Wage Bargaining and Partial Ownership

Juan Carlos Bárcena-Ruiz and María Luz Campo

Abstract

This work analyzes wage negotiation between firms and unions when cross-participation exists at ownership level. We consider two shareholders and two firms: one firm is jointly owned by the two shareholders and the other is owned by only one shareholder. Labor is unionized and firms produce substitute products. We show that partial ownership increases the bargaining strength of the firm owned by only one shareholder. Thus, in relation with the case in which each firm is owned by a single shareholder partial ownership reduces the wage paid by firms and the output of industry and therefore, the employment. It increases the utility of the union that negotiates with the firm owned by the two shareholders but reduces the utility of the other union and the aggregated utility of the two unions. Under partial ownership, the firm that is jointly owned by the two shareholders obtains greater profit. The other firm can increase or reduce its profit depending on the degree in which goods are substitutes.

Keywords: Partial Ownership; Wage Bargaining; Heterogeneous Goods
JEL Classification: L13, L21, J31

Departamento de Fundamentos del Análisis Económico I, Facultad de Ciencias Económicas y Empresariales, Universidad del País Vasco. Avenida Lehendakari Aguirre 83, 48015 Bilbao, Spain. Phone: 34 + 94 601 38 33; 34 + 94 601 38 33. Fax: 34 + 94 601 38 91. E-mails: juancarlos.barcena@ehu.es, mluz.campo@ehu.es.

1. Introduction

The factors that affect wage negotiations between firms and unions have been extensively studied by the literature on wage bargaining.[1] However, the theoretical analysis on this issue has not considered how partial ownership of firms influences wage bargaining between firms and unions. In this regard, the literature that analyzes partial ownership of firms usually assumes that production costs are exogenously given (see, for example, Malueg, 1992; Reitman, 1994). However, labor costs are by far the greatest component of costs in most corporations (see Bhagat *et al*., 1990). Therefore, it is important to assume explicitly a unionized labor since cross-ownership affects the bargaining position of firms and, thus, their profit.

The literature on wage bargaining has analyzed different factors that increase the bargaining strength of both firms and unions. From the unions side, it is well known that centralized bargaining (firms negotiate with independent unions at the firm level) results in higher wages than decentralized bargaining (each firm bargains with an industry-wide union) since the bargaining strength of the workers is greater. Under centralized bargaining, when the union bargains wages with one firm it has as disagreement payoff the rents obtained in the other firm, which increases the bargaining strength of the workers (see Horn and Wolinsky, 1988; Davidson, 1988). Dobson (1994) shows that when wage bargaining is centralized at the industry level, the union gains bargaining first with the firm that is in a relatively weak bargaining position or a firm with relatively large profits. He points out that, in some industries, strategic movements come from the union: the industry-wide union targets one firm with which it will negotiate first. The agreement is then used as the basis for future negotiations with other firms in the industry.

From the firms side, Bárcena-Ruiz and Garzón (2000) show that the decision of merging by firms together with the reorganization of production decisions, may allow

[1] See, for example, See Malcomson (1987), Farber (1986), Oswald (1985) and McDonald and Solow (1981).

employers to decrease union rents.[2] The decision of merging by firms, setting a multiproduct firm with two divisions, increases its bargaining strength since when the head of the multiproduct firm negotiates wages with the union of one division, its disagreement payoff is the profit of the other division when the first one does not produce.

In this paper we want to study another factor that influences on the bargaining strength of both firms and unions: the existence of cross-participation at ownership level.[3] We find this analysis quite relevant since in the literature that analyzes wage negotiations it is generally considered that each firm is owned by a different shareholder (see, for example, Dobson, 1994; Davidson, 1988). On the other hand, the literature that studies partial ownership usually assumes exogenous production costs.

The issue we analyze in this work can be illustrated by taking the automobile industry as an example. In this industry there are examples of partial ownership of rivals. One illustrative example is given by the French firm Renault, which created an alliance with the Japanese firm Nissan. Actually, Renault owns a 44.3% equity stake in Nissan Motor and Nissan Motor has a 15% stake in Renault (see www.renault.com). Besides, in the automobile industry in advanced countries firms negotiate wages with the workers' representatives.[4] We set our model in this context.

