown by Bárcena-Ruiz and Garzón (2007).

obtained by Lu and Poddar (2005) under quantity competition. Unlike Lu and Poddar (2005), we obtain that under price competition the private firm can choose under capacity and that the public firm can choose over capacity.

It remains to analyze whether firms prefer to take capacity decisions sequential or simultaneously. Solving the whole game we obtain that capacities are decided sequentially. Therefore, there are two equilibria: in one of them the public firm is the leader in capacities and, in the other, the follower. However, social welfare is greater in the second equilibrium than in the first one. Therefore, the public firm prefers to be the follower in capacities. But, the private firm wants to be the leader only if products are high substitutes; for the remaining cases, the private firm prefers to be the follower. We obtain that, when products are low substitutes, the government can give the private firm a payment great enough to achieve that this firm become leader in capacities. This means that both the private firm and the public firm are better in the equilibrium in which the private firm is leader in capacities.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 studies whether firms choose their prices simultaneously or sequentially and whether firms choose their capacities simultaneously or sequentially. Conclusions are drawn in Section 4.

2 The model

We consider a mixed duopoly market comprised by one private firm and one public firm, denoted by 1 and 2. The two firms produce a heterogeneous good. There is a continuum of consumers of the same type. The representative consumer maximizes $U(q_1, q_2) - p_1q_1 - p_2q_2$, where $q_i \geq 0$ is the amount of the good i and p_i is its price (i = 1, 2). The function $U(q_1, q_2)$ is assumed to be quadratic, strictly concave and symmetric in q_1 and q_2:

$$U(q_1, q_2) = a(q_1 + q_2) - \frac{1}{2}((q_1)^2 + 2bq_1q_2 + (q_2)^2),$$

where parameter b measures the degree to which goods are substitutes, $b \in (0, 1)$. Then, it is easy to see that demand functions are given by:

$$q_i = \frac{a(1-b) - p_i + bp_j}{1-b^2},\ i \neq j;\ \ i, j = 1, 2. \tag{1}$$

The two firms have the same technology represented by the cost function: $C(q_i, x_i)$, where q_i and x_i are, respectively, the production and capacity of firm i. Following Vives (1986), Nishimori and Ogawa (2004) and Lu and Podar (2005), we specify the cost function as:

$$C(q_i, x_i) = m\, q_i + (q_i - x_i)^2,\ i=1, 2.$$

This cost function shows that excess capacity or under-capacity would result inefficient. When quantity equals capacity the long-run average cost is minimized. We assume $a>m$ to assure that the production level of both firms is positive in all cases considered.

The profit of firm i is given by:

$$\pi_i = p_i\, q_i - mq_i - (q_i - x_i)^2,\ i = 1, 2, \tag{2}$$

where q_i is given by (1). As usual, we measure social welfare as the sum of consumer surplus (denoted by CS) and producer surplus (denoted by PS). Therefore, social welfare is given by:

$$W = CS + PS, \tag{3}$$

where $PS = \pi_1 + \pi_2$ and consumer surplus is given by:

$$CS = U(q_1, q_2) - p_1\, q_1 - p_2\, q_2 = \frac{(p_1)^2 - 2bp_1 p_2 + (p_2)^2 + 2a(1-b)(a - p_1 - p_2)}{2(1-b^2)}. \tag{4}$$

The objective of this paper is to analyze the order of moves in a mixed duopoly with capacity choice under price competition. Both price and capacity decisions can be taken sequentially or simultaneously. We propose a four stage game with the following timing. In the first stage, firms decide whether to choose their capacities simultaneously or sequentially. In the second stage, firms decide their capacities. In the third stage, firms decide whether to set prices simultaneously or sequentially. Finally, in the fourth stage firms decide their prices. We solve the game by backward induction from the last stage of the game to obtain a subgame perfect Nash Equilibrium.

3 Results

We analyze first whether firms set prices sequential or simultaneously.

3.1 Price decisions

In the fourth stage, given the capacity levels chosen in stage two, firms decide their prices either sequentially or simultaneously. Thus, there are three possible cases: firms decide prices simultaneously, the public firm decides its price before the private firm does and the private firm decides its price before the public firm does.

We analyze first the case in which firms set prices simultaneously. We denote this case by superscript *SP*. In the fourth stage, given the production capacities chosen by firms in the second stage, the private firm chooses the price, p_1, that maximizes its profit while the public firm chooses the price, p_2, that maximizes social welfare. Solving these problems we obtain the reaction functions in prices of the firms:

$$p_2 = \frac{2a(1-b)^2 - (1-b^2)(2x_2 - 2bx_1 - m(1-b)) + b(5-b^2)\,p_1}{3+b^2}, \tag{5}$$

$$p_1 = \frac{a(1-b)(3-b^2)-(1-b^2)(2x_1-m)+b(3-b^2)p_2}{2(2-b^2)}. \tag{6}$$

As firms compete on prices in the product market, if one firm raises its price the other firm reacts by raising its price too. Thus, prices are strategic complements. From (5) and (6) we obtain:

$$p_2^{SP} = \frac{a(8-b-3b^2-b^3+b^4)-4(2-b^2)x_2-2b(1+b^2)x_1+m(1+b)(4-3b+b^2)}{12-5b^2+b^4},$$

(7)

$$p_1^{SP} = \frac{a(3-b)(3-b^2)-2b(3-b^2)x_2-2(3-2b^2+b^4)x_1+m(1+b)^2(3-3b+b^2)}{12-5b^2+b^4}.$$

Substituting (7) in (2) and (3) we obtain the profit of the private firm, p_1^{SP}, and social welfare, W^{SP}. The expressions for p_1^{SP} and W^{SP} and the resolution of the cases where prices are set sequentially are relegated to Appendix 1. In the sequential games, we denote by superscript LP (FP) to the leader (follower) in prices. Thus, p_i^{LP} and p_i^{LP} (p_i^{FP} and p_i^{FP}) denote the profit of the firm i and its price, respectively, when firm i is the leader (the follower) in prices. Similarly, W^{LP} and W^{FP} denote social welfare when the public firm is the leader or the follower, respectively.

