Consumption-Leisure Trade-offs and Persistency in Business Cycles

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Abstract

This paper studies whether nonseparabilities between consumption and leisure may help to explain the observed persistence in GNP growth. We consider an extended version of Lucas' (1988) human capital investment model that includes labor adjustment costs and compare its performance under different utility specifications with different degrees of complementarity and substitutability between consumption and leisure. We find that when consumption and leisure are complements the model succeeds in matching not only the autocorrelation of output growth but also the important trend-reverting component found in US data. These results hold even if low adjustment costs of labor are considered. Hence, we conclude that an arguably simple margin not studied previously can provide useful insights into observed business cycle patterns.

JEL classification: E32; O41; C52.

Keywords: Real Business Cycle Models; Endogenous Growth; Propagation

Mechanism; Persistence.

1   Introduction

Even though preference specifications that are nonseparable in consumption and leisure have already been formally considered in fiscal policy studies (McGrattan, 1994; Finn, 1998), international RBC models (Baxter, 1995), the asset pricing literature (Mankiw, 1985; Eichenbaum, 1988), life-cycle models (Low, 2005) and monetary policy studies (Matheny, 1998), among other areas, most RBC models typically consider constant relative risk aversion preferences with leisure and consumption entering the utility function separably. Additively separable utility functions are used for the sake of analytic simplicity. As is already known, if utility is separable in consumption and labor, a logarithmic utility consumption is needed if a balanced growth path (BGP hereafter) is to exist, implying a relative risk aversion (measured as the inverse of the intertemporal elasticity of substitution in consumption) equal to one.[1] As a result, most RBC models assume an additively separable utility function in which case the RRA must equal one.

As is also known, in static models labor supply depends on the degree of complementarity or substitutability between consumption and leisure. An increase in consumption shifts to the right the short run labor supply curve when the marginal disutility of labor is a decreasing function of consumption (i.e., when consumption and leisure are substitutes). If consumption and leisure are complements the opposite effect occurs, and when consumption and leisure enter the utility function separably, no such labor supply effect arises. As noted by de Hek (1998, p. 255): "In a dynamic context of economic growth this acquires great significance as the nature of consumption leisure trade-off determines the intertemporal accumulation paths for the economy."

The empirical literature documents two stylized facts about U.S. output dynamics: first, GNP growth is positively autocorrelated over short horizons and has a weak

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[1]In the long run the effect of productivity growth on real wages and consumption must generate offsetting income and substitution effects to ensure the absence of a trend growth in per capita labor supply. King et al. (1988a) showed that for this to be achieved, when additively separable utility functions are considered they must be logarithmic in consumption implying a relative risk aversion (RRA hereafter) parameter equal to one, and for non-additively separable utility functions the RRA parameter can be greater or lower than one.

and possibly insignificant negative autocorrelation over longer horizons; second, GNP appears to have an important trend-reverting component that has a hump-shaped moving average representation. Several modeling strategies have been adopted in order to explain these patterns: capital and labor adjustment costs (Cogley and Nason, 1995), variable factor utilization rates (Burnside and Eichenbaum, 1996), the combination of habit formation in leisure and increasing returns to scale (Wen, 1998), external increasing returns and indeterminacy (Benhabib and Wen, 2004; Schmitt-Grohe, 2000), externalities and multiple equilibria (Perli, 1998), multisector models (Benhabib et al., 2006), 'skilled' and 'unskilled labor' with low elasticity of substitution in the production of human capital (Perli and Sakellaris, 1998) and job search (Andolfatto, 1996), among others.

All these papers can be broadly classified into three groups. Some papers rely on strong increasing returns. However, as Schmitt-Grohe (2000) finds these returns do not seem empirically plausible and as Wen (1998) shows they may even generate an upward sloping aggregate labor demand. Other papers such as Andolfatto (1996), Burnside and Eichenbaum (1996) and Cogley and Nason (1995) rely on 'rigidities' in the labor market. In Andolfatto (1996) these frictions arise endogenously, whereas in the other two they are an assumption. Among these papers only the job search approach by Andolfatto (1996) succeeds in replicating the observed dynamic pattern of output growth. Finally, other papers such as Benhabib et al. (2006) and Jones et al. (2005) emphasize the role played by intratemporal substitution across sectors, but they both fail to generate the autocorrelation found in the data.

Combinations of these three strategies have also been studied by Perli (1998) and Perli and Sakellaris (1998), among others. Perli (1998) considers a home sector coupled with increasing returns and Perli and Sakellaris (1998) rely on the low elasticity of substitution in the human capital sector relative to the physical sector which generates an adjustment cost effect. They both obtain autocorrelation properties similar to those observed in the data. Our paper pursues this line of research, but we consider an arguably simpler strategy: we study the role that intratemporal nonseparabilities between consumption and leisure may have in explaining the above mentioned patterns.

In particular, we consider an extended version of Lucas' (1988) human capital in-

vestment model that includes 'effective' labor adjustment costs.[2] We then generalize the utility function by including leisure. Further, we next study different utility specifications which satisfy the conditions needed to allow the existence of a BGP following King et al. (1988a), as is common in the RBC literature.

As mentioned above, even though nonseparable preference specifications have already been studied in different areas, there is no consensus about the best fitting preference specification. A general conclusion of this paper is that the dynamic features of the model under study joint with the ability to match important RBC features could drive the choice of the specification of the utility function. When the stochastic nature of the trend is exogenous (as is the case in the standard RBC model with exogenous permanent productivity shocks) changes in the RRA parameter will not affect the growth rate and therefore the degree of persistence will not vary. In this context, an additively separable utility function seems to be a good specification to analyze RBC features. However, in the context of an endogenous growth model, transitory shocks may have permanent effects on output, due to temporary changes in the amount of resources allocated to growth.[3] In this setting, one should expect the dynamics of growth and cyclical variables to be sensitive to alternative specifications of the utility function.

In our paper, we compare a log-separable specification of the utility function with a multiplicatively separable one in characterizing persistence in output growth, and assess the sensitivity of the results of the model to the degree of complementarity or substitutability between consumption and leisure. When the multiplicatively separable specification is considered, the RRA parameter determines the degree of complementarity or substitutability between consumption and leisure. Hence, standard RRA specifi-

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[2]In our model we impose labor adjustment costs following Cogley and Nason (1995) in order to achieve an improvement in the autocorrelation of GNP growth. As shown by Perli and Sakellaris (1998) this improvement could also be obtained endogenously without imposing labor market frictions by considering that labor is an aggregation of 'skilled' and 'unskilled labor' and that these two types of labor are substitutable in different degrees in each sector.