We consider in our paper that there are two firms that produce substitutes products. There are two shareholders, and one firm is jointly owned by the two

---

[2] It is usually argued that if a multiplant firm centralizes the wage negotiation its bargaining power increases, since when its head bargains the wage with the workers of one plant, it has as disagreement payoff the profits obtained in the other plants of the firm (see, Heywood and Peoples, 1994 or Mezzetti and Dinopoulos, 1991).

[3] An explanation of why partial ownership arrangements are formed can be seen in Alley (1997). One of the reasons is that it alters the degree of competition in the industry (see, for example, Reynolds and Snapp, 1986; Farrell and Shapiro, 1990; Malueg, 1992; Reitman, 1994).

[4] Bargaining structures in developed countries differ. In E.U. countries, in general, collective agreements are concluded between the relevant union and employers' association of an industry on a regional basis (see Layard *et al*., 1991; Addison and Siebert, 1993). In Japan, wages are negotiated simultaneously in the 'Spring offensive' and the basic structure of the Japanese labor union is mostly enterprise-based organization (see Sasajima, 1993).

shareholders while the other is owned by only one of them. The only factor of production is labor and all workers are unionized. There is an independent union in each firm. To determine the wage set in each firm, we consider the "right-to-manage" model of Nickell and Andrews (1983) where union and firms bargain over a uniform wage rate while the employment is set unilaterally by the firms.

We show that partial ownership reduces the wage paid by both firms, the output of industry and therefore, the employment. Under partial ownership one of the shareholders owns completely one firm and has part of the shares in the other firm. This shareholder has a positive disagreement payoff when bargaining with the union of the firm in which he is the single owner. And it is well known that the greater the disagreement payoff the stronger the bargaining position and the lower the wage paid by the firm. However, this shareholder takes into account how the wage paid by to the workers of the firm in which he is the single owner affects the profits of the other firm. This second effect weakens the bargaining position of the shareholder. We show that the first effect dominates the second effect and thus, compared with the case in which each firm is owned by a single shareholder, partial ownership increases the bargaining strength of the shareholder that has an investment in the two firms. Therefore, the firm owned only by one shareholder pays lower wages. And, as wages are strategic complements, if one firm pays a lower wage, the other firm pays a lower wage too. Besides, cross-ownership increases the utility of the union that bargains with the firm owned by the two shareholders but reduces the utility of the other union and the aggregated utility of the two unions. Under partial ownership, the firm that is jointly owned by the two shareholders obtains greater profit than if each firm is owned by a single shareholder. The other firm can obtain a greater or lower profit depending on the degree in which goods are substitutes.

The rest of the paper is organized as follows. Section 2 describes the general features of the model and Section 3 presents the results. Finally, Section 4 derives some conclusions.

2. The model

We consider a market consisting of two firms denoted by 1 and 2, that produce imperfect substitutes goods. Both firms have identical technology and face a linear demand:

$$p_i = a - q_i - b\, q_j,\ i?j;\ i, j=1, 2,$$

where p_i is the price of firm i and q_i, is the output level of firm i. We assume that the goods are substitutes so that $0=b=1$; parameter b measures the degree to which products are substitutes.

The only factor used in the production process is labor. Technology exhibits constant return to scale such that $q_i = L_i$. Each firm hires L_i workers with a uniform wage w_i. All workers are unionized and there is an independent union in each firm. The unions seek to maximize the wage bill and the utility function of the union of firm i (union i) is: $U_i(w_i, L_i) = w_i L_i$, $i=1, 2$. We consider a variant of the "right-to-manage" model of Nickell and Andrews (1983) where union and firms bargain over a uniform wage rate while the employment is set unilaterally by the firms.

There are two shareholders denoted by A and B. Firm 1 is owned by shareholder A while firm 2 is jointly owned by the two shareholders, though shareholder B owns the majority of shares in firm 2. We denote by a, $a<1/2$, the percentage of shares that shareholder A owns in firm 2. Therefore, the objective function of shareholders A and B, respectively, are: $p_A = p_1 + a\, p_2$ and $p_B = (1 - a)\, p_2$. The profit of firm i is given by $p_i = (p_i - w_i)\, q_i$, $i = 1, 2$, where w_i is the wage paid to the workers of this firm.