Given the capacity chosen by firms in the second stage, in the third stage, firms decide whether to set prices sequentially or simultaneously. Solving this stage of the game the following result is obtained.

Lemma 1. For a given production capacity: $p_1^{LP} > p_1^{SP} > p_1^{FP}$ and $W^{LP} > W^{SP} > W^{FP}$. Therefore, firms decide prices simultaneously.[5]

Proof. See Appendix 2.

It is easy to see that, for a given capacity: $p_1^{LP} > p_1^{SP} > p_1^{FP}$ and $p_2^{FP} > p_2^{SP} > p_2^{LP}$. As the objective of the public firm is social welfare it cares about the consumer surplus and, thus, it seeks to increase market competition. On the contrary, as the private firm cares about profits, it attempts to reduce market competition. Thus, when the private firm is the leader in prices sets a higher price than in the simultaneous case since, as prices are strategic complements, the follower (the public firm) sets a higher price too. When the public firm is the leader in prices, it lowers its price in comparison to the simultaneous case since in this way, as prices are strategic complements, the follower (the private firm) lowers its price too. As a result, when the private firm is the leader it sets a higher price than when decisions are set simultaneously and, in this last case, its price is higher than when it is the follower. When the public firm is the follower it sets a higher price than when decisions are set simultaneously and, in this last case, its price is higher than when it is the leader.

As the greater (lower) prices are set by firms when the private (public) firm is the leader in prices, the lower (greater) social welfare and the greater (lower) profit of the private firm is obtained in this case. Therefore, for a given capacity, both firms want to behave as leaders in prices since the greatest profit for the private firm and the greatest social welfare are obtained in this case. By contrast, neither of the two firms wants to behave as followers in prices since, in that case, the lowest profit for the private firm and the lowest social welfare are obtained. As a result, both firms decide prices simultaneously.

3.2 Capacity decisions

Taking into account that firms decide to set prices simultaneously in the third stage of the game, it remains to solve whether firms decide capacities sequential or simultaneously. In the second stage, firms decide their capacities either sequentially or simultaneously. Thus, there are

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[5] This result holds when products are complements.

three cases to be considered: firms decide capacities simultaneously, the public firm decides its capacity before the private firm does and the private firm decides its capacity before the public firm does.

We analyze first the case in which firms decide their capacities simultaneously (denoted by superscript S). In the second stage, given (7), the private firm chooses the capacity, x_1, that maximizes its profit and the public firm chooses the capacity, x_2, that maximizes social welfare. Solving these problems we obtain the following result.

Proposition 1. When firms take capacity decisions simultaneously, the public firm chooses over-capacity, $x_2>q_2$, and the private firm chooses under-capacity, $x_1<q_1$.[6]

Proof. See Appendix 3.

The explanation of this result is as follows. Given that the public firm maximizes social welfare it cares about the consumer surplus and, thus, about the output of industry. As a result, the public firm tries that the private firm increase its output (and, thus, reduce its price). However, as firms compete in prices, the private firm seeks to raise its price in order to reduce market competition and increase its profit.

The capacity level chosen by a firm not only affects its own price but also that of its rival. In fact, it can be shown from (7) that there is a negative relationship between the capacity level of the public (private) firm and the price level of the private (public) firm and, thus, a positive relationship with the output level of the private (public) firm. Thus, the public firm can improve social welfare by increasing its capacity while the private firm can reduce market competition by reducing its capacity. As a result, the private firm chooses under-capacity and the public firm chooses over-capacity.

[6] When products are complements, $b\in(-1,0)$, both firms choose under-capacity.

Next we consider that the public firm decides its capacity level before the private firm does. We denote by p_i^L, p_i^L and q_i^L (p_i^F, p_i^F and q_i^F) the profit, the price and the output level, respectively, of the firm *i* when it is the leader (the follower) in capacities. Similarly, W^L and CS^L (W^F and CS^F) denote social welfare and consumer surplus when the public firm is the leader (the follower) in capacities. In the second stage, given (7), the private firm chooses the capacity, x_1, that maximizes its profit. Solving this problem we obtain the reaction function in capacities of the private firm:

$$x_1 = \frac{6(2-b^2)((a-m)(3-b)-2bx_2)}{72-84b^2+49b^4-10b^6+b^8}. \tag{8}$$

It is easy to see from (8) that x_1 and x_2 are strategic substitutes ($\frac{\partial x_1}{\partial x_2} < 0$). Therefore, if the public firm increases its capacity, the private firm reacts by reducing its own capacity.

The public firm chooses the capacity, x_2, that maximizes social welfare knowing the reaction function of its rival (expression (8)) and taking advantage of this knowledge. Solving this problem we obtain the following result. Let b^* » 0.8050.