[3]It is a well-known result in the endogenous growth literature that temporary shocks may cause permanent effects on output by allowing temporary changes in the amount of resources allocated to growth. Thus, temporary shocks become persistent as they have effects on the growth rate. See King et al. (1988b) for a detailed discussion on this issue.

cations allow us to study the role that the degree of substitutability or complementarity may play by varying a single parameter. Interestingly, we find that when consumption and leisure are complements the model is able to reproduce the persistence of output growth found in the data, even with low adjustment costs of labor.

The rest of the paper is as follows: We briefly describe the endogenous growth model considered in Section 2. Section 3 analyzes the persistence in output growth, and Section 4 concludes.

2   The Model

We consider a stochastic discrete time version of Lucas' (1988) model in the absence of externalities with two modifications. First, we allow agents to derive utility not only from consumption but also from leisure.[4] Second, as suggested by Shapiro (1986) and Cogley and Nason (1995), labor adjustment costs are included. Here we study whether persistence results depend on the nature of how consumption and leisure enter the utility function. In particular, we consider two utility function specifications: a log-separable specification and a multiplicatively separable specification.

The economy consists of a large number of productive families which own both the production factors and the technology used in two productive activities: the production of the final good (market sector) and the production of new human capital (human capital sector).[5] Population size is assumed to be constant. At any point in time, individuals must decide what fraction of their time they devote to each of these activities, and how much time they set aside for leisure. The time endowment is normalized to one, so that l_t denotes the fraction of time given over to leisure and n_t the fraction of time devoted to the production of the consumption good.

The technology of the consumption good is described by a production function with

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[4]As shown by Ladrón de Guevara et al. (1999), the concavity of the representative agent's problem is not guaranteed. In the simulations reported below, existence and uniqueness of the BGP is verified for each calibration. For all models and parameter choices, the equilibrium was found to be saddlepath stable.

[5]The introduction of a non-market sector competing with the market sector has already been used in RBC literature. See for example Benhabib et al. (1991) and Greenwood and Hercowitz (1991).

constant returns to scale with respect to physical capital and efficient labor. As already mentioned, we also consider labor adjustment costs. In particular, the production function includes quadratic adjustment costs in labor input measured in efficiency units. Formally, the production technology is made (log) linear in the cost of adjustment,

$$
\begin{aligned}
\ln y_t &= \ln[F^m(k_t, Z_t, n_t h_t)] - \frac{\eta}{2}\left[\frac{\Delta(n_t h_t)}{n_{t-1}h_{t-1}}\right]^2 \\
&= \ln[A_m Z_t k_t^\alpha (n_t h_t)^{1-\alpha}] - \frac{\eta}{2}\left[\frac{\Delta(n_t h_t)}{n_{t-1}h_{t-1}}\right]^2,
\end{aligned}
\tag{1}
$$

where $n_t h_t$ represents the qualified labor units, the term in brackets represents the percentage change in efficient labor, A_m is the parameter which measures the productivity of this sector, k_t and h_t are the stocks of physical capital and human capital in per-capita terms, respectively, η is the labor adjustment cost parameter, and finally Z_t is a technology shock characterized by the following autoregressive process:

$$
\ln(Z_t) = \rho_1 \ln(Z_{t-1}) + (1 - \rho_1)\ln(\bar{Z}) + \varepsilon_t,
\tag{2}
$$

where $\ln(\bar{Z})$ is the mean of $\ln(Z_t)$ and ε_t is a serially independent innovation with standard deviation σ_Z.

The law of motion for physical capital is:

$$
k_{t+1} = y_t - c_t + (1 - \delta_k)k_t,
\tag{3}
$$

where c_t denotes consumption and δ_k represents the depreciation rate of physical capital, which is assumed to be constant.

New human capital is assumed to evolve according to the following process:

$$
h_{t+1} = A_h \theta_t (1 - l_t - n_t)h_t + (1 - \delta_h)h_t,
\tag{4}
$$

where A_h measures the productivity of this sector, δ_h denotes the depreciation rate of human capital and θ_t is a shock which follows a first order autoregressive process given by:

$$
\ln(\theta_t) = \rho_2 \ln(\theta_{t-1}) + (1 - \rho_2)\ln(\bar{\theta}) + \epsilon_t,
\tag{5}
$$

where $\ln(\bar{\theta})$ is the mean of $\ln(\theta_t)$ and ϵ_t follows a white noise process with standard deviation σ_θ. It is further assumed that θ_t is uncorrelated with the shock Z_t.

Consumers derive utility from the consumption of the final good and from leisure. Future utility is discounted at a rate β and preferences are described by the following utility function:[6]

$$U(c_t, l_t) = \begin{cases} \frac{(c_t^\lambda l_t^{1-\lambda})^{1-\gamma}-1}{1-\gamma}, & 0 \leq \lambda \leq 1, \quad \gamma > 0 \text{ and } \gamma \neq 1 \\ \lambda \ln c_t + (1-\lambda) \ln l_t & \text{for } \gamma = 1 \end{cases} \tag{6}$$

This utility function is increasing and concave in both arguments: $U_c > 0$, $U_l > 0$, $U_{cc} < 0$, $U_{ll} < 0$. Note that when $\gamma = 1$ we have that $U_{cl} = 0$ (i.e., the marginal utility of leisure is independent of consumption). However, for $\gamma \neq 1$, the marginal utility of leisure depends on the level of consumption, $U_{cl} \neq 0$. Further, depending on the value of γ, consumption and leisure will be substitutes or complements: for $\gamma > 1$ we have $U_{cl} < 0$, whereas for $\gamma < 1$ we have $U_{cl} > 0$.

As is well known, in the absence of externalities, public goods and distortionary taxation, the solution to the planner's problem is the competitive equilibrium allocation. The problem faced by the central planner is to choose the sequences of consumption, hours worked, leisure, physical capital and human capital that maximize the expected discounted stream of utility given by:

$$\max_{n_t, c_t, l_t, k_{t+1}, h_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t),$$

subject to (1), (2), (3), (4), (5), (6), the usual non-negativity constraints, $0 \leq l_t \leq 1$, $0 \leq 1 - l_t - n_t \leq 1$, and given an initial condition $(Z_0, \theta_0, k_0, h_0, n_0)$.