The timing of the game is the following. In the first stage, unions and firms negotiate the wage simultaneously and in the second stage firms' owners take output decisions. We solve the game by backward induction to get a subgame perfect Nash equilibrium.

3. Results

In the second stage, shareholders simultaneously choose the output level that maximize their objective functions. Solving, we obtain the Cournot-Nash equilibrium output (and therefore, employment) levels and firms' and shareholders' profits, as a function of wage rates:

$$q_1 = \frac{2(a-w_1)-b(1+a)(a-w_2)}{4-b^2(1+a)},\ q_2 = \frac{2(a-w_2)-b(a-w_1)}{4-b^2(1+a)}, \tag{1}$$

Substituting (1) in the profit of the firms, we obtain:

$$p_1 = \frac{(2(a-w_1)-b(1+a)(a-w_2))((a-w_1)(2-b^2a)-b(1-a)(a-w_2))}{(4-b^2(1+a))^2},$$

$$p_2 = \frac{(2(a-w_2)-b(a-w_1))^2}{(4-b^2(1+a))^2}. \tag{2}$$

In the first stage, unions bargain simultaneously the wage with firms' owners. The disagreement payoff of shareholder *A*, when bargaining the wage with union 1, is positive since this shareholder owns a positive percentage of the shares in firm 2 If the union 1 goes on strike shareholder *A* gets a percentage of firm 2's profits since union 2 does not go on strike. The disagreement payoff of shareholder *A* is the part that obtains in firm 2's profits operating this firm as a monopolist, denoted as D_A. It is easy to see that: $D_A = \frac{a(a-w_2)^2}{4}$. The disagreement payoff of each union is zero since we consider independent unions at firm level. The solution to the bargaining problem between shareholder *A* and union 1 is then given by:

$$w_1(w_2) = \underset{w_1}{arg\ max}\ [p_1 + a\,p_2 - D_A]\,[w_1\,q_1], \tag{3}$$

where p_1, p_2 and q_1 are given by (1) and (2), respectively. The disagreement payoff of shareholder B is zero since only owns shares in firm 2. The bargaining problem for shareholder B is:

$$w_2(w_1) = \underset{w_2}{arg\ max}\ [(1-a)\,p_2]\,[w_2\,q_2], \tag{4}$$

where p_2 and q_2 are given by (1) and (2), respectively. Solving (3) and (4) we get the reaction function in wages:

$$w_1 = \frac{a(2-b(1+a))+b(1+a)w_2}{8},\ w_2 = \frac{a(2-b)+bw_1}{8}. \tag{5}$$

As usual, wages are strategic complements, which means that if one firm pays a greater (lower) wage the other firm reacts by paying a greater (lower) wage too. From (5) we obtain the following result.

Lemma 1. *When there is cross-participation at ownership level, the wage paid to the workers, the output and employment levels of the firms, the profit of the firms and the utility of the unions are:*

$$w_1 = \frac{a(16-b(6+b)(1+a))}{64-b^2(1+a)},\ w_2 = \frac{a(16-6b-b^2(1+a))}{64-b^2(1+a)},$$

$$q_1 = L_1 = \frac{6a(16-b(6+b)(1+a))}{(64-b^2(1+a))(4-b^2(1+a))},\ q_2 = L_2 = \frac{6a(16-6b-b^2(1+a))}{(64-b^2(1+a))(4-b^2(1+a))},$$

$$p_1 = \frac{36a^2(16-b(6+b)(1+a))(16+2b(5a-3)-b^2(1+7a)-b^3a(1+a))}{(64-b^2(1+a))^2(4-b^2(1+a))^2},$$

$$p_2 = \frac{36a^2(16-6b-b^2(1+a))^2}{(64-b^2(1+a))^2(4-b^2(1+a))^2},$$

$$U_1 = \frac{6a^2(16-b(6+b)(1+a))^2}{(64-b^2(1+a))^2(4-b^2(1+a))},\ U_2 = \frac{6a^2(16-6b-b^2(1+a))^2}{(64-b^2(1+a))^2(4-b^2(1+a))}.$$

We consider as a benchmark the case in which there is no partial ownership. This happens when parameter a is equal to zero. In that case, there is not cross-

ownership and each firm is owned by a single shareholder. The results obtained in this case are that of Lemma 1 substituting a by zero. We shall denote the results obtained in this case without subscripts.