Proposition 2. When the public firm is the leader in capacities, this firm chooses overcapacity, $x_2>q_2$, if and only if $b<b^*$ and the private firm chooses under-capacity, $x_1<q_1$.[7]

Proof. See Appendix 4.

This result is in contrast to that obtained by Lu and Poddar (2005) assuming that firms producing a homogeneous good and compete in quantities. They show that, when the public firm is the leader in capacities and firms choose their output level simultaneously, the public firm chooses under-capacity while the private firm over-capacity.

[7] When products are complements the public firm chooses over-capacity if and only if $b<-0.8050$ and the private firm chooses under-capacity.

When the public firm is the leader in capacities, it chooses over-capacity ($x_2>q_2$), if and only if the degree in which products are substitutes is low enough ($b<b^*$). Therefore, if $b>b^*$ it is obtained a different result than in the simultaneously game. The explanation of this result is as follows. On the one hand, in contrast to the simultaneous case, as the public firm is the leader in capacities it takes into account that if it increases its capacity the private firm reacts by reducing its own capacity. In this way, the incentive of the public firm to choose over-capacity, in order to increase the output of industry, is weakened compared with the simultaneous case. On the other hand, market competition increases with parameter b;[8] thus, as b rises, the incentive of the public firm to choose over-capacity in order to increase market competition is weakened while the incentive that the private firm has to choose under-capacity in order to reduce market competition is reinforced. Therefore, when parameter b is great enough ($b>b^*$), as market competition is great, the public firm chooses under-capacity to avoid that the private firm select a low capacity (and, thus, a low output level). When parameter b is low enough, the public firm chooses over-capacity. As the private firm is the follower in capacities it chooses under capacity for all values of parameter b, in order to reduce market competition.

Finally we analyze the case in which the private firm decides its capacity before the public firm does, given that prices are set simultaneously. In the second stage, given (7), the public firm chooses the capacity, x_2, that maximizes social welfare. Solving this problem we obtain the reaction function in capacities of the public firm:

$$x_2 = \frac{(a-m)B_1 - 2b(15-10b^2+4b^4-b^6)x_1}{(48-50b^2+23b^4-6b^6+b^8}, \tag{9}$$

where $B_1 = (48-15b-35b^2+10b^3+13b^4-4b^5-2b^6+b^7)$. It is easy to see from (9) that x_1 and x_2 are strategic substitutes ($\frac{\partial x_2}{\partial x_1}<0$).

[8] Parameter b can be interpreted as an imperfect measure of the degree of competition in the product market since the higher the value of parameter b, the greater the substitutability between goods produced by the

The private firm chooses the capacity, x_1, that maximizes its profit knowing the reaction function of its rival (expression (9)) and taking advantage of this knowledge. Solving this problem we obtain the following result.

Proposition 3. When the private firm is the leader in capacities, both firms choose over-capacity, $x_i > q_i$, i=1, 2.[9]

Proof. See Appendix 5.

This result is also in contrast to that obtained by Lu and Poddar (2005). They show that, when the private firm is the leader in capacities, the public firm chooses no capacity and produces a positive quantity while the private firm chooses over-capacity.

Under price competition, as the private firm is the leader in capacities it takes into account that if it reduces its capacity the public firm reacts by increasing its own capacity. This implies that the incentive of the private firm to choose under-capacity is weaker than in the simultaneous case. In fact, the private firm wants to increase its capacity since this causes that the public firm reduces its capacity and, thus, its output level. This effect is stronger than the incentive that the private firm has to reduce its capacity in order to increase its price and, in this way, its profit. As a result, when the private firm is the leader in capacities it chooses over-capacity, the opposite result than in the simultaneous game. As the public firm is the follower in capacities, it chooses over-capacity in order to increase market competition. Thus, the same result is obtained than when capacities are chosen simultaneously.

3.3 First stage of the game

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firms and, as a result, the greater the competition in the product market.

[9] When products are complements the public (private) firm chooses under-capacity (over-capacity).

In stage one, firms decide whether to choose capacities sequential or simultaneously. Comparing the profit obtained by the private firm and social welfare in the different cases analyzed we obtain the following result.

Lemma 2. In equilibrium:

i) $x_1^F > x_1^S$, $x_1^L > x_1^S$, $x_2^F < x_2^S$ and $x_2^L < x_2^S$,

ii) $p_2^F > p_2^S$, $p_2^L > p_2^S$, $p_1^F > p_1^S$ and $p_1^L > p_1^S$,

iii) $W^L > W^S$ and $W^F > W^S$.

Proof. See Appendix 6.

As we have seen in proposition 1, when firms decide their capacities simultaneously the public firm chooses over-capacity to achieve that the private firm reduce its price increasing thus its output level. However, when the public firm is the leader in capacities, it takes into account that if it reduces its capacity the private firm reacts by reducing

increasing its own capacity. As a result, the capacity level chosen by the public firm (the leader) is lower than in the simultaneous case ($x_2^L < x_2^S$) and, as capacities are strategic substitutes, the capacity level of the private firm (the follower) is greater ($x_1^F > x_1^S$). Similarly, as it is shown in proposition 1, when firms take capacity decisions simultaneously, the private firm chooses under-capacity since it increases its price and, thus, its profit. However, when the private firm is the leader, it takes into account that if it reduces its capacity the public firm reacts by increasing its own capacity. As a result, $x_1^L > x_1^S$ and $x_2^F < x_2^S$.