In the steady state, n_t and l_t remain constant, since they are bounded between zero and one. Further, since consumption and investment are perfect substitutes $(y_t = c_t + i_t)$, the growth rates of consumption, investment, output and physical capital must be equal to the rate of growth of human capital. Therefore, stationary time series can be obtained by expressing growing variables in relation to the stock of human capital: $\hat{c}_t = \frac{c_t}{h_t}$, $\frac{h_{t+1}}{h_t}$ and $\hat{k}_t = \frac{k_t}{h_t}$. Hence, the FOC's can be expressed in ratios as:

$$U_1(\hat{c}_t, l_t) = \beta(\frac{h_{t+1}}{h_t})^{-\sigma} E_t\{U_1(\hat{c}_{t+1}, l_{t+1})[\frac{\alpha \hat{y}_{t+1}}{\hat{k}_{t+1}} + 1 - \delta_k]\}, \tag{7}$$

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[6]Note that this function satisfies the conditions needed to ensure the existence of a BGP. For further details, see King et al. (1988a, pp. 201-202).

$$U_2(\hat{c}_t, l_t) - U_1(\hat{c}_t, l_t) \left[\frac{1-\alpha}{n_t} - \eta \frac{n_t - n_{t-1}\frac{h_{t-1}}{h_t}}{n_{t-1}\frac{h_{t-1}}{h_t}} \frac{h_t}{n_{t-1}h_{t-1}} \right] \hat{y}_t =$$

$$\beta(\frac{h_{t+1}}{h_t})^{1-\sigma} E_t \{ U_1(\hat{c}_{t+1}, l_{t+1}) \eta \frac{n_{t+1} - n_t \frac{h_t}{h_{t+1}}}{n_t \frac{h_t}{h_{t+1}}} \frac{n_{t+1}h_{t+1}}{n_t h_t} \frac{\hat{y}_{t+1}}{n_t} \}, \tag{8}$$

$$\frac{U_2(\hat{c}_t, l_t)}{A_h \theta_t} = \beta(\frac{h_{t+1}}{h_t})^{-\sigma} E_t \{ \frac{U_2(\hat{c}_{t+1}, l_{t+1})}{A_h \theta_{t+1}} [A_h \theta_{t+1}(1 - l_{t+1}) + 1 - \delta_h] \}, \tag{9}$$

$$\frac{h_{t+1}}{h_t} = A_h \theta_t (1 - l_t - n_t) + 1 - \delta_h, \tag{10}$$

$$\hat{c}_t + \hat{k}_{t+1} \frac{h_{t+1}}{h_t} = \frac{A_m Z_t \hat{k}_t^\alpha n_t^{1-\alpha}}{\exp \left\{ \frac{\eta}{2} \left[\frac{n_t - n_{t-1}\frac{h_{t-1}}{h_t}}{n_{t-1}\frac{h_{t-1}}{h_t}} \right]^2 \right\}} + (1 - \delta_k)\hat{k}_t, \tag{11}$$

where $\hat{c}_t = \frac{c_t}{h_t}$ and $\hat{k}_t = \frac{k_t}{h_t}$.

From equation (10) it follows that the human capital growth rate depends on the resources allocated to this activity which will be constant in the steady state.

This endogenous growth model has some important properties for growth and fluctuations. On the one hand, although there is no steady state capital stock, there is a steady state ratio of physical to human capital. On the other hand, this model generates integrated time series, even when the underlying shocks are stationary. After the effects of these shocks vanish, output do not return to its trend level. That is, temporary shocks have permanent effects on output since they generate endogenous responses in the amount of resources allocated to growth. As a result, growth dynamics is an important component of the propagation mechanism in which the stochastic properties of the trend are endogenous. As argued by Fatás (2000), in this setting, output persistence is not simply equal to the persistence of disturbances, since shocks endogenously generate changes in the capital accumulation rate which result in persistent responses of output.

2.1 Calibration

Following the calibration procedure suggested by Kydland and Prescott (1982), we assign values to the parameters of the model based on either micro-evidence or long-

run properties of the economy. The steady state values of the variables are approached by averaging the corresponding US time series for the period 1955:3 to 1984:1. When the standard deviations of the shocks are set to 0, the model converges to a steady state. The model's nonstochastic BGP should satisfy the following equations:

$$\beta(1+v)^{-\sigma}\{\alpha\frac{\hat{y}}{\hat{k}} + 1 - \delta_k\} = 1,$$ (12)

$$(1-\lambda)\frac{\hat{c}}{\hat{y}} =$$

$$\lambda l\left[\frac{1-\alpha}{n} - \eta\frac{v(1+v)}{n}\right] + \beta(1+v)^{1-\sigma}\lambda l\eta\frac{v(1+v)}{n},$$ (13)

$$\beta(1+v)^{-\sigma}\{A_h(1-l)\bar{\theta} + 1 - \delta_h\} = 1,$$ (14)

$$v = A_h(1-l-n)\bar{\theta} - \delta_h,$$ (15)

$$\frac{\hat{c}}{\hat{y}} + \frac{\hat{k}}{\hat{y}}(v + \delta_k) = 1,$$ (16)

where v is the growth rate of all growing variables in the steady state, $\hat{y} = \frac{A_m\bar{Z}\hat{k}^\alpha n^{1-\alpha}}{\exp\{\frac{\eta}{2}v^2\}}$ and with $\bar{Z} = 1$ and $\bar{\theta} = 1$. Note that the real interest rate is equal to the rental price of capital net depreciation: $r = \alpha\frac{\hat{y}}{\hat{k}} - \delta_k$.

Looking at the market sector, the parameter α is set at 0.36, which is the capital's average share of per capita US GNP during the period under study. The rate of depreciation for physical capital, δ_k, is set equal to 0.025, which is equivalent to the 10% annual rate used by Kydland and Prescott (1982). The parameter A_m is normalized to unity.

The estimates for the parameters that describe the stochastic process which characterize the technology shock dynamics (Z_t) are provided by Prescott (1986), i.e., $\rho_1 = 0.95$ and $\sigma_Z = 0.007$.