Lemma 2. *When each firm is owned by an only shareholder, the wage paid to the workers of each firm, the output and employment levels of each firm, the profit of each firm and the utility of each union is:*

$$w = \frac{(2-b)a}{8-b},\ q = L = \frac{6a}{(2+b)(8-b)},\ p = \frac{36a^2}{(2+b)^2(8-b)^2},\ U = \frac{6(2-b)a^2}{(2+b)(8-b)^2}.$$

From Lemmas 1 and 2 it is easy to obtain the following result:

Proposition 1. *In equilibrium:*

i) $w > w_2 > w_1$;

ii) $q_2 = L_2 > q = L > q_1 = L_1$, $2L = 2q > q_1 + q_2 = L_1 + L_2$;

iii) $U_2 > U > U_1$, $2U > U_1 + U_2$.

Proposition 1 shows that $w > w_2 > w_1$. There are two effects that explain this result.[5] First, only shareholder A has a positive disagreement payoff when negotiate the wage with his union (disagreement payoff effect). Thus, when negotiate the wage with its union the bargaining position of shareholder A is stronger than that of shareholder B. And it is well known that the greater the disagreement payoff the stronger the bargaining position and the lower wage paid by the firm. The second effect is due to the fact that shareholder A takes into account how the wage paid by firm 1 affects the profits of firm 2 (bargaining effect). This effect weakens the bargaining position of shareholder A. This is due to the fact that the greater the wage paid by firm 1, the lower the market share of this firm and thus the greater the market share and profits of firm 2.

The objective function of shareholder A, taking into account the disagreement payoff, when bargain the wage of firm 1 is: $p_1 + a p_2 - D_A$. This can be rewritten as: $p_1 - (D_A - a p_2)$. Therefore, if we interpret $(D_A - a p_2)$ as the disagreement payoff of shareholder

[5] As we are considering independent unions, these effects come only from the firms' side not from the unions' side.

A when bargain the wage with union 1, it is easy to see which of the two effects dominate. As we have seen, D_A is the profit of firm 2 when this firm is a monopolist in the product market while p_2 is the profit of firm 2 acting as a duopolist. Thus, D_A is greater than p_2, and therefore, $(D_A - a p_2)$ is positive. As a result, the first effect dominates the second effect and firm 1 pays a lower wage than firm 2 ($w_2 > w_1$).

It is easy to see from Lemma 1 that the higher the value of parameter a the lower the wages paid by firms ($\frac{\partial w_2}{\partial a} = -\frac{6ab^2(8+b)}{(64-b^2(1+a))^2} < 0$, $\frac{\partial w_1}{\partial a} = -\frac{48ab(b+8)}{(64-b^2(1+a))^2} < 0$). Besides, the difference between the wages of the two firms increase with a ($\frac{\partial(w_2 - w_1)}{\partial a} = \frac{6ab(64-b^2)}{(64-b^2(1+a))^2} > 0$). The greater the percentage of the shares that shareholder *A* owns in firm 2, the lower the wages paid by the two firms and the greater the difference between the wages paid by the two firms.

When each firm is owned by a single shareholder neither of the two effects exists. Given that wages are strategic complements and that the disagreement payoff effect dominates the bargaining effect, under partial ownership both firms pay a lower wage than when each firm is owned by a single shareholder ($w > w_2 > w_1$)

Given that $w > w_2 > w_1$ we obtain that $q_2 = L_2 > q = L > q_1 = L_1$. This result is due to the fact that under partial ownership firm 1 pays a lower wage than firm 2; thus, the first firm obtains greater market share and hires more workers at the expense of the other firm. When there is no cross-ownership the output and employment level of the firms is between that obtained under partial ownership (and this although the wage pays by firms is the greater). We also obtain that the output of firm 1 decreases with a while the output of firm 2 increases with a.[6] Therefore, the higher the value of parameter a the