It is easy to see from (7) that there is a negative relationship between the capacities chosen by both firms and the price chosen by each firm. Thus, when the public firm is the leader in capacities it increases its price (p_2) compared to the simultaneous case since $x_2^L < x_2^S$. In this case, as the private firm is the follower, increases its capacity compared to the simultaneous case which reduces p_2. However, the first effect has a greater weight on p_2 than the second one and, as a result, when the public firm is the leader in capacities it chooses a greater price and, thus, a lower output than in the simultaneous case ($p_2^L > p_2^S$, $q_2^L < q_2^S$). As a result, the private firm gains market share at the expense of the public firm, producing more than in the simultaneous case ($q_1^F > q_1^S$) and obtaining greater profit ($p_1^F > p_1^S$). The public firm also obtains greater profit ($p_2^L > p_2^S$) since although it losses market share its price is greater than in the simultaneous case.

Similarly, the greater capacity level chosen by the private firm being the leader compared to the simultaneous case, has a greater effect on p_1 than the lower capacity chosen by the public firm. As a result, when the private firm is the leader in capacities it sets a lower price and, thus, a greater output level than in the simultaneous case ($p_1^L < p_1^S$, $q_1^L > q_1^S$). As the output level of each firm decreases with the output of its rival the public firm produces less than in the simultaneous case ($q_2^F < q_2^S$ and $p_2^F > p_2^S$). Therefore, when the private firm is the leader in capacities, both firms obtain greater profits than in the simultaneous case ($p_1^L > p_1^S$, $p_2^F > p_2^S$).

From the results shown in Lemma 2, it can be concluded that the producer surplus is greater when capacities are chosen sequentially rather than simultaneously. As $p_1^F > p_1^S$ and $p_2^L > p_2^S$, the producer surplus is greater when the public firm is the leader in capacities than when capacities are chosen simultaneously ($p_1^F + p_2^L > p_1^S + p_2^S$). Similarly, as $p_1^L > p_1^S$ and $p_2^F > p_2^S$ the producer surplus is greater when the private firm is the leader in capacities than when capacities are chosen simultaneously ($p_1^L + p_2^F > p_1^S + p_2^S$).

When the public firm is the leader in capacities both firms choose greater prices and, thus, market competition is lower than in the simultaneous case. As a result, the consumer surplus is lower than in the simultaneous case. When the private firm is the leader in capacities, it chooses a lower price than in the simultaneous case while the public firm chooses a greater one. This implies that the output level of the private (public) firm is greater (lower) than in the simultaneous case. When the parameter b increases, the private firm rises its output level while the public firm reduces yours. Thus, when market competition is high enough, the lower output level chosen by the public firm (the follower) has a greater weight than the higher output of the private firm (the leader), which implies a lower consumer surplus than in the simultaneous case. As a result, if products are highly substitutes (b>0.7091) the consumer surplus is greater when capacities are chosen simultaneously than when the private firm is the leader in capacities; if b<0.7091 the result is reversed. The producer surplus has a greater effect on social welfare than the consumer surplus and, as a result, social welfare is greater when capacities are chosen sequentially rather than simultaneously ($W^L > W^S$ and $W^F > W^S$).

From Lemma 2 the following result is obtained.

Proposition 4. In equilibrium, capacities are decided sequentially.[10]

Lemma 2 shows that the private firm obtains greater profits when capacities are chosen sequentially rather than simultaneously, independently on whether this firm is the leader or the follower in capacities. On the other hand, social welfare is greater when capacities are chosen sequentially rather than simultaneously, independently on whether the public firm is the leader or the follower in capacities. As a result, proposition 4 concludes that capacities are decided sequentially and, thus, there are two equilibria: in one of them the private firm is the leader in capacities and, in the other, the public firm.

It is easy to see that $p_1^L > p_1^F$ if and only if b>0.8946 and that $W^F > W^L$. Therefore, the public firm prefers the sequential equilibrium in which the public firm is the follower in capacities. However, the private firm wants to be the leader only if b>0.8946. For the remaining values of parameter b the private firm prefers to be the follower. It can be shown that $W^F - W^L > p_1^F - p_1^L$ if and only if b<0.8946; thus, the government can give the private firm a payment great enough to achieve that this firm become the leader in capacities. As a result, both

[10] The same result is obtained when products are complements.

the private firm and the public firm are better in the equilibrium in which the private firm is the leader in capacities.

4 Conclusions

The literature that analyzes the capacity choice of firms in a mixed market considers that the order of moves in which private firms and public firms take decisions (either sequentially or simultaneously) is exogenously given. However, a sequential order of moves may give rise to significantly different results from those obtained in a simultaneous game. Thus, in contrast to the assumption made by this literature, in this paper we endogenize the order of moves in a mixed duopoly with capacity choice under price competition, when price decisions can be taken either sequentially or simultaneously and when capacity decisions can be taken either sequentially or simultaneously.

We show that, for a given capacity level, both firms want to behave as leaders in prices since the greatest profit of the private firm and the greatest social welfare are obtained in that case. Thus, both firms choose prices simultaneously and, as a result, we analyze three cases. In the first one, capacity decisions are taken simultaneously and it is obtained that the public (private) firm chooses over (under) capacity. In the second one, the public firm decides its capacity before the private firm obtaining that the public firm chooses over-capacity if and only if products are low substitutes ($b<0.8050$) and the private firm chooses under-capacity. Finally, in the third one, the private firm decides its capacity before the public firm and it is obtained that both firms choose over-capacity. These results are in contrast to those obtained in the literature that analyzes a mixed duopoly in which firms producing a homogeneous good compete in quantities. Unlike this literature we obtain that the private firm can choose under capacity and that the public firm can choose over capacity.