The labor adjustment parameter, η, has been calibrated from estimates in Shapiro (1986). Following Cogley and Nason (1995) we take $\eta = 0.36$ as the baseline value. These authors point out that this value probably overstates the size of aggregate labor adjustment costs. Here, labor is measured in efficiency units and, as a consequence, not only the hours worked but also human capital are subject to adjustment costs. Hence, the same baseline value seems to be more suitable when human capital is included.

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Given that the human capital sector here is a non-market sector, the calibration of the parameters involved is no trivial. The quarterly growth rate of output is set to $v = 0.0036$ which is consistent with the observed average growth rate of per capita output in the US (1.44% per annum). Parameter A_h is chosen to match the 1.44% annual growth rate and is inferred from (14) and (15). Estimates for human capital depreciation rate, δ_h, range from approximately 0.6% to 13.3% per year (Heckman, 1976; Rosen, 1976). We consider $\delta_h = 0.005$ which is equivalent to 2% per year. Coefficient ρ_2 has been assigned the same value as the one assigned to the technology shock (0.95). In order to examine the robustness of the results to changes in σ_θ, we have experimented with different values for this parameter. The results are shown in Appendix A. It is assumed that both shocks are uncorrelated.

The discount factor (β) is chosen so that the steady state real interest rate (r) is equal to 1% per quarter. The value for β is derived from (12). The capital to (quarterly) output ratio implied by (12) is $\frac{k}{y} = 10.28$, which is consistent with the observed value. Combining this information with (16) results in a consumption to output ratio of about $\frac{c}{y} = 0.70$, which is close to what is observed.

Looking at household preferences, since the utility function is multiplicatively separable, it can be written as $U(c, l) = u(c)v(l)$, where $u(c)$ is homogeneous of degree $1 - \sigma$. Note that $(1 - \gamma)\lambda = 1 - \sigma$. When $\sigma = 1$ (i.e., $\gamma = 1$) the utility function is logarithmic and $U_{cl} = 0$. But, for $\sigma > 1$ (i.e., $\gamma > 1$) then $U_{cl} < 0$, and for $\sigma < 1$ (i.e., $\gamma < 1$) then $U_{cl} > 0$, respectively. Mehra and Prescott (1985) consider σ in the interval [0, 10]. We consider a smaller interval [0.9, 2] to show how sensitive certain statistics displayed by the model are to changes in the degree of complementarity or substitutability between consumption and leisure.[7] The parameter λ is established to guarantee that the steady state fraction of time allocated to the market sector (n) is 0.24, which is equal to the average fraction of time spent working by the U.S. working-age population over the

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[7]Most empirical studies on the RRA coefficient suggest a moderate range for σ that includes the [0.5, 3] interval. For example, Hansen and Singleton (1982) estimate that σ ranges between 0.3502 and 0.9903, σ being less than 1. But, Friend and Blume (1975) estimate that σ lies between 1 and 3. As mentioned above, in this paper all parameter choices for σ are consistent with the evidence cited by Mehra and Prescott (1985).

sample.

A list of the calibrated parameter values is contained in Table 2. This model has no closed-form solution. Hence, a numerical approximation method is required to solve it. The resolution method we follow is Uhlig's (1999) Log-linear Method (see the technical appendix for more details).

3 Persistence in output growth

The simulation procedure we employ to study the dynamic properties can be summarized as follows. We generate artificial time series for output by simulating various RBC models. This allows us to evaluate the contribution of each utility specification in explaining the persistence in output growth. In particular, the performance of the model is compared with U.S. quarterly data from 1955:3 to 1984:1. The autocorrelation function and impulse response functions are estimated for each artificial sample which is 115 periods long (each model was simulated 1,000 times) and the corresponding empirical probability distributions are computed.

3.1 Autocorrelation Functions

We analyze whether all the different models replicate the sample autocorrelation function (ACF) for output growth. Given a model, we have computed the average c and the covariance matrix V_c of the ACF's of the artificial time series. We apply the following generalized test statistic to analyze the goodness of fit between the actual and the theoretical ACF's:

$$Q_{acf} = (\widehat{c} - c)'V_c^{-1}(\widehat{c} - c),$$

where $\widehat{c}$ stands for the actual ACF and c for the model-generated one. A high value of Q_{acf} indicates a poor fit between the theoretical ACF and the actual ACF. The generalized Q_{acf} statistic is approximately $\chi^2(p)$, where p is the number of lags in c.

Following Cogley and Nason (1995) (CN hereafter), we compute the generalized statistic for the first $p = 8$ autocorrelations. The results are summarized in Table 3 and Figure 1. The first three columns of Table 3 report values of the Q_{acf} statistic for

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each choice of σ with probability values in parentheses. Lines 1-5 show how sensitive the results are to changes in the labor adjustment costs parameter, η.

As σ becomes higher, the p-values decrease. We find the same result as η decreases. Numerical results show that regardless of the value of σ, the introduction of labor adjustment costs is crucial for obtaining realistic ACF results for output growth. However, when $\sigma = 0.9$ (i.e., consumption and leisure are complements in the sense that $U_{cl} > 0$) the model is not rejected even for $\eta = \frac{0.36}{4}$. Figure 1 shows that the lower the RRA parameter σ, the higher the first autocorrelation coefficients are. The internal propagation mechanism of the model provides some intuition for these results.

We have also analyzed the sensitivity of the ACF results to changes in the standard deviation of the human capital shock, σ_θ. As shown in Appendix A, the ACF results do not depend on human capital accumulation process being stochastic since when $\sigma = 0.9$ and $\eta = \frac{0.36}{3}$ the model is not rejected even for a human capital shock that is half as large.

3.2   Impulse Response Functions

We also analyze whether these models replicate observed impulse response functions (IRF's). The IRF's are obtained by using the structural VAR model with long-run restrictions developed by Blanchard and Quah (1989). Note that, although the specification considered for both shocks is an AR(1) process, either shock affect the level of the long-run growth path of output. As a consequence, we cannot identify demand shocks and supply shocks as Blanchard and Quah (1989) did. But as argued by Collard (1999), the Blanchard and Quah (1989) approach is still valid to obtain a trend-cycle (permanent-transitory) decomposition and compute the trend-reverting component of output.[8] To implement this technique, a third-order VAR for per-capita output growth and ln(hours) is estimated.[9] We analyze whether the theoretical IRF is close to the

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[8]Collard (1999) studies the persistence properties of an endogenous growth model with temporary disturbances altering the long-run growth of output, as is the case in our model. In such a setting, they use the Blanchard and Quah (1989) approach to identify trend versus cyclical components.