---

[6] $\frac{\partial q_1}{\partial a} = -\frac{6ab(1536 - 832b + 32b^3(1+a) - (6b^4 + b^5)(1+a)^2)}{(256 - 68b^2(1+a) + b^4(1+a)^2)^2} < 0;$

$\frac{\partial q_2}{\partial a} = \frac{6ab^2(832 - 408b - (32b^2 - 12b^3)(1+a) + b^4(1+a)^2)}{(256 - 68b^2(1+a) + b^4(1+a)^2)^2} > 0.$

higher the difference of the outputs will be. Besides, cross-ownership decreases the output of the industry and therefore, the employment: $2L=2q>L_1+L_2= q_1+q_2$.[7]

Regarding the utility obtained by unions, we get that $U_2>U>U_1$. Therefore, partial ownership increases the utility of the union that belongs to the firm owned by the two shareholders but reduces the utility of the union of the other firm. The firm owned by only one shareholder pays the lower wage ($w>w_2>w_1$) and hires the lower employment ($L_2>L>L_1$) and thus the utility obtained by its union is the lower possible. The rival firm pays an intermediate wage and hires the greater employment which implies that its union obtains the greater utility. Besides, it can be shown that $2U>U_1+U_2$; therefore, cross-ownership reduces the aggregated utility obtained by the two unions. This is due to the fact that under cross-ownership both firms pay lower wages and total employment is lower.

Let b^* the value of parameter b such that $p_1=p_2$, and b^{**} the value of parameter b such that $p_1=p$. It has to be noted that both b^* and b^{**} depend of parameter a (see Appendix). By comparing the profits of the firms with and without cross-ownership we obtain the following result that is illustrated in Figure 1.

Proposition 2. *In equilibrium:*

i) $p_2q_2>pq>p_1q_1$, $p_1+p_2>2p$,

ii) $p_1 \geq p_2>p$ *if and only if* $b\leq b^*$,

iii) $p_2>p_1 \geq p$ *if and only if* $b^*<b\leq b^{**}$,

iv) $p_2>p>p_1$ *if and only if* $b^{**}<b$.

Proof. See Appendix

[INSERT FIGURE 1 AROUND HERE]

[7] The fact that cross-ownership reduces market competition and thus the output of industry is a well known result (see, for example, Malueg, 1992).

We have seen in proposition 1 that $q_2>q>q_1$, which implies that $p_2q_2>pq>p_1q_1$. It is shown in the appendix that $p_1>p_2>p$; therefore, the preceding result depends mainly on the market share obtained by firms.

It is easy to see that if there is no wage bargaining and thus production costs are exogenous: $p_2>p>p_1$ and $p_2+p_1>2p$. In this case, firm 2 obtains greater profit at the expense of firm 1. Shareholder *A* tries to reduce market competition by reducing the output level of firm 1 because he takes into account how the output level of firm 1 affects the profit of firm 2. As a result the profit of industry is greater under cross-ownership. Proposition 2 shows that when production costs are endogenously determined the profit of industry is greater under cross-ownership; in that case, firm 1 can obtain greater profit than firm 2. This last result depends on the degree in which goods are substitutes.

To explain the result shown in this proposition it has to be noted that the utility obtained by unions are the production costs of the firms. And we have seen that firm 2 obtains the greater incomes and has the greater production costs and that firm 1 obtains the lower incomes and has the lower production costs ($p_2q_2>pq>p_1q_1$ and $U_2>U>U_1$).

Proposition 2 shows that firm 2 obtains greater profit under partial ownership ($p_2>p$ for all b). This result, as when production costs are exogenously given, is due to the reduction in the competition in the product market caused by partial ownership of firms. As we have seen, this benefits firm 2, the firm that is jointly owned by the two shareholders. In the reminder cases, the result obtained can be different than when production costs are exogenous.

When parameter b is great enough ($b>b^{**}$), it is obtained that firm 1 gets lower profits under cross-ownership ($p>p_1$). In this case, as parameter b is great enough competition in the product market is strong and the result is due to the market share (and thus to the income) obtained by firms: the firm that obtains greater market share obtains greater profits, and firm 1 obtains greater market share when there is no partial ownership. However, when $b<b^{**}$ the lower production costs of firm 1 under partial ownership make this firm to obtain greater profits ($p_1> p$).