In spite of prices are chosen simultaneously we show that capacities are decided sequentially. Therefore, there are two equilibria: in one of them the public firm is the leader in capacities and, in the other, the follower. However, in the second equilibrium social welfare is greater than in the first one. Therefore, the public firm prefers to be the follower in capacities. The private firm wants to be the leader only if products are high substitutes. When products are low substitutes the private firm prefers to be the follower. We obtain that the government can give the private firm a payment great enough to achieve that the private firm become leader in capacities when products are low substitutes. In this last case, both the private firm and the public firm are better in the equilibrium in which the private firm is leader in capacities.

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Appendix

1. Price decisions

First case: firms set prices simultaneously. The profit of the private firm, p_1^{SP}, and social welfare, W^{SP}, are:

$$p_1^{SP} = \frac{1}{(12-5b^2+b^4)^2}((a-m)^2(3-b)^2(2-b^2) + 4(a-m)(3-b)(2-b^2)(3x_1 - bx_2) +$$

$$4b(2-b^2)x_2(bx_2 - 6x_1) - (72-84b^2+49b^4-10b^6+b^8)x_1^2),$$

$$W^{SP} = \frac{1}{2(12-5b^2+b^4)^2}((a-m)^2(93-30b-65b^2+20b^3+25b^4-8b^5-5b^6+2b^7)+$$

$$4(48-15b-35b^2+10b^3+13b^4-4b^5-2b^6+b^7)x_2(a-m)+$$

$$4x_1(15-10b^2+4b^4-b^6)((a-m)(3-b)-2bx_2)-$$

$$2(48-50b^2+23b^4-6b^6+b^8)x_2^2 - 2(54-60b^2+25b^4-4b^6+b^8)x_1^2).$$

Second case: the public firm is the leader in prices and the private firm the follower. In the fourth stage, given the production capacities chosen by firms in the second stage, the private firm chooses the price, p_1, that maximizes its profit. Solving, we obtain the reaction function in prices of the private firm (expression (6)).

The public firm chooses the price, p_2, that maximizes social welfare knowing the reaction function of its rival and taking advantage of this knowledge. Solving this problem we obtain:

$$p_2^{LP} = (a(32-7b-17b^2+b^3+3b^4)-4(8-6b^2+b^4)x_2-2b(7-b^2)x_1+$$
$$m(1+b)(16-9b-3b^2+2b^3))\,/\,(48-29b^2+5b^4)\,,$$
$$p_1^{FP} = (a(3-b)(4-b^2)(3-b^2)-2b(4-b^2)(3-b^2)x_2-4(6-4b^2+b^4)x_1+$$
$$m(1+b)(12-8b^2+b^3+b^4))\ /(48-29b^2+5b^4)\,.$$

Substituting the above prices in (2) and (3) we obtain the profit of the private firm, p_1^{LP}, and social welfare, W^{LP}:

$$p_1^{FP} = \frac{1}{(48-29b^2+5b^4)^2}((a-m)^2(3-b)^2(4-b^2)^2(2-b^2)+(4(4-b^2)^2(2-b^2))$$
$$((3-b)(3x_1-bx_2)(a-m)+bx_2(bx_2-6x_1))-(1152-1632b^2+961b^4-254b^6+25b^8)x_1^2)\,,$$

$$W^{LP} = \frac{1}{2(48-29b^2+5b^4)}((a-m)^2(31-10b-14b^2+4b^3+b^4)+$$
$$4(a-m)((16-5b-8b^2+2b^3+b^4)x_2+(3-b)(5-2b^2)x_1)-$$
$$2x_2^2(16-13b^2+3b^4)-8bx_2x_1(5-2b^2)-2x_1^2(18-17b^2+5b^4)).$$

Third case: the private firm is the leader in prices and the public firm the follower. In the fourth stage of the game, given the production capacities chosen by firms in the second stage, the public firm chooses the price, p_2, that maximizes social welfare. Solving, we obtain the reaction function in prices of the public firm (expression (5)).

The private firm chooses the price, p_1, that maximizes its profit knowing the reaction function of its rival and taking advantage of this knowledge. Solving this problem we obtain:

$$p_2^{FP} = (a(24-3b-7b^2+3b^3-b^4)-2(12-5b^2+b^4)x_2-6b(1-b^2)x_1+$$
$$m(12+3b-5b^2-3b^3+b^4))\,/\,(12(3-b^2)),$$
$$p_1^{LP} = \frac{a(3+b)(3-b)^2-2b(9-b^2)x_2-18(1-b^2)x_1+m(9+9b-9b^2-b^3)}{12(3-b^2)}.$$

Substituting the above prices in (2) and (3) we obtain the profit of the private firm, p_1^{LP}, and social welfare, W^{FP}:

$$p_1^{LP} = \frac{(a-m)^2(3-b)^2+4(a-m)(3-b)(3x_1-bx_2)-24bx_2x_1-12(3-2b^2)x_1^2+4b^2x_2^2}{24(3-b^2)},$$

$$W^{FP} = \frac{1}{288(3-b^2)^2}((a-m)^2(837-270b-513b^2+180b^3+27b^4-6b^5+b^6)+$$
$$4(432-135b-243b^2+90b^3+18b^4-3b^5+b^6)x_2(a-m)-4(216-189b^2+54b^4-b^6)x_2^2-$$
$$36(27-18b^2+7b^4)x_1^2+12(45-30b^2+b^4)\;x_1((a-m)(3-b)-2bx_2)).$$