[9] We select the order of the VAR model following the model selection criteria given by Lütkepohl (1991).

actual IRF by using the following test statistic:

$$Q_{irf} = (\widehat{r} - r)'V_r^{-1}(\widehat{r} - r),$$

where $\widehat{r}$ is the actual IRF and r is the model-generated one, which is computed by the average of IRF's across the artificial time series. The matrix V_r denotes the covariance matrix of the IRF's of the Monte Carlo draws.

A high value of Q_{irf} indicates that the performance of the theoretical model is not consistent with actual data. We compute this statistic with coefficients up to lag 8. Table 3 reports the sensitivity of the Q_{irf} statistics to changes in σ, with Monte Carlo probability values in parentheses. Figure 2 illustrates the transitory IRF's for this generalized endogenous growth model with different values of σ.

Let us consider our benchmark value $\sigma = 1$ in which case the utility function is logarithmic. In this case, the results are partially successful since the model is able to generate the pattern of the transitory IRF found in the data, but fails to reproduce the permanent IRF. In contrast with well-known transitory IRF results on standard RBC models, our setting is not rejected at conventional significance levels even when $\eta = 0$ (see the fifth column of Table 3). Hence, this human capital investment model not only generates the right qualitative response to transitory shocks but it is also able to match the magnitude of the transitory IRF.

The results are very sensitive to changes in the value of the RRA parameter. When $\sigma = 2$ labor adjustment costs are needed in order to generate the transitory IRF found in the data. As σ decreases the performance of the model improves. Note that when $\sigma = 0.9$ the model is not rejected at conventional significance levels even in the absence of labor adjustment costs (i.e., when $\eta = 0$). Hence, when consumption and leisure are complements there is no need to include labor adjustment costs with endogenous growth in order to match not only the large transitory IRF, but also the permanent IRF found in the data. Figure 2 shows that as the RRA decreases (i.e., as the degree of complementarity between consumption and leisure increases) the hump displayed by the transitory IRF increases.

As is well known, if the model were driven by a single shock its bivariate VAR would have stochastic singularities. The IRF results are sensitive to changes in σ_θ but,

as shown in Appendix A, when $\sigma = 0.9$ (i.e., consumption and leisure are complements) and $\eta = \frac{0.36}{3}$, the model is able to match the observed transitory IRF even for $\sigma_\theta = 0.002$.

3.3   The dynamic response functions

In order to assess the importance of those properties of the model that amplify the effect of the shocks and cause the deviation from the steady state to persist, and also to get some intuition for the results, we analyze how the internal propagation mechanism is affected by changes in σ.

Figures 3 and 4 report the responses of certain interesting variables to both technology (Z_t) and human capital (θ_t) shocks, respectively. As shown by these figures, the dynamic response functions depend on the value of σ, which determines the degree of complementarity or substitutability between consumption and leisure.

When a favorable technology shock (Z_t) takes place, the greater the degree of risk aversion is, the fewer the resources that are devoted to producing goods. This, in the end, leads to a smaller reduction of the growth rate (see g_t in Figure 3). Due to the existence of labor adjustment costs, output not only rises at the time of the impact but also in subsequent periods. As shown in this figure, not only the impact effect on output of Z_t but also the lagged effects become smaller the greater σ is. This generates not only a smaller serial correlation in output growth but also a smaller hump in the transitory IRF (see Figure 2). In addition to this effect, we must take into account that which results from considering human capital shocks (θ_t). In that case, the greater the degree of risk aversion, the fewer the resources that are devoted to human capital accumulation, which will lead to a smaller increase of the growth rate (see g_t in Figure 4). Output falls during some periods and subsequently rises back toward its initial trend, but the greater σ is, the smaller is the response. Hence, this second effect also generates a smaller correlation in output growth.[10] As a result, increasing RRA leads

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[10]Note that this second effect is only observed when the human capital production sector is considered. As noted by Barañano and Moral (2003), this effect explains why introducing endogenous growth in non-standard RBC model with labor adjustment costs enhances the model's ability to reproduce the observed persistence in US output.

to decreasing the serial correlation in output growth. Figure 3 illustrates how the hump displayed by hours increases as σ decreases, since hours worked not only increase at the time of the impact but also in subsequent periods, and the same holds for output.

These sensitivity results are consistent with the results obtained in a recent paper by Jones et al. (2005) (JMS hereafter), in a similar endogenous growth model.[11] They show that in contrast to well-known results for exogenous growth models, the RRA parameter plays a major role in determining the second moment properties of several macroeconomic time series in a human-capital based endogenous growth model.[12] As argued by JMS (2005), these sensitivity results arise in human capital investment models since the total share of all capital is large relative to the share in otherwise usual RBC models, which has important consequences for the response to a shock.[13]

From this discussion it is clear that in human capital based growth models the RRA parameter plays an important role in generating the kind of internal propagation mechanism needed to obtain realistic output dynamics of GNP. Further, our results highlight how the degree of complementarity or substitutability between consumption

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[11]Their human capital investment model differs from ours in many aspects. First, they not only consider that physical capital enters the human capital production function, but also that both capitals are generated by the same technology, implying that they are perfect substitutes. In our setting the production of human capital involves no physical capital. Second, they do not include labor adjustment costs and, as a consequence, their model fails to generate the ACF found in the data.

[12]JMS (2005) show how sensitive several second moments statistics are to changes in the RRA parameter. They study: (i) the standard deviations of the growth rate of output, the growth rate of labor productivity, the investment-output ratio and labor; (ii) the first-order autocorrelations of the growth rate of output, the growth rate of labor productivity and labor; and (iii) several cross-correlations. Note that we study the whole ACF as suggested by CN (1995), whereas JMS consider only the first autocorrelation coefficient of output growth. Maury and Tripier (2003) show the importance of preserving the whole CN's empirical setup. These authors analyze the properties of JMS's model from the viewpoint of CN's analysis and conclude that although this model improves the first positive value of the ACF over the standard RBC model, it is however unable to reproduce the negative values of the ACF for higher orders.