Finally, when the degree to which goods are substitutes is low enough ($b \leq b^*$), it is obtained that under cross-ownership firm 1 obtains greater profits than firm 2 ($p_1 \geq p_2$). In this case, as parameter b is low enough competition in the product market is weak and firm 1 obtains greater profit than firm 2 due to its lower production costs. When $b > b^*$, firm 2 obtains greater profits ($p_2 > p_1$) due to its greater market share.

4. Conclusions

The literature on wage bargaining between firms and unions has studied different factors that affect the bargaining strength of both negotiators, but it has not considered how partial ownership of firms affects wage negotiation. In this paper, we have analyzed wage negotiations when one firm is jointly owned by the two shareholders and the other is owned by only one shareholder. We have compared this case with that in which each firm is owned by a single shareholder.

The results obtained show that partial ownership reduces the wage paid by both firms, the output of industry and therefore, the employment. Besides, it increases the utility of the union that bargains with the firm owned by the two shareholders but reduces the utility of the other union due to this firm owned by only one shareholder pays the lower wage and hires the lower employment. We show that cross-ownership affects the bargaining position of firms and, thus, their profit. The firm that is jointly owned by the two shareholders obtains greater profit than when each firm is owned by a single shareholder. The profit of the other firm depends on the degree in which goods are substitutes.

One possible extension of this paper would be to consider industry wide unions. We left this issue for future work.

Acknowledgements

Financial support from Ministerio de Ciencia y Tecnología and FEDER (SEJ2006-05596) and Departamento de Educación, Universidades e Investigación del Gobierno Vasco (IT-223-07) is gratefully acknowledged.

Appendix

Proof of Proposition 2.

We prove firs that $p_1>p_2>p$.

i) $p_1 - p_2 = \dfrac{36a(1-b)ba}{256-68b^2(1+a)+b^4(1+a)^2} > 0$, since $b\in[0,1]$ and α <1/2.

ii) $p_2 - p = \dfrac{6ab^2a\,(44-10b+b^2(1+a)+b^3(1+a))}{(16+6b-b^2)(256-68b^2(1+a)+b^4(1+a)^2)} > 0$, since $b\in[0,1]$ and α<1/2.

Therefore, $p_1>p_2>p$.

Next we prove that $p_1 \geq p_2$ if $b=b^*$. It can be proved that:

$$p_1 - p_2 = \frac{36a^2ba\,(64+16b^2(1+a)+b^4(1+a)^2-60b(2+a)+6b^3(2+3a+a^2))}{(256-68b^2(1+a)+b^4(1+a)^2)^2}.$$

The denominator of this expression is positive. We have to analyze the sign of the numerator. Let denote D_1 as:

$$D_1 = (64+16b^2(1+a)+b^4(1+a)^2-60b(2+a)+6b^3(2+3a+a^2)).$$

Then:

$$\frac{\partial D_1}{\partial b} = 32b(1+a)+4b^3(1+a)^2-60(2+a)+18b^2(2+3a+a^2),$$

$$\frac{\partial^2 D_1}{\partial b^2} = 4(8(1+a)+3b^2(1+a)^2+9b(2+3a+a^2)) > 0,$$

Therefore, D_1 is strictly concave in b. On the other hand, if $b=0$ we obtain that D_1=64>0, while that if b=1 we obtain that $D_1 = -27-24a+7a^2 < 0$. Then, there

exists a value of parameter b, denoted by b^* ($0<b^*<1$), such that if $b<b^*$ we obtain that $D_1>0$, while that if $b>b^*$ we obtain that $D_1<0$. As a result, $p_1 \geq p_2$ if $b=b^*$.

It is easy to see that if $a=0$ we get that $p_1 = p_2$ for $b^*=0.6055$, while that if $\alpha=1/2$ we get that $p_1 = p_2$ for $b^*=0.4812$. As a result, $0.4812<b^*<0.6055$.