2. Proof of Lemma 1

In the third stage, firms decide whether to set prices sequential or simultaneously. Comparing the prices set by the private and public firms in the three cases, we obtain:

$$p_1^{LP}-p_1^{SP} = \frac{b^2(5-b^2)(3+b^2)b_1}{b_2},\; p_1^{SP}-p_1^{FP} = \frac{b^2(1-b^2)(3-b^2)^2b_1}{b_3},$$
$$p_2^{FP}-p_2^{SP} = \frac{b^3(5-b^2)^2b_1}{b_2},\; p_2^{SP}-p_2^{LP} = \frac{2b(1-b^2)(3-b^2)(2-b^2)b_1}{b_3},$$

where $b_1 = ((a-m)(3-b) - 2bx_2 + 6x_1)$, $b_1 > 0$ to assure that the firms produce a positive output level; $b_2 = 12(3-b^2)(12-5b^2+b^4)$ and $b_3 = (12-5b^2+b^4)(48-29b^2+5b^4)$. Thus: $p_1^{LP} - p_1^{SP} > 0$, $p_1^{SP} - p_1^{FP} > 0$, $p_2^{FP} - p_2^{SP} > 0$ and $p_2^{SP} - p_2^{LP} > 0$ since $b \in (0, 1)$. Therefore: $p_1^{LP} >$ $p_1^{SP} > p_1^{FP}$ and $p_2^{FP} > p_2^{SP} > p_2^{LP}$.

Comparing the profits obtained by the private firm in the three cases, we obtain:

$$p_1^{SP} - p_1^{FP} = \frac{(b_1)^2 b^2 (1-b^2)(3-b^2)(2-b^2)(96-61b^2+14b^4-b^6)}{(b_3)^2},$$

$$p_1^{LP} - p_1^{SP} = \frac{b^4 (5-b^2)^2 (b_1)^2}{2b_2(12-5b^2+b^4)},$$

where $p_1^{SP} - p_1^{FP} > 0$ and $p_1^{LP} - p^{SP} > 0$ since $b \in$ (0, 1). Therefore: $p_1^{LP} > p_1^{SP} > p_1^{FP}$.

Comparing the social welfare obtained in the three cases, we obtain:

$$W^{SP} - W^{FP} = \frac{b^2 (5-b^2)(3+b^2)(72-51b^2+10b^4+b^6)(b_1)^2}{2(b_2)^2},$$

$$W^{LP} - W^{SP} = \frac{b^2 (1-b^2)^2 (3-b^2)^2 (b_1)^2}{2b_3(12-5b^2+b^4)},$$

where $W^{SP} - W^{FP} > 0$ and $W^{LP} - W^{SP} > 0$ since $b \in (0, 1)$. Therefore: $W^{LP} > W^{SP} > W^{FP}$.

3. Firms decide their capacities simultaneously

In the second stage, given (7), the private firm chooses the capacity, x_1, that maximizes its profit and the public firm chooses the capacity, x_2, that maximizes social welfare. Solving these problems we obtain:

$$x_2^S = b_4(24-15b-23b^2+10b^3+13b^4-4b^5-2b^6+b^7),$$

$$x_1^S = 6b_4(1-b)(2-b^2), \text{ where } \quad b_4 = \frac{(a-m)}{24-38b^2+23b^4-6b^6+b^8}.$$

From (1), (6) and the above expressions we obtain:

$$q_2^S = b_4(24-18b-20b^2+14b^3+9b^4-5b^5-b^6+b^7),\ q_1^S = b_4(1-b)(12-5b^2+b^4),$$

$$x_2^S - q_2^S = b_4 b(1-b)^2(1+b)(3-b^2),\ x_1^S - q_1^S = -b_4 b^2(1-b)(1+b^2),$$

where $x_2^S - q_2^S > 0$ and $x_1^S - q_1^S < 0$ since $b \in (0, 1)$.

The profit of the private firm and social welfare are:

$$p_1^S = (b_4)^2(1-b)^2(2-b^2)(72-84b^2+49b^4-10b^6+b^8),$$

$$W^S = \frac{(b_4)^2}{2}(1008-864b-2334b^2+1884b^3+2571b^4-1930b^5-1639b^6+1136b^7+$$
$$682b^8-434b^9-192b^{10}+114b^{11}+35b^{12}-20b^{13}-3b^{14}+2b^{15}).$$

4. The public firm decides its capacity before the private firm does

In the second stage, given (7), the private firm chooses the capacity, x_1, that maximizes its profit. Solving this problem we obtain the reaction function in capacities of the private firm (expression (8)). The public firm chooses the capacity, x_2, that maximizes its profit knowing the reaction function of its rival and taking advantage of this knowledge. Solving this problem and substituting in (8) we obtain:

$$x_2^L = b_5(1728-1296b-3600b^2+2376b^3+3912b^4-2295b^5-2459b^6+$$
$$1264b^7+987b^8-431b^9-239b^{10}+99b^{11}+33b^{12}-14b^{13}-2b^{14}+b^{15}),$$

$$x_1^F = 6b_5(1-b)(2-b^2)(72-84b^2+49b^4-10b^6+b^8),$$

where $b_5 = \dfrac{(a-m)}{(1728-4896b^2+6288b^4-4754b^6+2251b^8-670b^{10}+132b^{12}-16b^{14}+b^{16})}$.