[13]As they explain, in a non-stochastic version of this endogenous growth model when a surprise increase in capital stocks takes place, output increases at impact but there is no dampened return for output in levels to the original time path, as occurs in the exogenous version. For a detailed discussion see JMS (2005, p. 816).

and leisure is a key factor in determining the properties of the model.[14] The numerical results show that dynamics of the variables of the model may vary substantially. For instance, let us consider the dynamic response of leisure to a 1% technology shock. As shown by Figure 3, when $\sigma = 1$ individuals maintain a smooth path for leisure, implying that they respond to fluctuations by varying the time allocated to each sector. As we move from this bechmark value, the behavior of leisure changes. When $\sigma > 1$ the time devoted to leisure falls at the impact period and remains under the steady state value for longer than fifty periods. As Figure 3 shows, the hump displayed by consumption is smaller, since consumption and leisure are substitutes. However, when $\sigma < 1$ (i.e., $U_{cl} > 0$), although the technology shock causes leisure to decrease at the impact period, it rises in subsequent periods and remains over the steady state value for longer than fifty periods. As also shown in Figure 3, the hump displayed by consumption increases, since an increase in leisure raises the marginal utility of consumption and induces individuals to consume more.

To sum up, this subsection shows how the degree of complementarity or substitutability between consumption and leisure may be a crucial determinant of the performance of this generalized endogenous growth model. We find that both ACF of output growth and IRF results are sensitive to changes in the RRA parameter. Our ACF results depend on the size of the labor adjustment costs, although the smaller the RRA parameter, the smaller the costs required. However, our IRF results do not depend on the size of these costs when leisure and consumption are complements.

These results naturally raise the question of whether this complementarity between consumption and leisure has a counterpart in the data. There are different types of leisure activities.[15] Following Hurd and Rohwedder (2003), some types of leisure such as watching TV seem to be neutral with consumption, others such as home production seem to be substitutes, and others such as travel and entertainment would seem to be

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[14]In our case, due to the specification considered, this degree of complementarity or substitutability depends on the value considered for σ.

[15]For a detailed description of different definitions of leisure see Aguiar and Hurst (2007). These definitions range from the narrowest one, which include any activity that yield direct utility, to the broadest one, that is the residual of market plus non-market work.

complements:"Everyday observation and introspection say that we have all types, and it is an empirical question as to which dominates"(p. 8). Hence, complementarities between consumption and leisure seem empirically plausible, although the extent of their magnitude is an open empirical question which, unfortunately, exceeds the scope of this paper.

4   Conclusions

This paper studies the role that nonseparabilities between leisure and consumption may have in explaining the persistence in output growth in postwar US data. It considers an extended version of Lucas' (1988) human capital investment model which includes 'effective' labor adjustment costs.

Even though nonseparable preference specifications have already been studied in different areas, there is no consensus about the best fitting preference specification. This paper shows that the dynamic features of the model under study joint with the ability to match important RBC features could drive the choice of the specification of the utility function. In particular, when the stochastic nature of the trend is exogenous (as in the standard RBC model) a log-separable utility function seems to be a good specification to analyze RBC features, since the RRA plays a minor role in characterizing both growth and cyclical features displayed by the model. This is the most common specification used in the RBC literature. However, when the stochastic properties of the trend are endogenous (as is the case in some endogenous growth models), transitory disturbances cause permanent effects on the level of output due to temporary changes in the amount of resources allocated to growth. In this context, the dynamics of growth and cyclical variables are sensitive to alternative specifications of the utility function. We argue that in this setting, the dynamic features of the model joint with its ability to replicate business cycle features could drive the choice.

In our paper we compare a log-separable specification with a multiplicatively separable specification in explaining the persistence in output growth. As argued by King et al. (1988a), in the latter case the RRA can be greater or lower than one, implying that the marginal utility of consumption can be a decreasing or an increasing function

of leisure, respectively. In this sense, given the specification considered, the RRA parameter determines the degree of complementarity or substitutability between leisure and consumption. We find that the persistence in GNP growth is sensitive to changes in the RRA parameter. Hence, the link between consumption and leisure built into the utility function may help to explain its persistence. In particular, when consumption and leisure are complements the model succeeds in matching not only the autocorrelation of output growth but also the trend-reverting component found in US data, even if low adjustment costs of labor are considered. We conclude that a seemingly simple margin not studied previously, but which occupies an important role in other areas, offers the potential to provide useful insights into observed patterns of business cycles.

Technical appendix

In a recent paper Uhlig (1995) (see also Uhlig (1999)) describes a simple log-linear method to solve for the dynamics of a nonlinear DSGEM. His procedure can easily be summarized in the following four steps:

Step 1: Obtain the necessary conditions characterizing the equilibrium. The FOC's are bellow.

Step 2: Choose the parameter values of the model and find steady state values. Parameter values are shown in Table 2. In this framework, although there is no steady state capital stock, there is a steady state ratio of physical to human capital. Hence, stationary time series are obtained by expressing growing variables in relation to the stock of human capital.

Step 3: Log-linearize the first-order conditions, which characterize the equilibrium of the model, in order to make all the equations approximately linear in the log-deviations from the steady state.

Step 4: Solve for the recursive equilibrium law of motion using the method of undetermined coefficients suggested by Uhlig (1995), this is simple and of general applicability (that is, it can be implemented in models where there are more endogenous state variables than expectational equations). Uhlig's method of undetermined coefficients is closely related to Blanchard and Khan's (1980) approach for solving linear difference equations.

Steps 1-3 involve many tedious, though simple, computations. After Step 3, it is convenient to rewrite the system of log-linearized first-order conditions in matrix form. Then, using Uhlig's notation, we have a matrix system where there is one vector of endogenous state variables x_t (size mx1), another vector containing other endogenous variables y_t (size nx1) and a third vector of exogenous stochastic variables z_t (size kx1):

$$Ax_t + Bx_{t-1} + Cy_t + Dz_t = 0, \tag{20}$$

$$E_t[Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t] = 0, \tag{21}$$

$$z_{t+1} = Nz_t + \epsilon_{t+1}, \tag{22}$$

where $E_t(\epsilon_{t+1}) = 0$. It is assumed that C is of size lxn, $l \geq n$ and of rank n, l is the number of deterministic equations (i.e., the number of equations involved in (20)), F is of size $(m+n-l)xm$, and N has only stable eigenvalues. In our model, we have three endogenous state variables $\hat{k}_t$, $\frac{h_{t+1}}{h_t}$ and u_t, where $u_t = n_{t-1}$[16], two exogenous variable z_t and θ_t and three other (non-state) endogenous variables: consumption-human capital ratio, $\hat{c}_t$, the fraction of time allocated to the leisure, l_t and output $\hat{y}_t$.