Next, we prove that $p_2 \geq p$, $\forall\ b$:

$$p_2 - p = (36a^2b^2a(52-6b-b^2-b^2a)(512-136b^2+2b^4- 84b^2a-6b^3a+3b^4a+b^4a^2))/((8-b)^2(2+b)^2(64-b^2-b^2a)^2(4-b^2-b^2a)^2).$$

The denominator is positive and the numerator is positive since $(52-6b-b^2-b^2a)>0$ and $(512-136b^2+2b^4-84b^2a-6b^3a+3b^4a+b^4a^2)>0$, $\forall\ b \in [0, 1]$ and $a \in (0, 1/2)$.

Next, we prove that $p_1 \geq p$ if $b=b^{**}$. It is easy to check that:

$$p_1 - p = (36a^2ba(16384+8192b-21760b^2-2688b^3+4944b^4+168b^5- 136b^6-2b^7+b^8-15360ba-7424b^2a+2560b^3a+4944b^4a+616b^5a- 204b^6a-11b^7a+2b^8a+1536b^3a^2+1408b^4a^2+352b^5a^2-68b^6a^2-10b^7a^2+ b^8a^2-b^7a^3))/((8-b)^2(2+b)^2(64-b^2-b^2a)^2(4-b^2-b^2a)^2).$$

The denominator of this expression is positive. We have to analyze the sign of the numerator. Let denote D_2 as:

$$D_2 = (16384+8192b-21760b^2-2688b^3+4944b^4+168b^5-136b^6-2b^7+b^8- 15360ba-7424b^2a+2560b^3a+4944b^4a+616b^5a-204b^6a-11b^7a+2b^8a+ 1536b^3a^2+1408b^4a^2+352b^5a^2-68b^6a^2-10b^7a^2+b^8a^2-b^7a^3).$$

This expression can be rewritten as:

$$D_2 = (16384 + 8192b - 21760b^2 - 2688b^3 + 4944b^4 + 168b^5 - 136b^6 - 2b^7 + b^8) -$$
$$b(15360 + 7424b - 2560b^2 - 4944b^3 - 616b^4 + 204b^5 + 11b^6 - 2b^7)a +$$
$$b^3(1536 + 1408b + 352b^2 - 68b^3 - 10b^4 + b^5)a^2 - b^7a^3.$$

The first and the third terms are positive while the second and the fourth terms are negative. It can be proved that if b=0.9 the first term is higher than the second one, while the third term is higher than the fourth one. It remains to see what happen if b>0.9. In order to analyze this, we check whether D_2 increases or decreases with b:

$$\frac{\partial D_2}{\partial b} = 2(4096 - 21760b - 4032b^2 + 9888b^3 + 420b^4 - 408b^5 - 7b^6 + 4b^7) -$$
$$(15360 + 14848b - 7680b^2 - 19776b^3 - 3080b^4 + 1224b^5 + 77b^6 - 16b^7)a +$$
$$2b^2(2304 + 2816b + 880b^2 - 204b^3 - 35b^4 + 4b^5)a^2 - 7b^6a^3.$$

If b>0.9, both the first as second terms are negative. The third term is positive and the fourth term is negative. It easy to see that the first term is higher than the second if b>0.9. As a result, D_2 decreases with b. When b=0.9 we obtain that:

$$D_2 = \frac{744180661741 - 1447651405548a + 221089479021a^2 - 47829690a^3}{10^8} > 0.$$

If b=1, we obtain that: $D_2 = 5103 - 14877a + 3219a^2 - a^3$, and this expression can be positive or negative; its sign depends on the value of parameter a. It can prove that this expression is positive if and only if a <0.3731. Therefore, there exists a value of the parameter b denoted by b^{**} ($0.9<b^{**}<1$), such that if $b<b^{**}$ it is obtained that $D_2>0$, while if $b>b^{**}$ it is obtained that $D_2<0$. Therefore, $p_2 \geq p$ if $b=b^{**}$.

It is easy to see that if a =1/2, then D_2=0 for b=0.9316; as a result, $0<b^{**}<1$. As b^* is such that $0.4812<b^*<0.6055$, then $b^*<b^{**}$.