From (1), (7) and the above expressions we obtain:

$$q_2^L = b_5(1728-1440b-3456b^2+2784b^3+3504b^4-2706b^5-2048b^6+$$
$$1502b^7+749b^8-511b^9-159b^{10}+113b^{11}+19b^{12}-15b^{13}-b^{14}+b^{15}),$$
$$q_1^F = b_5(1-b)(864-1368b^2+1080b^4-449b^6+111b^8-15b^{10}+b^{12}),$$
$$x_2^L - q_2^L = b_5 b(1-b)(3-b^2)(48-120b^2+97b^4-47b^6+11b^8-b^{10}),$$
$$x_1^F - q_1^F = b_5 b^2(-1+b)(1+b^2)(72-84b^2+49b^4-10b^6+b^8).$$

It is easy to see that $x_1^F - q_1^F$<0 since $b \in$ (0, 1). On the other hand, $x_2^L - q_2^L$>0 if 0<b<b^*, b^* ≈0.8050. It easy to see that the sign{ $x_2^L - q_2^L$ }= sign{ $48-120b^2+97b^4-47b^6+11b^8-b^{10}$ }. This last expression is positive for b=0 and negative for b=1. Besides: $\dfrac{\partial(48-120b^2+97b^4-47b^6+11b^8-b^{10})}{\partial b}$ = $-240b+388b^3-282b^5+88b^7-10b^9$<0 since $b \in$ (0, 1). Therefore, there exists a value of parameter b, b^*, such that $x_2^L - q_2^L$>0 if 0< b<b^*, where b^* ≈0.8050.

The profit of the private firm and social welfare are:

$$p_1^F = (b_5)^2(1-b)^2(2-b^2)(72-84b^2+49b^4-10b^6+b^8)^3,$$
$$W^L = \frac{(a-m)b_5}{2}(3024-2592b-5976b^2+4752b^3+6207b^4-4590b^5-3723b^6+$$
$$2528b^7+1418b^8-862b^9-338b^{10}+198b^{11}+47b^{12}-28b^{13}-3b^{14}+2b^{15}).$$

5. The private firm decides its capacity before the public firm does

In the second stage, given (7), the public firm chooses the capacity, x_2, that maximizes social welfare. Solving this problem we obtain the reaction function in capacities of the public firm given by (9). The private firm chooses the capacity, x_1, that maximizes its profit knowing the reaction function of its rival and taking advantage of this knowledge. Solving this problem and substituting in (9) we obtain:

$$x_2^F = b_6(1152+432b-2112b^2-882b^3+1724b^4+687b^5-876b^6-$$
$$308b^7+301b^8+84b^9-71b^{10}-14b^{11}+11b^{12}+b^{13}-b^{14}),$$
$$x_1^L = 2b_6(2-b^2)(12-5b^2+b^4)^2,$$

where: $b_6 = \dfrac{(a-m)}{(1+b)(1152-2112b^2+1724b^4-876b^6+301b^8-71b^{10}+11b^{12}-b^{14})}$.

From (1), (7) and the above expressions we obtain:

$$q_2^F = b_6(1152+228b-2112b^2-540b^3+1724b^4+370b^5-876b^6-$$
$$148b^7+301b^8+34b^9-71b^{10}-4b^{11}+11b^{12}-b^{14}),$$
$$q_1^L = b_6(576-840b^2+574b^4-237b^6+65b^8-11b^{10}+b^{12}),$$
$$x_2^F - q_2^F = b_6(1+b)b(1-b)(3-b^2)(48-50b^2+23b^4-6b^6+b^8),$$
$$x_1^L - q_1^L = b_6(1+b)b^2(1-b)(12-5b^2+b^4)(6-3b^2+b^4).$$

where $x_1^L - q_1^L > 0$ and $x_2^L - q_2^L > 0$ since $b \in (0, 1)$.

The profit of the private firm and social welfare are:

$$p_1^L = (a-m)b_6(1-b)(2-b^2)(12-5b^2+b^4)^2,$$

$$W^F = \frac{(b_6)^2(1+b)}{2}(2322432+331776b-8391168b^2-1340928b^3+14406336b^4+2458944b^5$$
$$-15817272b^6-2783688b^7+12517732b^8+2214076b^9-7591574b^{10}-1319338b^{11}+$$
$$3651955b^{12}+608989b^{13}-1420673b^{14}-221183b^{15}+450863b^{16}+63427b^{17}-116687b^{18}-$$
$$14237b^{19}+24387b^{20}+2443b^{21}-4023b^{22}-305b^{23}+501b^{24}+25b^{25}-43b^{26}-b^{27}+2b^{28}).$$