The log-linear procedure is looking for a recursive equilibrium law of motion of the following form:

$$x_t = Px_{t-1} + Qz_t, \tag{23}$$

$$y_t = Rx_{t-1} + Sz_t, \tag{24}$$

that is, finding P, Q, R and S so that the equilibrium described by these rules is stable. In our case, $l = n$, then (see Corollary 1 of Uhlig (1995)).

(i) P satisfies the following quadratic equation:

$$(F - JC^{-1}A)P^2 - (JC^{-1}B - G + KC^{-1}A)P - KC^{-1}B + H = 0. \tag{25}$$

Solving for P in this equation requires the use of Theorem 2 in Uhlig (1995, pp. 14-15).

(ii) R is given by

$$R = -C^{-1}(AP + B). \tag{26}$$

(iii) Q satisfies

$$vec(Q) = (N' \otimes (F - JC^{-1}A) + I_k \otimes (JR + FP + G - KC^{-1}A))^{-1}$$

$$vec((JC^{-1}D - L)N + KC^{-1}D - M), \tag{27}$$

where $vec(.)$ denotes columnwise vectorization.

(iv) S is given by

$$S = -C^{-1}(AQ + D). \tag{28}$$

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[16] $\frac{h_{t+1}}{h_t}$ is considered as a state variable to apply the log-linear approximation, although it is not a state variable in a proper sense.

In the model under study, the first-order conditions (FOC's) associated with the planner's problem are:

$$U_2(c_t, l_t) =$$

$$U_1(c_t, l_t)[\tfrac{1-\alpha}{n_t} - \eta \tfrac{\Delta(n_t h_t) h_t}{(n_{t-1} h_{t-1})^2}] y_t + \beta E_t\{U_1(c_{t+1}, l_{t+1}) \eta \tfrac{\Delta(n_{t+1} h_{t+1}) n_{t+1} h_{t+1} h_t}{(n_t h_t)^3} y_{t+1}\}, \quad [1]$$

$$U_1(c_t, l_t) = \beta E_t\{U_1(c_{t+1}, l_{t+1})[\frac{\alpha}{k_{t+1}} y_{t+1} + 1 - \delta_k]\}, \qquad [2]$$

$$\frac{U_2(c_t, l_t)}{A_h \theta_t h_t} = \beta E_t\{\frac{U_2(c_{t+1}, l_{t+1})}{A_h \theta_{t+1} h_{t+1}}[A_h \theta_{t+1}(1 - l_{t+1}) + 1 - \delta_h]\}, \qquad [3]$$

$$h_{t+1} = A_h \theta_t(1 - l_t - n_t) h_t + (1 - \delta_h) h_t, \qquad (17)$$

$$k_{t+1} + c_t = \frac{A_m Z_t k_t^\alpha (n_t h_t)^{1-\alpha}}{\exp\left\{\frac{\eta}{2}\left[\frac{\Delta(n_t h_t)}{n_{t-1} h_{t-1}}\right]^2\right\}} + (1 - \delta_k) k_t,$$

$$\lim_{t \to \infty} E_t \beta^t U_1 k_{t+1} = 0,$$

$$\lim_{t \to \infty} E_t \beta^t \frac{U_2}{A_h \theta_t h_t} h_{t+1} = 0,$$

where E_t is an operator whose expectations are conditional on the information available up to period t.

Equation [1] shows the optimal way of determining the fraction of time devoted to the production of goods. At the margin, the marginal utility reported by an additional labor unit has to be equal to its marginal disutility. On the one hand, labor adjustment costs reduce current marginal utility but, on the other hand, labor adjustment costs increase the expected utility via future output. Hence, due to the presence of labor adjustment costs, firms do not adjust labor input completely in the current quarter. Their optimal response is to defer a part to the subsequent quarter.

Equation [2] governs the accumulation of physical capital and establishes that on the margin, the expected return obtained by acquiring an additional unit of physical capital must equal the cost it causes in utility terms today.

Equation [3] governs the accumulation of human capital. Given that $1 - l_t$ denotes the fraction of time not allocated to leisure, this equation establishes that, at the margin, the expected return in current period utility obtained by acquiring an additional unit of human capital must equal the cost it causes.

In the steady state, the variables k_t, y_t and c_t grow at a constant rate (v) which is equal to the human capital accumulation growth rate, while n_t and l_t remain constant. Therefore, stationary time series are obtained by expressing growing variables in relation to the stock of human capital: $\hat{c}_t = \frac{c_t}{h_t}$, $\frac{h_{t+1}}{h_t}$ and $\hat{k}_t = \frac{k_t}{h_t}$. This also facilitates the use of the computational technique, as it reduces the number of state variables. Hence, the FOC's can be rewritten as:

$$U_1(\hat{c}_t, l_t) = \beta(\frac{h_{t+1}}{h_t})^{-\sigma} E_t\{U_1(\hat{c}_{t+1}, l_{t+1})[\frac{\alpha \hat{y}_{t+1}}{\hat{k}_{t+1}} + 1 - \delta_k]\}, \qquad [5]$$

$$U_2(\hat{c}_t, l_t) =$$

$$U_1(\hat{c}_t, l_t)\left[\frac{1-\alpha}{n_t} - \eta\frac{n_t - n_{t-1}\frac{h_{t-1}}{h_t}}{n_{t-1}\frac{h_{t-1}}{h_t}}\frac{h_t}{n_{t-1}h_{t-1}}\right]\hat{y}_t + \beta(\frac{h_{t+1}}{h_t})^{1-\sigma}E_t\{U_1(\hat{c}_{t+1}, l_{t+1})\eta\frac{n_{t+1} - n_t\frac{h_t}{h_{t+1}}}{n_t\frac{h_t}{h_{t+1}}}\frac{n_{t+1}h_{t+1}}{n_t h_t}\frac{\hat{y}_{t+1}}{n_t}\},$$

$$[6]$$

$$\frac{U_2(\hat{c}_t, l_t)}{A_h\theta_t} = \beta(\frac{h_{t+1}}{h_t})^{-\sigma}E_t\{\frac{U_2(\hat{c}_{t+1}, l_{t+1})}{A_h\theta_{t+1}}[A_h\theta_{t+1}(1 - l_{t+1}) + 1 - \delta_h]\}, \qquad [7]$$