It remains to compare $p_1+p_2>2p$. We get the following result:

$$p_1+p_2-2p = 36a^2ba((16384+34816b-24832b^2-10272b^3+5760b^4+$$
$$408b^5-148b^6-4b^7+b^8)-2b(7680+3712b+1160b^2-2568b^3-514b^4+$$
$$108b^5+8b^6-b^7)a+b^3(1536+1408b+488b^2-68b^3-14b^4+$$
$$b^5)a^2-2b^7a^3)/((16+6b-b^2)^2(256-68b^2(1+a)+b^4(1+a)^2)^2).$$

The denominator of this expression is positive. To obtain the sign of the numerator, it can be proved that both the first as third terms are positive while both the second as fourth terms are negative. Besides, the first term is higher than the second one and the third term is higher than the fourth one. Therefore, $p_1+p_2-2p>0$.

References

Addison, J, and W. S. Siebert (1993): "The U. K.: Labour Market Institutions, Law and Performance", en *Labour Markets Contracts and Institutions*, editado por J. Hartog y J. Theeuwes, North Holland.

Alley, W. (1997): "Partial Ownership Arrangements and Collusion in the Automobile Industry." *Journal of Industrial Economics* 45: 191-205.

Bhagat, S.; Shleifer A.; Vishny R. (1990): "Hostile Takeovers in the 1980s: the Return to Corporate Specialization". *Brooking Papers on Economic Activity*, Microeconomics: 1-84.

Bárcena Ruiz, J. C., and M. B. Garzón (2000): "Corporate Merger and Control of Labor", *Spanish Economic Review*, 2, 129-144.

Davidson, C. (1988): "Multiunit Bargaining in Oligopolistic Industries", *Journal of Labor Economics*, 6, 397-422.

Dobson, P. (1994): "Multiform unions and the Incentive to Adopt Pattern Bargaining in Oligopoly", *European Economic Review*, 38, 87-100.

Farber, H. (1996): "The Analysis of Union Behavior". In *Handbook of Labor Economics*, Vol. II, 1039-1089. Edited by O. Ashenfelter and R. Layard, Elsevier Science Publishers.

Farrell, J., and Shapiro, C. (1990): "Asset Ownership and Market Structure in Oligopoly". *RAND Journal of Economics* 21: 275-292.

Heywood, J. S. and Peoples, J. H. (1994): "Unions and the Pattern of Corporate Mergers: U.S. Evidence", *Labour Economics*, vol. 1, pp. 203-221.

Horn, H., and A. Wolinsky, (1988): "Bilateral Monopolies and Incentives for Merger", *Rand Journal of Economics*, 19, 408-419.

Layard, R., Nickell, S. and Jackman, R. (1991): *Unemployment macroeconomic performance and the labour market*, Oxford University Press.

Malcomson, J. (1987): "Trade Union Labour Contracts. An Introduction", *European Economic Review*, 31, 139-148.

Malueg, D. (1992): "Collusive Behaviour and Partial Ownership of Rivals". *International Journal of Industrial Organization* 10 (1): 27-34.

Mezzetti, C. and Dinopoulos, E. (1991): "Domestic Unionization and Import Competition", *Journal of International Economics*, vol. 31, pp. 79-100.

McDonald, I., Solow, R. (1981): Wage bargaining and employment. *American Economic Review* 71, 896-908.

Nickell, S. J., and M. Andrews (1983): "Unions, Real Wages and Employment in Britain 1951-1979", *Oxford Economic Papers,* 35 (supplement), 183-206.

Oswald, A.J. (1985): "The Economic Theory of Trade Unions: an Introductory Survey", *Scandinavian Journal of Economics*, 87, 160-183.

Reitman, D. (1994): "Partial Ownership Arrangements and the Potential for Collusion." *International Journal of Industrial Organization* 4: 313-322.

Reynolds, R.J., and Snapp, B.R. (1986): "The Competitive Effects of Partial Equity Interests and Joint Ventures." *International Journal of Industrial Organization* 4: 141-153.

Sasajima, Y. (1993): "The Japanese Labour Market: its Institutions and Performance", en *Labour Markets Contracts and Institutions*, editado por J. Hartog and J. Theeuwes, North Holland.

Figure 1. Illustration of proposition 2.