6. Proof of Lemma 2

Comparing the profit obtained by the private firm in the three cases we obtain:

i) $x_1^F - x_1^S = 144b^2 \dfrac{b_4 b_5}{(a-m)}(2-b^2)^2(3-3b-4b^2+4b^3+4b^4-4b^5-b^6+b^7)$,

ii) $x_1^L - x_1^S = \dfrac{b_4 b_6}{(a-m)} 4b^2(2-b^2)(15-10b^2+4b^4-b^6)(48-50b^2+23b^4-6b^6+b^8)$,

iii) $x_2^F - x_2^S = \dfrac{-b_4 b_6}{(a-m)} 8b^3(2-b^2)(15-10b^2+4b^4-b^6)^2$,

iv) $x_2^L - x_2^S = \dfrac{b_4 b_5}{(a-m)} 12b(2-b^2)(1-b)(3-b^2)(-1+b^2-b^4)\ (72-84b^2+49b^4-10b^6+b^8)$,

v) $$p_2^F - p_2^S = 8b^3\frac{(b_4 b_6)^2}{(a-m)^2}(1+b)(1-b)(2-b^2)(15-10b^2+4b^4-b^6)(165888-103680b- \\ -649728b^2+438480b^3+1199808b^4-850608b^5-1390992b^6+1024416b^7+ \\ 1138128b^8-867066b^9-698464b^{10}+549618b^{11}+332416b^{12}-270502b^{13}- \\ 124883b^{14}+105265b^{15}+37244b^{16}-32570b^{17}-8771b^{18}+7961b^{19}+1600b^{20}- \\ 1506b^{21}-217b^{22}+211b^{23}+20b^{24}-20b^{25}-b^{26}+b^{27})\ ,$$

vi) $$p_2^L - p_2^S = 24b\frac{(b_4 b_5)^2}{(a-m)^2}(1-b)(3-b^2)(2-b^2)(-1+b^2-b^4)(-995328+974592b+ \\ 4748544b^2-4675968b^3-10806912b^4+10615104b^5+15511104b^6-15082800b^7- \\ 15639600b^8+14954532b^9+11682044b^{10}-10929064b^{11}-6648424b^{12}+6074473b^{13}+ \\ 2921007b^{14}-2616213b^{15}-992713b^{16}+881205b^{17}+259081b^{18}-232039b^{19}-51047b^{20}+ \\ 47229b^{21}+7361b^{22}-7251b^{23}-727b^{24}+799b^{25}+43b^{26}-57b^{27}-b^{28}+2b^{29})\ ,$$

vii) $$p_1^S - p_1^F = \frac{(b_4 b_5)^2}{(a-m)^2}48b^2(1-b)^2(-3+b^2)(2-b^2)^2(1-b^2+b^4)\ (72-84b^2+49b^4- \\ 10b^6+b^8)\ (1728-4824b^2+6156b^4-4610b^6+2179b^8-658b^{10}+132b^{12}-16b^{14}+b^{16})\ ,$$

viii) $p_1^L - p_1^S = \dfrac{b_6 (b_4)^2}{(a-m)} 16b^4(1-b)(2-b^2)^2(15-10b^2+4b^4-b^6)^2$,

ix) $W^L - W^S = \dfrac{b_5 (b_4)^2}{(a-m)} (12b(1-b)(3-b^2)(2-b^2)(1-b^2+b^4))^2$,

x) $$W^S - W^F = -\frac{(b_6 b_4)^2}{(a-m)^2}8(1+b)(1-b)b^2(2-b^2)\ (15-10b^2+4b^4-b^6) \\ (48-50b^2+23b^4-6b^6+b^8)\ (3456-12024b^2+20208b^4-20792b^6+ \\ 14333b^8-7003b^{10}+2493b^{12}-650b^{14}+121b^{16}-15b^{18}+b^{20})\ ,$$

where: $x_1^F - x_1^S > 0$, $x_1^L - x_1^S > 0$, $x_2^F - x_2^S < 0$, $x_2^L - x_2^S < 0$, $p_2^F - p_2^S > 0$, $p_2^L - p_2^S > 0$, $p_1^S - p_1^F < 0$, $p_1^L - p_1^S > 0$, $W^L - W^S > 0$ and $W^S - W^F < 0$ since $b \in (0, 1)$.

By computing the consumer surplus in the three cases considered, it can be seen that:

i) $$CS^L - CS^S = \frac{(b_4 b_5)^2}{(a-m)^2}24b(1-b)(3-b^2)(2-b^2)(1-b^2+b^4)\ (-995328+539136b+ \\ 5184000b^2-2747520b^3-12735360b^4+6571584b^5+19554624b^6-9806712b^7- \\ 20915688b^8+10200372b^9+16436204b^{10}-7817558b^{11}-9759930b^{12}+4553805b^{13}+$$

$$4441675b^{14}-2052167b^{15}-1556759b^{16}+721091b^{17}+419195b^{18}-197359b^{19}-85727b^{20}+41613b^{21}+12977b^{22}-6597b^{23}-1381b^{24}-749b^{25}+93b^{26}-55b^{27}-3b^{28}+2b^{29}),$$

ii) $CS^F - CS^S = \dfrac{(b_6 b_4)^2}{(a-m)^2} 8b^2(1+b)(1-b)(2-b^2)(15-10b^2+4b^4-b^6)$ $(165888-$

$$165888b-715392b^2+649728b^3+1419552b^4-1199808b^5-1747848b^6+1390992b^7+1506648b^8-1138128b^9-966308b^{10}+698464b^{11}+476593b^{12}-332416b^{13}-183633b^{14}+124883b^{15}+55483b^{16}-37244b^{17}-13031b^{18}+8771b^{19}+2323b^{20}-1600b^{21}-299b^{22}+217b^{23}+25b^{24}-20b^{25}-b^{26}+b^{27}),$$

where $CS^F - CS^S > 0$ if and only if $b < 0.7092$ and $CS^L - CS^S < 0$ since $b \in (0, 1)$.

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