$$\frac{h_{t+1}}{h_t} = A_h\theta_t(1 - l_t - n_t) + 1 - \delta_h, \qquad [8]$$

$$\hat{c}_t + \hat{k}_{t+1}\frac{h_{t+1}}{h_t} = \frac{A_m Z_t \hat{k}_t^\alpha n_t^{1-\alpha}}{\exp\left\{\frac{\eta}{2}\left[\frac{n_t - n_{t-1}\frac{h_{t-1}}{h_t}}{n_{t-1}\frac{h_{t-1}}{h_t}}\right]^2\right\}} + (1 - \delta_k)\hat{k}_t, \qquad [9]$$

where $\hat{c}_t = \frac{c_t}{h_t}$ and $\hat{k}_t = \frac{k_t}{h_t}$.

Appendix A

Table 1: Sensitivity analysis to changes in σ_θ and η

Costs \ Shock	Q_{acf} [a]		$Q_{irf}(Y_P)$ [b]		$Q_{irf}(Y_T)$ [c]	
	$\sigma_\theta = 0.004$	$\sigma_\theta = 0.002$	$\sigma_\theta = 0.004$	$\sigma_\theta = 0.002$	$\sigma_\theta = 0.004$	$\sigma_\theta = 0.002$
$\eta = 0.36$	9.45	10.55	18.88	36.05	8.61	18.88
	(0.305)	(0.228)	(0.084)	(0.019)	(0.318)	(0.094)
$\eta = \dfrac{0.36}{2}$	11.68	12.90	18.39	36.02	8.96	19.43
	(0.165)	(0.115)	(0.088)	(0.023)	(0.302)	(0.088)
$\eta = \dfrac{0.36}{3}$	13.56	14.81	18.24	36.31	9.82	20.57
	(0.093)	(0.063)	(0.09)	(0.023)	(0.268)	(0.075)

(a) Q statistic for the autocorrelation function of GNP growth.

(b) Q statistic for the permanent impulse-response function of GNP.

(c) Q statistic for the transitory impulse-response function of GNP.

Probability values in parenthesis.

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Table 2: Benchmark parameter and steady state values [a]

Parameter	Value[b]	Interpretation
α	0.36	Share of physical capital in the final good technology
δ_k	0.025	Depreciation rate of physical capital
δ_h	0.005	Depreciation rate of human capital
A_m	1	Scale parameter in the final good technology
A_h	0.0266666	Scale parameter in the human capital production function
λ	0.3769	Consumption weight in utility function
η	0.36	Size of labor adjustment costs
σ_Z	0.007	Standard deviation of ε_t
σ_θ	0.004	Standard deviation of ϵ_t
ρ_2	0.95	Persistence of θ_t
ρ_1	0.95	Persistence of Z_t
v	0.0036	Quarterly growth rate (based on average US data)
n	0.24	Hours worked (based on average US data)
r	0.01	Real interest rate (based on average US data)
β	$\beta = \frac{(1+v)^\sigma}{1+r}$	Subjective discount factor
γ	$(1-\gamma)\lambda = 1-\sigma$	Risk aversion parameter

[a] For parameters with a time dimension, the unit of time is a quarter of a year.

[b] Following RBC tradition, when changing σ we recalibrate other parameters such as β and γ.

Table 3: Sensitivity analysis to changes in σ and η

Costs \ RRA	Q_{acf}			$Q_{irf}(Y_P)$			$Q_{irf}(Y_T)$		
	$\sigma = 0.9$	$\sigma = 1$	$\sigma = 2$	$\sigma = 0.9$	$\sigma = 1$	$\sigma = 2$	$\sigma = 0.9$	$\sigma = 1$	$\sigma = 2$
$\eta = 0.36$	9.45	10.09	11.99	18.88	25.60	47.84	8.61	11.65	25.71
	(0.305)	(0.258)	(0.151)	(0.084)	(0.048)	(0.015)	(0.318)	(0.203)	(0.058)
$\eta = \dfrac{0.36}{2}$	11.68	12.46	14.63	18.39	25.26	48.82	8.96	12.11	27.38
	(0.165)	(0.131)	(0.066)	(0.088)	(0.05)	(0.012)	(0.302)	(0.198)	(0.051)
$\eta = \dfrac{0.36}{3}$	13.56	14.40	16.71	18.24	25.17	49.22	9.82	13.11	29.32
	(0.093)	(0.071)	(0.033)	(0.09)	(0.052)	(0.013)	(0.268)	(0.169)	(0.048)
$\eta = \dfrac{0.36}{4}$	15.06	15.95	18.33	18.19	25.16	49.49	10.55	13.95	30.81
	(0.057)	(0.043)	(0.018)	(0.09)	(0.050)	(0.012)	(0.236)	(0.157)	(0.045)
$\eta = 0$	28.58	28.66	32.45	18.47	25.69	51.36	15.94	19.91	40.13
	(0.0003)	(0.0003)	(0.00008)	(0.09)	(0.046)	(0.008)	(0.117)	(0.087)	(0.034)

(a) Q statistic for the autocorrelation function of GNP growth.

(b) Q statistic for the permanent impulse-response function of GNP.

(c) Q statistic for the transitory impulse-response function of GNP.

Probability values in parenthesis.

Figure 1: ACF for output growth

Figure 2: Transitory IRF function for $\ln(GNP)$ from the Blanchard-Quah technique
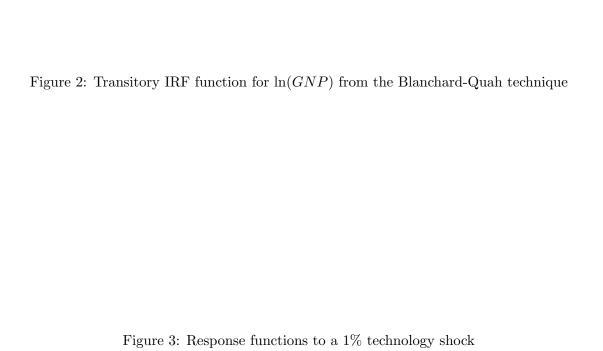
Figure 3: Response functions to a 1% technology shock

Figure 4: Response functions to a 1% human capital shock